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THE
ASTROPHYSICAL JOURNAL
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THE CONSTRUCTION OF A SENSITIVE GALVANOMETER FOR SPECTRO-BOLOMETRIC PURPOSES.

By C. G. Abbot.[1]

Since 1896 the galvanometers used for bolographic purposes at the Smithsonian Astrophysical Observatory have been more or less improved every year at the hands of the Observatory staff, with the encouragement and approval of the director, Secretary Langley, who has so kindly introduced this paper, which is a description of the results so far reached. For bolographic purposes reflecting galvanometers of low resistance and comparatively short time of swing are best suited. Solar-energy spectra are now being taken at the Smithsonian Observatory with a galvanometer of 1.6 ohms resistance, usually employed at 1½ seconds time of single swing of the needle; though for certain very delicate experiments the time of single swing is sometimes as high as 10 seconds. It may then be said that it is aimed to measure the least possible current with a galvanometer of 1.6 ohms resistance at 10 seconds single swing of the needle. It is well to make

[1] A paper read before Section B of the American Association for the Advancement of Science, December 30, 1902, after introductory remarks by S. P. Langley, Secretary of the Smithsonian Institution. It is here published with his permission.

here a distinction between computed sensibility and working sensitiveness, for it may happen that an instrument of less computed sensitiveness is capable of measuring smaller currents than another whose high figure of sensibility is overbalanced by unsteadiness of its needle system. This consideration leads me to suggest that possibly it might be fairer to compare galvanometers on a basis of the least current which can be read with them, or at least to give this in addition to the usual statement of computed sensibility.

In common with other laboratories, the Smithsonian Observatory has greatly profited by the introduction of Professor Boys' quartz fibers and by diminishing the size of the needle system and mirror of the galvanometer. These improvements were introduced about 1893 in the galvanometer described by Mr. Wadsworth[1] and used for bolographic purposes up to the autumn of 1896.[2]

In September 1896 the Observatory was visited by Professor Kayser, of Bonn, who expressed the belief that a galvanometer of considerably greater sensitiveness might be employed. He referred especially to the excellent instrument of Paschen, and to a galvanometer which he had seen at Baltimore, of somewhat similar design to Paschen's. Acting upon this suggestion, Mr. Langley addressed a letter of inquiry to Professor Paschen, and he was so good as to send in reply quite an account of his instrument. I also went to Baltimore and saw the galvanometer being set up by Messrs. Mendenhall and Saunders. These instruments seemed to gain in sensitiveness chiefly by diminishing the dimensions of the needle system, and as an aid to this the dimensions of the coils were also small.

Beginning with the autumn of 1896 experimental and theoretical work has gone on in the Smithsonian Observatory at frequent intervals, with the object of improvement in all the

[1] *Phil. Mag.*, **38**, 553–558, 1894.

[2] As last employed with a needle system of 31 milligrams weight, constructed by the writer, this instrument of about 20 ohms resistance gave a deflection of 1 millimeter on a scale at 1 meter with a current of 6×10^{-10} ampères when the time of single swing was 4 seconds. At this time of swing the needle was ideally steady, so that deflections of 0.1 millimeter could readily be observed.

branches of galvanometer construction, including the coils, the needle system, the case, and the accessories. In what follows the principal results of this study will be stated.

THE COILS.

Best resistance.—The bolometric circuit with which the galvanometer is connected, consists, as is well known, of a balanced Wheatstone's bridge. In our practice one of the four resistances of this bridge is a fine flattened platinum wire subjected to changes of temperature through the absorption of radiation. For the sake of symmetry a second one of the four resistances is a similar strip of platinum close to the first, but protected from radiation by means of diaphragms. The other two resistances are coils of platinoid wire. The battery is connected between the junction of the two coils and the junction of the two strips, while the galvanometer is connected between the junction of one strip with one coil and the other corresponding point. It is shown in the *Annals of the Astrophysical Observatory of the Smithsonian Institution*, Vol. I, p. 246, that the condition of maximum galvanometer deflection requires the resistance of the two coils to be large in comparison with the strips.

Let a and $\left(a + \frac{a}{m}\right)$ be the resistances of the two bolometer strips; (m being a large number); na the resistance of each balancing coil; G the resistance of the galvanometer; C the current in the bolometer strip whose resistance is a; g the current flowing through the galvanometer; Δ the deflection of the galvanometer, and k a constant.

Then

$$g = \frac{C}{m} \frac{1}{2 + \frac{G}{a}\left(\frac{1+n}{n}\right)}. \qquad (1)$$

When, as with the coils shortly to be described,

$$\Delta = kgG^{.45}, \qquad (2)$$

we have by substitution

$$\Delta = k\frac{C}{m} \frac{G^{0.45}}{2 + \frac{G}{a}\left(\frac{1+n}{n}\right)}. \qquad (3)$$

In this expression Δ is a maximum with respect to n when n is infinitely great, and with respect to G when

$$\left(\frac{1+n}{n}\right)\frac{G}{a}=1.64\ . \tag{4}$$

If n is large, this becomes approximately

$$\frac{G}{a}=1.64\ . \tag{5}$$

The two following tables indicate how much is lost by departure from these conditions of maximum deflection:

n	¼	1	2	3	4	∞
Δ	44	71	82	87	89	100

$\left(\frac{1+n}{n}\right)\frac{G}{a}$	0.0164	0.164	0.82	1.64	3.28	16.4	164
Δ	23	60	95	100	94	56	18

It appears that the conditions of maximum sensitiveness of the bolometric circuit as used here are closely approximated when the balancing coils are upward of four times the resistance of the bolometer strips, and the galvanometer resistance is not less than six-tenths, or more than four times the resistance of the bolometer strip.

At the Smithsonian Observatory best results have been obtained with bolometers of comparatively low resistance, for, unless in an air-tight case, very thin strips are much affected by air-currents, so that a perpetual tremor of the galvanometer is observed when there is moderate wind. At present a bolometer with strips of 4 ohms resistance is chiefly employed, but occasionally a wide bolometer of only 0.8 ohms resistance is substituted. Accordingly a galvanometer of from 2 to 3 ohms resistance would be most serviceable, but the actual resistance of the galvanometers now in use is 1.6 ohms.

A four-coil galvanometer of a given total resistance G, may have its separate coils of resistance $4G$, G, or $\frac{G}{4}$, according to

the manner in which they are connected; and for coils whose force at the center varies with the 0.45 power of the resistance, the relative efficiency of these three ways of producing the total resistance G is in the ratio of the numbers 87, 93, and 100. Thus it appears that the resistance of a four-coil galvanometer may be varied in three steps from 1 to 16 without greatly altering its efficiency. Again a galvanometer of a given resistance may be made by connecting in series two, four, eight, sixteen, or even more such coils. The efficiency of these several arrangements of the given resistance, so far as the force at the center of the coils is concerned, is in the ratio of the numbers 100, 68, 49, 32. There is a slight compensation for this diminution of efficiency as the coils get smaller, arising from the greater effect of the outer windings of each coil upon the magnets at the center of its neighbor, but the only considerations which warrant a multiplication of coils are some which concern the efficiency of the magnet system.

Best form of coils.— Maxwell[1] has shown that the contour of the cross-section of the coil should be of the form determined by the equation

$$r^2 = H^2 \sin\theta \,, \tag{6}$$

where r is the length of a radius making the angle θ with the axis of the coil, and H the value of r when $\theta = 90°$. Although the efficiency of the winding is not very greatly impaired by considerable variations from the best form of coil section, it is so easy to follow it closely that this form has always been employed here. In order to leave room for the needle system, a space is left unwound at the center and front of each coil as indicated by dotted lines in the accompanying diagram (Fig. 1).

Best sizes of wire.— The reader is referred to *Annals of the Astrophysical Observatory of the Smithsonian Institution*, Vol. I, p. 248, for the derivation of equations required to determine the radius of a coil of the above form containing a given length of wire of a given diameter and gain of diameter by insulation, and the force such a coil exerts at its center. Maxwell has shown[2]

[1] MAXWELL, *Electricity and Magnetism*, Vol. II, paragraph 718.

[2] *Ibid.*, paragraph 719.

that the wire should increase in diameter from the center outward, and it has been customary therefore to wind the coils in several sections, with larger sizes toward the outside of the coil. Although there would be a slight gain by using a larger number, it has been the practice here to wind the coils in three sections as shown in the accompanying diagram (Fig. 1), which is drawn to scale to illustrate the best dimensions for a three-section coil of 5 ohms resistance. The following table is for the most part abridged from Table 32 of the Astrophysical Observatory *Annals*, Vol. I, and shows the diameters of wire, gain of diameter by single white silk insulation,[1] lengths of wire, external radii of sections and forces exerted at the center by the most efficient coils of various given resistances, when wound of wire of a single size or in three sections of wire of different diameters. The coils of maximum efficiency have been selected from a large number whose constants have been computed.

Total Resistance	Diameter of Wire in Sections			Gain of Diameter by Insulation	Length of Wire in Sections			External Radii of Sections			Forces at Center of Coil			Total Force at Center of Coil
	d_1	d_2	d_3	t	L_1	L_2	L_3	H_1	H_2	H_3	F_1	F_2	F_3	F
ohms	cm	cm	cm	cm	cm	cm	cm	cm	cm	cm				
0.1	0.0405	......	.....	0.0038	79.1	.....	....	0.55	....	....	....	....	....	104
"	0.0202	0.0405	0.0511	"	6.6	26.4	41.6	0.36	0.42	0.60	54	45	42	141
5	0.0101	0.0202	0.0405	"	81	328	1318	0.32	0.60	1.30	298	321	301	920
"	0.0101	0.0255	0.0644	"	81	521	3332	0.32	0.75	2.26	298	359	290	947
25	0.0160	.....	......	"	3092	.....	.....	1.00	....	....	....	....	....	1467
"	0.0080	0.0160	0.0321	"	256	1031	4144	0.38	0.74	1.63	671	670	605	1946
"	0.0101	0.0202	0.0405	"	407	1639	6589	0.46	0.95	2.16	754	653	564	1971
50	0.0160	......	......	"	6185	.....	.....	1 24	....	....	...	....	....	1907
"	0.0080	0.0160	0.0321	"	513	2062	8288	0.45	0.91	2.03	991	913	784	2688
"	0.0101	0.0202	0.0405	"	815	3278	13180	0.55	1.17	2.70	1095	898	717	2710
250	0.0101	......	......	"	12225	.....	.....	1.23	...	....	....	....	....	4007
"	0.0050	0.0101	0.0202	"	1017	4075	16390	0.46	0.91	1.96	1871	1798	1645	5314
"	0.0050	0.0127	0.0321	"	1017	6483	41440	0.46	1.14	3.38	1871	2017	1617	5505
500	0.0101	......	.....	"	24450	.. .	.	1 53	....	....	....	..	....	5398
"	0.0050	0.0101	0.0202	"	2033	8150	32780	0.555	1.12	2.45	2731	2396	2126	7253
"	0 0050	0 0127	0.0321	"	2033	12970	82880	0.555	1.42	4.23	2731	2677	2068	7470

Taking the best coils given in the table, the total force exerted at the center is closely proportional to the 0.45 power

[1] Wire of the diameters given (which are those of standard sizes of the B & S [illegible]e gauge) can be procured with white silk insulation of the given thickness from the [illegible]rm of A. F. Moore & Co., of Philadelphia.

of the total resistance. Coils composed of three sections of best sizes of wire give about 1.4 times the force at center given by a coil of the best single size of wire of the same total resistance.

Method of winding. — The method I have employed in winding such coils is as follows: A brass mandrel is prepared as shown in the diagram (Fig. 1). Where coils of large resistance are to be wound the ends of wires of the several sections are brought out at the back of the coil and soldered after its completion, but for 0.1-ohm coils it is better to solder the wires together before winding. The wire is wound in a solution of shellac, and the sections are formed correctly and held in place till the shellac dries by winding thread against the back of the section as it grows in radius. After the coil is complete it is boiled in carnauba wax to make it rigid, after which the front plate of the mandrel is warmed and removed, and then the mandrel itself is warmed while the coil is held downward in cold water till it can be drawn from the mandrel.

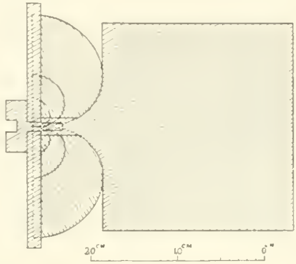


FIG. 1.

THE NEEDLE SYSTEM.

The great sensitiveness of recent galvanometers of the Thomson type in comparison with those of twenty years ago is much more due to the small dimensions of the needle system, than to improvements in the coil construction. Paschen [1] has emphasized this point, and he and others had constructed galvanometers of great sensitiveness prior to the time when the present investigations began. More recently Mendenhall and Waidner,[2] in a valuable paper on galvanometer construction, have described more thorough studies of the construction of needle systems, and point out that the useful diminution of dimensions cannot proceed as far as would appear from Paschen's paper.

[1] *Zeitschrift fur Instrumentenkunde*, 13. 13, 1893.

[2] *Am. Jour. Sci.* (4), 12, 249, 1901.

In their construction of magnet systems these authors are guided by extensive experiments on magnets of different forms, but they have not given exact directions for the dimensions and arrangement of the whole needle system to give highest sensitiveness. In what follows I shall endeavor to furnish exact information of this kind.

If we suppose a single short magnet suspended horizontally at the center of a coil to be influenced in a direction parallel to the plane of the coil by a magnetic field of strength H, and to be influenced at right angles to the plane of the coil by a current field of strength F, it will take up a direction making an angle θ with the plane of the coil such that

$$\tan\theta = \frac{F}{H}. \tag{7}$$

Let T be the time of swing of the suspension (undamped), M its magnetic moment, and I its moment of inertia, and let F be small as compared with H, then

$$T^2 \propto \frac{I}{MH}. \tag{8}$$

Combining (7) and (8)

$$\tan\theta \propto \frac{FMT^2}{I}. \tag{9}$$

For a given current and time of swing therefore

$$\tan\theta \propto \frac{M}{I}. \tag{10}$$

It is usual for purposes of astaticism to employ more than one group of magnets, with poles in the several groups oppositely directed. Suppose n such groups each composed of p magnets of moment M and let ΣMp be their algebraically combined magnetic moment tending to direct the system in a given sense, when $F = 0$, while nMp is their total magnetic moment without regard to sign. Let each group hang at the center of a coil whose field strength is f, and let H, T, and I have the same significance as before.

Then
$$\tan\theta = \frac{npfM}{H\Sigma Mp}. \tag{11}$$

If, as before, $npfm$ is small as compared with $H\Sigma Mp$

$$T^2 \propto \frac{I}{H\Sigma Mp} \cdot \tag{12}$$

Whence for a given current and time of swing

$$\tan\theta \propto \frac{npM}{I} \cdot \tag{13}$$

The moment of inertia I falls naturally in two portions, viz., the non-magnetic part which may be called I_n, and the magnetic part which is equal to npI_m where I_m is the moment of inertia of each separate magnet. Let $np = N$ the total number of magnets.

Thus the expression (13) becomes

$$\tan\theta \propto \frac{NM}{NI_m + I_n} \cdot \tag{14}$$

Of I_n all that need be said at present is that obviously it should be as small as practicable, and may be regarded as a known constant. If then M and I_m can be expressed in common terms the conditions of maximum deflection can be determined. It is easy to express I_m in terms of the linear dimensions of the magnets; thus for a magnet of length $2a$, weight w, and radius r cemented to a thin vertical glass stem, the moment of inertia is approximately $\frac{wa^2}{3}$ when $\frac{a}{r}$ is not less than 7.

Magnetic moment in terms of linear dimensions.—In order to find an expression for the magnetic moment in terms of the linear dimensions of the magnet, many experiments have been conducted here, which have led to a useful expression of this kind. As the method of measuring relative magnetic moments is useful in testing magnets used in needle systems, I shall briefly describe it. A single short magnet provided with a mirror is suspended in a field controlled in such a way that the time of swing of the suspension is raised to one or two seconds. The magnets to be tested are then brought up from the rear, end on, at right angles to the suspended magnet, at the same height with it and to a fixed distance which should be large compared with the length

of the magnets used. In this case the deflections of the suspension are directly proportional to the magnetic moments of the magnets approached.

In the experiments the relative magnetic moments of numerous magnets of different kinds, forms, and sizes were thus determined. The magnets varied in weight between 368 mg and 0.06 mg; in length between 22 mm and 0.8 mm; in diameter between 1.7 mm and 0.075 mm. Some were flat, some short and thick, others as much as fifty times as long as their diameters. Not only were they tried separately, but in combinations of from two to twenty, separated by spaces of from zero to five diameters, sometimes magnetized separately and then approached, at other times magnetized together.

The following table illustrates the advantage of dividing a magnet of a given weight into a number of thinner ones. The magnets were all from the same bar of steel tempered at once and magnetized to saturation:

TABLE I.

Designation of Magnet	Length mm	Width mm	Thickness mm	Weight	Magnetic Moment (Arbitrary Unit)
A_1	6.1	3.0	0.35	0.0425	7.3
A_2	6.1	0.88	0.35	......	4.9
A_3	6.1	0.88	0.35	Total weight	
A_4	6.1	0.88	0.35	of four	
A_5	6.1	0.88	0.35	0.0507	
A_2 and A_3 placed parallel and a little apart					9.3
A_2 A_3 and A_4 do. do.					13.4
A_2 A_3 A_4 and A_5 do. do.					18.0

We see that the four magnets had a combined magnetic moment two and one-half times as great as the single one of five-sixths their combined weight. Other experiments showed that a given weight of steel continued to increase in magnetic moment with increasing ratio of length to mean diameter even when the ratio was 50 to 1. A round or square cross-section appears preferable to a flat one.

Thoroughly to determine the connection between magnetic

moment and dimensions, a certain piece of steel was drawn into wire of four different sizes. Pieces were taken from each wire and all hardened at the same time. These were then magnetized to saturation together in a long helix. After determining the magnetic moment of each they were broken up into short lengths and again magnetized as before and the magnetic moment of each piece determined separately. The results are given in the following table:

TABLE II.

Designation	Length $2a$ cm	Diameter $2r$ cm	Weight W gr	Magnetic Moment M (Arbitrary Unit)	Ratio $\frac{2a}{2r}$	Ratio $10^5 \frac{W}{M} \frac{2a}{2r}$	Deviations from Mean
A_1	0.80	0.0266	0.00297	130.0	30.1	68.7	−29.2
A_2	0.30	0.0266	0.00111	9.7	11.3	117.4	+19.5
A_3	0.13	0.0266	0.00048	1.8	4.9	130.0	+32.1
B_1	0.73	0.0181	0.00138	59.0	40.3	93.5	− 4.4
B_2	0.35	0.0181	0.00066	19.0	19.4	67.0	−30.9
B_3	0.13	0.0181	0.00025	1.3	7.2	138.0	+40.1
B_4	0.08	0.0181	0.00015	0.7	4.4	94.6	− 3.3
C_1	0.33	0.0137	0.00037	8.4	24.1	94.4	− 3.5
C_2	0.08	0.0137	0.00009	0.5	5.8	96.9	− 1.0
D_1	0.50	0.0075	0.00015	9.4	66.6	106.4	+ 8.5
D_2	0.19	0.0075	0.00006	2.1	25.4	70.0	−27.9
					Mean	97.9	18.1

It will be seen that in this series of experiments the weight of the magnets varied fifty fold, the ratio of length to diameter varied from 66.6 to 4.4, and the magnetic moment varied 250 fold. In all this range of values the magnetic moment was proportional to the product of the weight by the ratio of length to diameter, within an average deviation of about 18 per cent. While this average deviation is large, it does not indicate a departure from the numerical relation just pointed out, for there appears on the whole no tendency for the deviations to become positive or negative either for the heavier or lighter needles or for those longest or shortest in proportion to their diameter. The large deviations may reasonably be explained as caused by differences in the magnetic quality of the steel, as will appear in a later page. Numerous experiments not here given have con-

firmed this general result, and in some cases series have been considerably more accordant than the one here given. It will be interesting to see whether the results of Mendenhall and Waidner support this relation. The following table is roughly computed from the series of magnets H and A given by them.[1]

TABLE III.

Designation	Length $2a$	Mean Diameter[2] $2r$	Weight W	Magnetic Moment M	Ratio $\frac{2a}{2r}$	Ratio $100\ \frac{W}{M}\ \frac{2a}{2r}$	Deviations from Mean
	cm	cm	g				
H_1	0.530	0.010	0.000430	0.0258	53.0	88.5	+23.6
H_2	0.415	0.010	0.000337	0.0184	41.0	75.2	+10.3
H_3	0.316	0.010	0.000251	0.0122	32.0	65.8	+ 0.9
H_4	0.195	0.010	0.000155	0.0048	19.0	61.3	− 3.6
H_5	0.124	0.010	0.000100	0.00215	12.0	55.9	− 9.0
H_6	0.102	0.010	0.000081	0.00143	10.0	56.5	− 8.4
H_7	0.065	0.010	0.000053	0.00054	6.5	62.9	− 2.0
					Mean	64.9	8.3
A_1	0.335	0.024	0.00157	0.0407	13.0	50.0	+ 2.2
A_2	0.229	0.024	0.00105	0.0198	8.8	46.9	− 0.9
A_3	0.167	0.024	0.00078	0.0100	6.4	50.0	+ 2.2
A_4	0.080	0.024	0.00036	0.0031	3.2	37.7	−10.1
A_5	0.153	0.017	0.00034	0.0049	9.0	59.9	+12.1
					Mean	47.8	5.5

These results show closer agreement to the relation above given than does the series of observations before quoted. There appears, however, a tendency to departure from the formula for magnets more than forty and for those less than five times as long as their diameter. I shall therefore restrict my statement as follows: For magnets of a given steel tempered and magnetized to saturation under like conditions, and whose length is between five and forty times their diameter, the magnetic moment is proportional to the product of the weight by the ratio of length to diameter.

Mutual action of magnets.—My observations have led me to a

[1] *Am. Jour. Sci.*, **12**, 256, 1901.

[2] The square root of the product of width and thickness.

different view of the seriousness of mutual action between magnets in needle systems from that arrived at by Mendenhall and Waidner, who incline to think it of little consequence. In the table of experimental results which they give the distance between centers of the magnets was never less than 1.0 mm and the magnets were flat and were 0.8 mm wide. Had they been turned up edgewise and approached closer, I think the mutual effect would have greatly increased. The following experiments were made with magnets of round wire about 1.4 mm long and 0.08 mm in diameter, being of the same dimensions that I am accustomed to use for needle systems. Ten of these were magnetized to saturation in a long helix, and gave (in arbitrary units) magnetic moments of 15, 13, 15, 12, 14, 13, 14, 13, 11, and 17, respectively, when measured separately, so that the arithmetical sum of their moments was 137 units. When placed in a bunch parallel and touching each other, the combined moment was only 35 units. The magnets were then again tested separately and had permanently decreased in magnetic moment to only five units each. They were then built into a system with spaces of three diameters between them, and were re-magnetized in the helix. This system gave a combined moment of 105 units. At two diameters and one diameter separation the moment became 95 and 74 units, respectively. Further experiments showed that similar results were reached whether the magnets were first approached and then magnetized, or *vice versa*. With spaces of five diameters between the magnets the loss of combined moment by mutual effect was only about 3 per cent.

Comparative magnetic moment of different samples of steel.— Different kinds of steel are unequally efficient for magnet systems. The best kind tried here was a small bar of tungsten steel of a composition unknown to me, which was given to me at Johns Hopkins University in Baltimore. As compared with Stubs' tool steel, this made up into magnets of twice the magnetic moment for equal weight and similar form. I have used it exclusively in recent galvanometers. But it is not alone samples of steel of different composition, but different samples

from the same bar, that show unequal magnetic value. Thus among 140 magnets of tungsten steel wire 0.008 cm in diameter and all between 0.125 and 0.135 mm long, broken from the same wire, tempered at the same time, and magnetized in a long helix at once, the magnetic moment was found to vary between the limits 13 and 35. Sixty-four, however, were selected whose average magnetic moment was 30 on this arbitrary scale, and most of the remainder were between the limits 20 and 25. It is undoubtedly to this variation of small samples from the same bar that the large deviations in Table II are due.

The following is a summary of the general results reached in these experiments on magnetic moment.

1. Rowland's tungsten steel made the strongest magnets of all the kinds tried, and proved twice as good as Stubs' tool steel.

2. A given weight of steel used to compose a single magnet has a greater magnetic moment if square or round in cross-section than if flat, and the magnetic moment is greater the greater the ratio of the length to the diameter.

3. A given weight of steel of a given length is increased in magnetic moment the more it is subdivided lengthwise to form separated magnets.

4. For magnets of the same steel between five and forty times as long as their diameter, the magnetic moment is given by the relation $M = K' \frac{Wa}{r}$, where M is the magnetic moment, W the weight, a the half length, r the radius, and K' a constant for the given kind of steel. This important relation was found to hold throughout the great range of weights and forms of magnets examined.

5. A number of magnets placed parallel in a group reduce each other's magnetic moments by about one-thirtieth if separated by spaces of 5 diameters, and the loss becomes about one-third at two diameters. It is undesirable to have the spaces less than three diameters.

6. It is immaterial whether the magnets are first magnetized and then approached, or first approached and then magnetized, for the loss is not recovered when the magnets are separated.

We now are in position to determine the best construction of needle systems; for, remembering that the magnetic moment is proportional to $\frac{NWa}{r}$, the moment of inertia to

$$\frac{NWa^2}{3} + I_N\ ,$$

and the deflection of the galvanometer to their quotient we have

$$\tan\theta \propto \frac{NW\frac{a}{r}}{\frac{NWa^2}{3} + I_N}\ . \tag{15}$$

I have used this formula to compute numerical values for a series of systems, selecting for construction one which is as heavy as possible without sacrificing more than one-tenth in sensitiveness as compared with the most sensitive. In this way better results are reached than by substituting for W in terms of the linear dimensions and specific gravity, and determining analytically the conditions of maximum sensitiveness.

We see by the formula that if the total weight of the system, the length of the needles, and the non-magnetic moment of inertia are all kept constant, the deflection increases indefinitely by diminishing the diameter of the magnets. But a practical limit is soon reached in this course of procedure, for in order to remain effective, the magnets must be separated by three or four diameters, and thus the space available for them is soon taken up. In practice I now employ magnets 0.008 centimeters in diameter.

By inspection of the formula it is also apparent that if the length and radius of the magnets and the non-magnetic moment of inertia all remain unchanged, an increase in the number of magnets increases the deflection. The great advantage of increasing the number is, however, that the non-magnetic moment of inertia can also be increased at the same time, and thus both the weight and rigidity of the needle system is augmented without loss of sensitiveness, so far as the needle system is concerned, and with a real gain in steadiness. But the limitation of space at the center of the coils bars indefinite progress in this

direction. In the two galvanometers now in use at the Observatory this increase in number of magnets has been effected by increasing the number of groups of magnets from two to eight, with an attendant increase of the number of coils from four to sixteen.[1] As shown at a former page, this increase in the number of coils diminishes the effectiveness of the current in the ratio of 68 to 32.

The "constant" of one of these sixteen-coil instruments expressed in ampères for a deflection of 1 mm on scale at 1 meter with time of single swing of 10 seconds and total resistance only 1.6 ohms is 5×10^{-11}.[2]

It is well to remark that, while the sixteen-coil instrument involves a deliberate sacrifice in the efficiency of the coils, it gains greatly in its better astaticism and the steadiness of the needle system. The least current which can be read with it as now arranged is about 1×10^{-12} ampères.

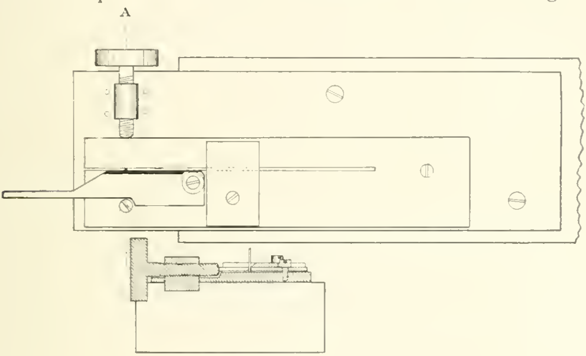
For a region free from prejudicial magnetic disturbances it would be better to retain the four-coil form of galvanometer. By making the coils rather large the interior unwound space could be a centimeter or more in diameter without great sacrifice of current effectiveness, and a large number of needles could be placed in each group.

Building of needle systems.—The method of building up a needle system of fifty or more magnets deserves a little explanation. In the first place the steel wire is cut off in three-quarter-inch lengths and hardened glass hard between thin

[1] A needle system of 10 mg weight having 48 needles each 0.132 cm long and 0.0082 cm in diameter, fixed to one side only of a stem 12 cm long and weighing complete with magnets and mirror 8 mg, is now in use. The several moments of inertia of its materials are: of magnets, 0.0000045 gr cm; of mirror, 0.0000005; of stem, 0.0000012; of shellac, 0.0000002. The practice of putting needles on both sides of the stem far enough apart to avoid loss of magnetic moment by mutual action increases the moment of inertia of the system too much to be of advantage.

[2] One needle system of 2 mg weight for a four-coil instrument of 1.4 ohms at 1.5 seconds single swing gave a millimeter deflection on the scale at one meter distance, with a current of 4×10^{-10} ampères. If we compute from this to a ten-second single swing the result is 9×10^{-12}, and if to a ten-second complete period the result is 24000 on the Ayrton-Mather scale. But both these computed values are illusory, for with atmospheric pressure the deflection of light systems is not even approximately proportional to the square of the time of swing. As shown a little later, this relation holds closely for galvanometers from which the air is exhausted to a pressure of 0.2 mm of mercury.

plates of steel. I have never softened the wire at all, although I have heard that there is a slight advantage in tempering it. The needles are then cut off from the glass-hard wire by means of a simple shears, illustrated in the diagram (Fig. 2), which cuts all of nearly equal length. These short pieces, more in number than are actually required, are next measured under the microscope. Each piece is then stuck like a pin in the end of a little square of cardboard, on which is marked its length;



Section at A.

FIG. 2.

all the cardboards are put upon a rod and inserted together in a magnetizing helix, and the magnets are strongly magnetized to saturation. The magnetic moment of each is then determined and the weak ones are eliminated, and, from the remainder, the groups are selected so that total magnetic moment and total weight of all the groups shall be equal.

The groups are built up on a form shown in the accompanying diagram (Fig. 3), consisting of two straight-edged glass strips of equal thickness separated by a space of about 0.8 mm and stuck down to a piece of plate glass. In the crack between the straight edges runs a wire connected to binding posts, one of which is movable and connected to a spring so as to stretch the wire. The little magnets are laid in sugar syrup across the crack from edge to edge, and their distance apart is carefully

adjusted while the sugar is kept moist by breathing upon it; after which the syrup is allowed to harden. A straight glass stem is now drawn of the desired weight, and upon this at intervals corresponding with the distance between the groups of needles are placed little portions of shellac. The stem is now laid upon the needles and a proper current of electricity is passed through the wire underneath till it is seen with a hand glass that the shellac has run around every magnet, but without running out toward

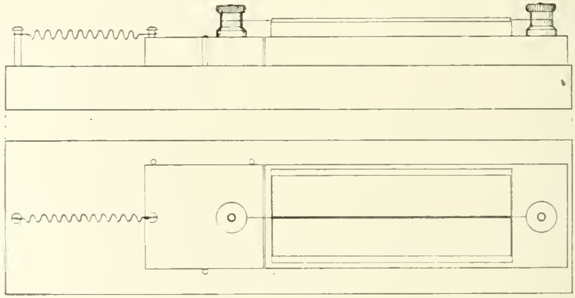


FIG. 3.

their ends. After the shellac cools, the needle system is dissolved off from the sugar bed with water, carefully washed, and is ready for the mirror. Such a needle system is almost mechanically perfect. I find no occasion with eight group needles to astaticize, but I do re-magnetize by the aid of an electro-magnet of sixteen poles.

The mirror is a fragment of microscope cover glass, selected from a large number of pieces of glass about 1×1.5 mm cut out with a dividing engine. These are all coated with metal on both sides, and their optical quality is tested by mounting each on a wad of cotton and reflecting with each mirror in succession the rays from a slit. Both surfaces are tested and each with both long sides and short sides vertical, so that the chance of obtaining a pretty good figure is quite considerable.

Galvanometer support and accessories.—In a city two well-known

kinds of disturbance are ever active, namely, mechanical jarring and electro-magnetic influences. The former of these may be communicated either by sound-waves in air or by direct ground vibrations. To remove the latter the Julius three-wire suspension is efficient, but may be combined advantageously with mercury flotation. Such a combination is employed for both the galvanometers at the Astrophysical Observatory, and its chief features will be found described in Vol. I of the Observatory *Annals*, and in the Appendix to the *Report of the Secretary of the Smithsonian Institution* for 1900 and 1901, pp. 100 and 122 respectively. I will merely add here that as a resnlt of a series of experiments it was found best to use glycerine to damp the horizontal vibrations of the three-wire suspension, as this liquid appears to damp best in proportion to its transmission of ground tremors.

For the destruction of the disturbing vibrations of sound within the galvanometer case, and for a further purpose shortly to be spoken of, the most recent galvanometer case has been made air tight and is used at an air pressure of less than ½ mm of mercury.

Magnetic shielding.—Magnetic shielding is now employed for the most sensitive galvanometer. Three shields are made use of, whose dimensions are in general accord with the recommendations of Du Bois.[1] The outer one is a two-inch-thick cast-iron box open at top and bottom and about six feet high and three feet square. Within this hangs the heavy table upon which rests the galvanometer. Upon this table are two concentric soft cast-iron cylindrical shields. The outer is 30 cm high, 15 cm outside, and 13 cm inside diameter. The inner, symmetrically placed with regard to the outer and fastened to it, is 20 cm high, 8.7 cm in exterior, and 7.5 cm in inside diameter. Both shields are lifted up by means of a cord when one wishes to examine or adjust within the galvanometer. The latter is in the basement of the laboratory and the beam of light for reading it is brought down to the mirror from a slit 3 m above. A small side mirror within the galvanometer case at an angle of 45° to the vertical reflects

[1] *Wied. Ann.*, **65**, 8, 1898.

the beam upon the needle system, and returns it to the scale or photographic plate 4 m away in the room above.

A number of strong control magnets within the outer iron box outside the inner shield are used to adjust the time of swing and position of zero. As a constant temperature is essential to freedom from drift when the galvanometer is used at a high time of swing, automatic temperature regulation and other precautions are resorted to.

With this galvanometer with its relatively heavy needle system — protected so thoroughly from shaking by a mercury-floated Julius suspension system weighing nearly a ton, and from magnetic disturbances by three iron shields — great steadiness is of course obtained, and consequently it is practicable to use a high time of swing. But, as is well known, the damping of the air generally prevents the deflection of a modern galvanometer from increasing anything like the square of the time of swing, when the time of single swing is above 1 second, and the air has therefore been exhausted. It was found that little gain resulted till a pressure of less than 1 cm was reached, but at 0.8 mm mercury pressure the law of squares held to 2.5 seconds single swing, and at 0.2 mm the deflection remained proportional to the square of the time of swing up to a time of single swing of 5.5 seconds. The needle system was found to be almost surprisingly steady.

This brings the account down to September, 1902. Since then attention has been wholly diverted from the great galvanometer. It will illustrate the working sensitiveness of an instrument measuring currents of 1×10^{-12} ampères, when I say that the difference of radiating power between an observer's black coat distant 2 m from the balanced bolometer, and the walls of the room which his coat momentarily hid, threw the spot of light off the scale some 40 cm, while the observer's naked hand within a meter of the bolometer turned the galvanometer needle round and round.

In conclusion I wish to say that in the work above described I have had the co-operation of Mr. F. E. Fowle, Jr., of the Astrophysical Observatory.

WASHINGTON, D. C.,
May 1903.

ON FORMULÆ FOR SPECTRUM SERIES.

By A. FOWLER and H. SHAW.

IN spite of the admirable work on spectrum series which has been done by Rydberg and by Kayser and Runge, the formulæ which they have employed to express the relation between the lines which constitute a series can, in many cases, be regarded only as rough approximations, more particularly for series of the principal and second subordinate types. The present paper gives the results of an attempt to bring the results of calculation into better accordance with those of observation by employing modified formulæ.

After several trials, the most accurate representations of various series were obtained by further generalizing the modifications of Balmer's formula which have already been suggested or employed by others. Two of these formulæ seem to merit a somewhat detailed discussion.

Employing Rydberg's convenient notation so far as possible, the first may be written:

$$n = n_\infty - \frac{C}{m^p - m_0}, \tag{I}$$

where m takes successively the values 2, 3, 4, $\cdots$, n is the oscillation frequency corresponding to the different values of m, and n_∞, C, m_0, and p are four constants to be determined from four lines for which n and m are known; n_∞ is the "convergence frequency" of the series, or the value which n assumes when m is infinite. This formula may be considered as a variation of the generalization of Balmer's formula suggested by Ames,[1] which may be written

$$n = n_\infty - \frac{C}{m^2 - m_0},$$

in which m is taken to be a whole number.

The second equation investigated is

$$n = n_\infty - \frac{C}{(m+\mu)^2 - m_0}. \tag{II}$$

[1] *Phil. Mag.*, 30, 47, 1890.

where n_∞, C, μ, and m_0 are the four constants to be calculated for each series. This may be regarded as a combination of the formula of Ames with that of Rydberg,[1] the latter being

$$n = n_\infty - \frac{C}{(m+\mu)^2},$$

in which m is a whole number, but μ is usually a fraction, and C has the value 109675 for all series.

Formula II may be regarded geometrically as the equation to a hyperbola of the second degree with respect to axes parallel to the asymptotes, the co-ordinates being represented by the values of n and $(m+\mu)^2$, while n_∞ and m_0 are the co-ordinates of the point of intersection of the asymptotes. Similarly, Formula I may be looked upon as the equation to a hyperbola of the pth degree.

In terms of wave-lengths, the two formulæ may be written:

$$\lambda = \lambda_0 + \frac{C'}{m^p - m'}. \qquad (\mathrm{I}a)$$

$$\lambda = \lambda_0 + \frac{C''}{(m+\mu)^2 - m''}. \qquad (\mathrm{II}a)^2$$

DETERMINATION OF THE CONSTANTS.

The determination of the constants of the two formulæ is less tedious than might at first appear. Designating the four given lines by n_1, n_2, n_3, n_4, and the corresponding values of m by m_1, m_2, m_3, m_4, the index p in Formula I may be derived by a few trials from the relation

$$\frac{(n_4 - n_1)\ (n_3 - n_2)}{(n_2 - n_1)\ (n_4 - n_3)} = \frac{(m_4^p - m_1^p)\ (m_3^p - m_2^p)}{(m_2^p - m_1^p)\ (m_4^p - m_3^p)}.$$

If many series are to be investigated, it is convenient to construct curves showing the values of the expression on the right

[1] "Recherches sur la constitution des spectres d'émission des éléments chimiques," *Kgl. Svenska Vet.-Akad. Handl.*, **23**, 11, 1890.

[2] Since completing the present paper we have found that this formula has been previously employed by Rummel (*Proc. Roy. Soc. Victoria*, **10**, 75, 1897; **12**, 15, 1899), who, however, does not appear to have noticed that the equation is of the same form whether expressed in wave-lengths or frequencies.

The work of Rummel appears to have received far less attention than it deserves. This is perhaps partly due to the fact that in an earlier paper (*Proc. Roy. Soc. Vict.*, **9**, 260, 1897) the formula which he employed was simply that of Ames transposed to wave-lengths (which applies with reasonable accuracy to only a few series), and to his continued use of the symbol n (corresponding with m in the above formulæ) in his later papers for a number which may have a fractional value.

for various values of p and for different groups of values of m. From such curves the value of p may usually be read with a sufficient degree of accuracy, and when p has been determined the other constants are easily derived.

The value of $(m_1 + \mu)$ in Formula II is determined directly from the relation

$$(m_1 + \mu) = \sqrt{\frac{a}{a-3}} - 1.5 ,$$

where

$$a = \frac{(n_4 - n_1)\ (n_3 - n_2)}{(n_4 - n_3)\ (n_2 - n_1)} .$$

and n_1, n_2, n_3, n_4 refer to four consecutive lines. If four lines which are not consecutive are employed in the calculation of the constants, the above equation will not hold, but another of similar character may easily be obtained. When μ has been ascertained, the other constants readily follow; or, if desired, n_∞ may be determined independently.

In view of the possible physical significance of the term μ, it may eventually be desirable to adjust its value by the addition of an integer, when necessary, so that $m = 1$ always gives the first positive value of n, or, as in the case of hydrogen, makes $n = 0$. For the present this has not been considered necessary.

EXAMPLES.

As already remarked, it is chiefly in the case of series of the principal and second subordinate types that the older formulæ are most defective, and it may now be added that this is especially marked when it is attempted to include the least refrangible line of a series. The following examples illustrating the two formulæ under investigation have accordingly been chiefly selected from series of these types. In the first instance, the equations given are those calculated from the first four lines of each series.

Unless otherwise stated, the frequencies given in the tables have been derived from Kayser and Runge's wave-lengths (as given in Watts's *Index of Spectra*), corrected to vacuum by the revised table drawn up by the same observers.[1]

[1] *Abh. Konigl. Preuss. Akad. Wissensch.*, Berlin, 1893; WATTS's *Index*, App. E, p. 52.

For the helium series, the frequencies already corrected to vacuum, are those given by Runge and Paschen.[1]

In the tables the numbers in brackets refer to the values of m; $O-C$ indicates observed minus computed values.

The differences $O-C$ in the case of Kayser and Runge's calculations, unless otherwise stated, are quoted from Professor Kayser's recent book (*Handbuch der Spectroscopie*, Vol. II), where the formulæ and tables are conveniently brought together.[2] Those resulting from the use of Rydberg's formula are taken directly from Rydberg's general work ("Recherches," etc.), but it should be pointed out that these would probably be slightly modified if recalculated with the later value of the constant C, namely 109675 in place of 109721.6.

THE PRINCIPAL SERIES OF SODIUM.

For the less refrangible components of the double lines which constitute the principal series of sodium lines, the equations derived from the first four lines are:

Formula I: $n = 41510.37 - \dfrac{92513.4}{m^{1.917} - 0.02613}$.

Formula II: $n = 41449.92 - \dfrac{110875.4}{(m + 0.1935)^2 - 0.284956}$.

Oscillation Frequency in Vacuo	$O-C$ Formula I In Frequency	$O-C$ Formula I In Tenth-Meters	$O-C$ Formula II In Frequency	$O-C$ Formula II In Tenth-Meters	$O-C$ K. and R.'s Formula (In Tenth-Meters)	$O-C$[4] K. and R. (In Tenth-Meters)	$O-C$ Rydberg (In Tenth-Meters)
16955.15	(2) +0.14	−0.049	(2) +0.03	−0.010	+77.46	+1.0	+3.0
30265.61	(3) −0.09	+0.010	(3) −0.01	+0.001	0.00	−1.6	−7.8
35041.12	(4) −0.03	+0.002	(4) 0.00	0.000	0.00	+1.0	+0.7
37295.34	(5) −0.02	+0.001	(5) 0.00	0.000	0.00	+0.7	+1.2
38539.66[3]	(6) −0.04	+0.002	(6) +1.81	−0.121	+ 0.06	+0.1	+2.9
39299.06[3]	(7) −1.15	+0.074	(7) +3.67	−0.237	+ 0.17	−0.4	.....
39793.70[3]	(8) −5.90	+0.370	(8) +2.39	−0.160	+ 0.65	−0.4	.....

[1] *Sitz. d. K. Akad. der Wiss.*, Berlin, July, 1895, pp. 639, 759; ASTROPHYSICAL JOURNAL, January 1896.

[2] The equation generally employed by Kayser and Runge is $N = A\ B\dfrac{\beta}{m^2} - \dfrac{C}{m^4}$, where A, B, C, are three constants to be separately determined for each series, and m is a whole number.

[3] Observed as single lines.

[4] The errors in this column are those resulting from a formula in which all the lines are used in the calculation of constants (KAYSER, *Handbuch der Spect.*, Vol. II, p.521).

PRINCIPAL SERIES OF POTASSIUM.

For the less refrangible components of the principal series of double lines of potassium the formulæ and results are as follows:

$$\text{Formula I: } n = 34927.31 - \frac{195630.4}{m^{2.2182} - 2.5224}.$$

$$\text{Formula II: } n = 35047.96 - \frac{115321.9}{(m + 0.459)^2 - 0.81980}.$$

Oscillation Frequency in Vacuo	$O-C$ Formula I: In Frequency	$O-C$ Formula I: In Tenth-Meters	$O-C$ Formula II: In Frequency	$O-C$ Formula II: In Tenth-Meters	$O-C$ K. & R.'s Formula In Tenth-Meters	$O-C$ Rydberg In Tenth-Meters
12984.69	(3) 0.00	0.000	(2) −0.03	+0.020	+161.30	−17.0
24700.46	(4) −0.01	+0.002	(3) +0.02	−0.003	0.00	−22.7
28998.41	(5) 0.00	0.000	(4) 0.00	0.000	0.00	−6.6
31068.72	(6) 0.00	0.000	(5) 0.00	0.000	0.00	+1.6
32224.22	(7) −0.91	+0.087	(6) −4.06	+0.390	+0.27	+4.0
32940.18	(8) +4.53	−0.417	(7) −4.01	+0.369	+0.23	+4.3
33409.17	(9) +6.58	−0.588	(8) −8.45	+0.755	+0.68	+6.6
33736.46	(10) +11.18	−0.978	(9) −10.67	+0.934	+1.05	+7.3
33971.45	(11) +14.24	−1.234	(10) −14.33	+1.242	+1.45	+6.7

THE PRINCIPAL SERIES OF LITHIUM.

For this series the formulæ and results are as follows:

$$\text{Formula I: } n = 43446.07 - \frac{116717.5}{m^{2.032} - 0.00056}.$$

$$\text{Formula II: } n = 43470.35 - \frac{108764.1}{(m + 0.931)^2 + 0.07855}.$$

Oscillation Frequency in Vacuo	$O-C$ Formula I: In Frequency	$O-C$ Formula I: In Tenth-Meters	$O-C$ Formula II: In Frequency	$O-C$ Formula II: In Tenth-Meters	$O-C$ K. & R. In Tenth-Meters	$O-C$ K. & R.[2] In Tenth-Meters	$O-C$ Rydberg In Tenth-Meters
14903.08	(2) +0.25	−0.111	(1) −0.09	+0.040	+108.12	0.00	+0.4
30924.54	(3) −0.16	+0.017	(2) +0.06	−0.006	0.00	0.00	+2.2
36467.48	(4) −0.05	+0.004	(3) +0.02	−0.001	0.00	0.00	+0.5
39011.60	(5) −0.03	+0.002	(4) +0.02	−0.001	0.00	+0.66	−0.8
40389.99	(6) +5.45	−0.350	(5) +4.68	−0.287	−0.20	+0.95	−0.3
41215.53	(7) +7.77	−0.457	(6) +5.57	−0.327	−0.01	+1.44	−0.4
41749.29	(8) +9.54	−0.550	(7) +5.92	−0.338	+0.29	+1.80	−0.4
42112.35[1]	(9) +9.41	−0.530	(8) +4.25	−0.238	+0.75	+2.18	−0.6

[1] Based on observations by Messrs. Liveing and Dewar.

[2] The errors in this column are those which result when the constants are calculated from the first three lines.

THE SERIES OF MAGNESIUM TRIPLETS.

For the middle components of the triplets which constitute the first subordinate series of magnesium lines the following formulæ have been derived from the first four lines of the series:

$$\text{Formula I}: n = 39762.68 - \frac{126484.3}{m^{2.058} - 0.34427}.$$

$$\text{Formula II}: n = 39790.41 - \frac{109684.5}{(m + 0.840)^2 - 0.06227}.$$

Oscillation Frequency in Vacuo	$O-C$ Formula I		$O-C$ Formula II		$O-C$ K. & R.'s Formula	$O-C$ Rydberg's Formula
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters	In Tenth-Meters	In Tenth-Meters
26085.57	(3) 0.00	0.000	(2)+0.02	−0.003	−0.01	+0.5
32320.40	(4)+0.01	−0.001	(3)−0.01	+0.001	−0.01	−1.1
35095.68	(5) 0.00	0.000	(4) 0.00	0.000	+0.31	−0.1
36568.50	(6)+0.01	−0.001	(5) 0.00	0.000	0.00	−0.6
37444.55	(7)+2.29	−0.162	(6)+1.67	−0.118	−0.41	+0.9
38004.23	(8)+1.71	−0.118	(7)+0.12	−0.008	−0.46	+0.4

For the least refrangible components of the second subordinate series of triplets the formulæ are:

$$\text{Formula I}: n = 39699.68 - \frac{176665.9}{m^{2.175} - 2.25379}.$$

$$\text{Formula II}: n = 39794.75 - \frac{115447.6}{(m + 0.555)^2 - 0.89949}.$$

Oscillation Frequency	$O-C$ Formula I		$O-C$ Formula II		$O-C$ K. & R.'s Formula (In Tenth-Meters)	$O-C$ Rydberg's Formula (In Tenth-Meters)
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters		
(b1) 19285.44	(3) 0.00	0.000	(2)+0.03	−0.008	+53.22	+3.9
29960.21	(4)−0.01	+0.001	(3)−0.02	+0.002	−0.03	−13.0
33978.46	(5)−0.01	+0.001	(4)+0.01	+0.001	−0.05	−2.2
35941.23	(6)+0.04	−0.003	(5) 0.00	0.000	−0.02	+2.5
37047.87	(7)−0.18	+0.013	(6)−2.64	+0.198	+0.11	+3.8
37734.99	(8)+1.80	−0.127	(7)−4.76	+0.336	+0.27	+4.8

RYDBERG'S SERIES OF MAGNESIUM.

There is another series of magnesium lines to which attention was first drawn by Rydberg[1] for which the formulæ of Rydberg

[1] *Öfversigt af Kongl. Vet.-Akad. Forhandl.* Stockholm, 1893.

and of Kayser and Runge give very unsatisfactory results. For this series Rydberg has employed the modified formula:

$$n = n_\infty - \frac{B}{(m+\mu)^2} - \frac{C}{(m+\mu)^4}\,.$$

It is interesting to find that the new formulæ may be applied to the series without change. As the lines are somewhat nebulous, it is possible that the wave-lengths of the more refrangible members of the series, which are also feeble lines, are not very precise, and in fact for the last two lines the limit of error stated by Kayser and Runge is 1.0 tenth-meter.

Formula I: $n = 26633.03 - \dfrac{88847.1}{m^{1.925} + 2.10247}\,.$

Formula II: $n = 26601.49 - \dfrac{107071.4}{(m + 0.2304)^2 + 2.13282}\,.$

Oscillation Frequency in Vacuo	$O-C$ Formula I		$O-C$ Formula II		$O-C$ Rydberg's Modified Formula
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters	
18082.33	(3)−0.01	+0.003	(3) 0.00	0.00	+0.019
21255.70	(4) 0.00	0.000	(4) 0.00	0.00	−0.009
22970.71	(5) 0.00	0.000	(5) 0.00	0.00	−0.004
23986.85	(6)−0.01	+0.002	(6) 0.00	0.00	0.000
24633.15	(7)−1.01	+0.170	(7)−0.53	+0.09	+0.200
25074.10	(8)+3.61	−0.570	(8)+5.00	−0.79	−0.480

THE TWO SUBORDINATE SERIES OF HELIUM DOUBLES.

For the less refrangible components of the first subordinate series, the formulæ and results are as follows:

Formula I: $n = 29222.11 - \dfrac{110093.6}{m^{2.0015} + 0.00332}\,.$

Formula II: $n = 29222.89 - \dfrac{109697.9}{(m + 0.996)^2 + 0.009151}\,.$

Oscillation Frequency in Vacuo	$O-C$ Formula I		$O-C$ Formula II		$O-C$[1] K. & R.'s Formula		$O-C$[2] Rydberg's Formula	
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters
D_3 17014.13	(3) 0.00	0.000	(2)+0.02	−0.007	0.00	0.00	−9.43	+3.263
22356.97	(4) 0.00	0.000	(3)−0.01	+0.002	0.00	0.00	+2.40	−0.480
24829.56	(5) 0.00	0.000	(4) 0.00	0.000	0.00	0.00	+0.74	−0.119
26172.44	(6) 0.00	0.000	(5) 0.00	0.000	−0.13	+0.02	+0.25	−0.036
26981.92	(7)−0.08	+0.011	(6)−0.10	+0.014	−0.39	+0.05	−0.01	+0.001
27507.23	(8)−0.11	+0.013	(7)−0.16	+0.021	−0.60	+0.08	−0.12	+0.014
27867.34	(9)−0.12	+0.015	(8)−0.20	+0.026	−0.78	+0.10	−0.17	+0.022
28124.73	(10)−0.02	+0.003	(9)−0.43	+0.054	−1.09	+0.14	−0.36	+0.046
28315.49	(11)−0.05	+0.006	(10)−0.21	+0.026	−1.00	+0.13	−0.16	+0.020
28460.56	(12)+0.29	−0.045	(11)−0.08	+0.010	−0.94	+0.12	−0.01	+0.001
28573.37	(13)+0.19	−0.023	(12)−0.06	+0.007	−0.99	+0.12	+0.03	−0.004
28662.74	(14)+0.10	−0.012	(13)−0.17	+0.021	−1.16	+0.14	−0.08	+0.010
28735.00	(15)+0.20	−0.024	(14)−0.10	+0.012	−1.14	+0.14	0.00	0.000
28794.33	(16)+0.48	−0.057	(15)+0.15	−0.018	−0.93	+0.11	+0.26	−0.031
28843.27	(17)+0.50	−0.060	(16)+0.12	−0.014	−0.99	+0.12	+0.24	−0.029
28881.93	(18)−1.86	+0.222	(17)−2.25	+0.268	−3.39	+0.41	−2.12	+0.258
28919.43	(19)+0.94	+0.112	(18)+0.53	−0.063	−0.64	+0.08	+0.66	−0.079

For the second subordinate series the formulæ for the less refrangible components, as calculated from the first four lines, are as follows:

$$\text{Formula I:} \quad n = 29176.95 - \frac{142019.0}{m^{2.103} - 0.62765} .$$

$$\text{Formula II:} \quad n = 29227.70 - \frac{110377.6}{(m+0.724)^2 - 0.099836} .$$

Oscillation Frequency in Vacuo	$O-C$ Formula I		$O-C$ Formula II		$O-C$ K. & R.'s Formula		$O-C$[4] Rydberg's Formula	
	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters	In Frequency	In Tenth-Meters
14149.48	(3)+ 0.01	−0.005	(2)−0.02	+0.011	−47.85	+24.00	−44.86	+22.660
21210.94	(4) 0.00	0.000	(3)+0.01	−0.002	0.00	0.00	− 6.70	+ 1.500
24259.39	(5) 0.00	0.000	(4)−0.01	+0.002	0.00	0.00	− 1.41	+ 0.241
25848.55	(6) 0.00	0.000	(5) 0.00	0.000	0.00	0.00	− 0.16	+ 0.024
26780.61	(7)+ 0.84	−0.117	(6)−0.36	+0.050	+ 1.38	− 0.19	+ 0.11	− 0.015
27373.71	(8)+ 1.27	−0.168	(7)−0.78	+0.103	+ 3.44	− 0.46	+ 0.18	− 0.024
27774.08	(9)+ 4.04	+0.524	(8)−1.44	+0.187	+ 5.37	− 0.70	− 0.03	+ 0.004
28057.39	(10)+ 6.35	−0.807	(9)−1.75	+0.222	+ 7.51	− 0.95	+ 0.05	− 0.006
28264.92	(11)+ 8.55	−1.064	(10)−2.40	+0.299	+ 9.28	− 1.16	− 0.03	+ 0.004
28421.47	(12)+10.64	−1.324	(11)−2.62	+0.326	+10.76	− 1.33	− 0.17	+ 0.024
28543.27	(13)+13.41	−1.634	(12)−2.25	+0.274	+12.82	− 1.57	+ 0.47	− 0.057
28638.93	(14)+15.31	−1.865	(13)−2.43	+0.296	+14.10	− 1.72	+ 0.52	− 0.063
28714.37[3]	(15)+15.99	−1.948	(14)−3.97	+0.484	+13.85	− 1.69	− 0.82	+ 0.100

[1] For all the helium series Runge and Paschen use the formula:

$$n = A - \frac{B}{m^2} - \frac{C}{m^3} .$$

[2] Rydberg does not appear to have published calculations for the helium lines, but he has published statements of the convergence frequencies of the series (Astrophysical Journal, **4**, 94, 1896) and of the values of the term μ in his equation (*Ibid.*, **6**, 234, 1897). By making use of these we get for this series the formula

$$n = 29222.70 - \frac{109675}{(m+0.996084)^2} ,$$

and it is from this that the tabulated errors have been derived.

[3] Marked "doubtful" in Runge and Paschen's table.

[4] See note to previous table; the errors stated have been calculated from the equation

$$n = 29222.63 - \frac{109675}{(m+0.701464)^2} .$$

GENERAL DISCUSSION.

The foregoing examples suffice to demonstrate that series which present great departures from calculation when the older formulæ are employed can be better represented by either of the two formulæ investigated, but few of them permit a final comparison of their relative merits or of the ultimate accuracy of which they are capable. In some of the series there are probably small errors of observation which account in part for differences between the observed and calculated frequencies, and in most of them comparatively few lines have been traced. For helium, however, the case is different; many lines have been observed, and as they are also well defined, the positions determined by Runge and Paschen are probably accurate to a high degree.

All the helium series are not, however, well adapted to test the different formulæ. The first subordinate series of doubles, for example, is very closely represented even by the simplest generalization of Balmer's formula for hydrogen, the equation calculated from lines for which m has the values 3, 4, 5, being

$$n = 29225.58 - \frac{109901.37}{m^2}.$$

With this formula the differences between the observed and calculated frequencies for values of m as far as 17 do not differ by more than 2.13. As all the formulæ under investigation are attempted generalizations of Balmer's law, it is clear that such a series as this cannot be effectively employed for their comparison, and it will be seen by reference to the table that the differences between the observed and calculated frequencies are very small throughout. It is the second subordinate series of double lines of helium which at present permits the best comparison of the formulæ.

From the table already given for this series it appears that Formula I, when calculated for the first four lines, gives errors comparable with those of Kayser and Runge's formula for lines corresponding to the greater values of m, but has the advantage of correcting the large error in the case of the first line. Still, this cannot be considered an accurate formula for the series,

especially as the convergence frequency is notably smaller than that derived from the first subordinate series. Nevertheless it is instructive to observe the effect of calculating the constants from lines more widely separated than the first four; taking lines for which $m = 3, 4, 6$, and 8, we get equation I*b* stated at the head of the following table. Again, reducing p to 2.09 and calculating the other constants from the first three lines, we get equation I*c*.

The results are as follows:

$$\text{Formula I}a: \quad n = 29176.95 - \frac{142019.0}{m^{2.103} - 0.62765}.$$

$$\text{Formula I}b: \quad n = 29189.83 - \frac{140337.9}{m^{2.0947} - 0.65602}.$$

$$\text{Formula I}c: \quad n = 29200.77 - \frac{139489.5}{m^{2.09} - 0.66775}.$$

Oscillation Frequency in Vacuo	$O-C$ Formula I*a* In Frequency	$O-C$ Formula I*b* In Frequency	$O-C$ Formula I*c* In Frequency
14149.48	+ 0.01[1]	−0.02[1]	0.00[1]
21210.94	0.00[1]	+0.02[1]	0.00[1]
24259.39	0.00[1]	+0.61	0.00[1]
25848.55	0.00[1]	−0.01[1]	−1.66
26780.61	+ 0.84	−0.36	−3.13
27373.71	+ 1.27	0.00[1]	−3.76
27774.08	+ 4.04	−1.62	−3.96
28057.39	+ 6.35	+1.98	−3.38
28264.92	+ 8.55	+3.31	−2.67
28421.47	+10.64	+4.64	−1.86
28543.27	+13.41	+6.67	−0.20
28638.93	+15.31	+8.23	+0.89
28714.37	+15.99	+8.26	+0.59

The errors may in this way be generally reduced and differently distributed, but it is clear that the series cannot be represented by Formula I with an accuracy equal to the probable accuracy of the observations.

For the same series, Formula II, as derived from the first four lines, is decidedly more accurate than Formula I derived from the same four lines, and the convergence frequency is also very nearly equal to that of the first subordinate series. The regular increase in the differences $O - C$ suggests that the formula might

[1] Used in calculation of constants.

be improved by taking a value of μ slightly different from that given by the first four lines. Thus, by calculating the constants from lines for which $m = 2$, 3, 5, and 7, the value of μ becomes 0.716, and the equation derived is

$$n = 29224.24 - \frac{110040.6}{(m + 0.716)^2 - 0.07766} \ .$$

Here the convergence frequency differs very slightly from that of the first subordinate series, and the differences $O - C$ are greatly reduced, as shown in the following table:

Observed Frequency in Vacuo	$O - C$ Formula II Above	
	In Frequency	In Tenth-Meters
14149.48	−0.01	+0.005
21210.94	0.00	0.000
24259.39	−0.24	0.041
25848.55	0.00	0.000
26780.61	+0.03	−0.004
27373.71	0.00	0.000
27774.08	−0.28	+0.036
28057.39	−0.32	+0.041
28264.92	−0.49	+0.061
28421.47	−0.72	+0.091
28543.27	−0.17	+0.021
28638.93	−0.20	+0.024
28714.37	−1.61	+0.196

The differences between calculation and observation thus become almost insignificant, never amounting to more than 0.1 tenth-meter, except in the case of the last line, which is marked "doubtful" by Kayser and Runge.

Of the other examples given, the principal series of potassium is about the only one which is sufficiently extended to be used in a minute comparison of the formulæ, but in this case the wave-length determinations are probably much less precise than those of helium. From the table already given it will be seen that when the constants are determined from the first four lines, the two formulæ investigated in this paper represent the series with almost equal accuracy, and much more closely than the older formulæ. As in the case of the second subordinate series of helium, however, it is found that changes in the constants of Formula I do not result in quite so accurate a representation of

the series as can be obtained by a change of constants in Formula II. Thus by calculating the constants of the latter from the first, second, fourth, and six lines, we get the equation:

$$n = 35031.60 - \frac{113919.3}{(m + 0.430)^2 - 0.7380};$$

and the differences $O - C$ compared with the limits of error of observation stated by Kayser and Runge are as follows:

Observed Frequency in Vacuo	$O-C$ Formula Above		Limit of Error of Observation in Tenth-Meters
	In Frequency	In Tenth-Meters	
12984.69	+0.99	−0.58	5.00
24700.46	−0.06	+0.01	0.03
28998.41	−1.59	+0.19	0.03
31068.72	−0.04	+0.004	0.03
32224.22	−1.96	+0.19	0.10
32940.18	+0.11	−0.01	0.10
33409.17	−2.58	+0.23	0.15
33736.46	−3.35	+0.29	0.20
33971.45	−5.80	+0.50	1.00

When the errors are distributed in this way, it will be seen that they are on the whole scarcely too great to be considered as arising from errors of observation.

The general result of the comparison, therefore, is that, while both the formulæ investigated represent the observed series very closely, the most accurate general formula appears to be:

$$n = n_\infty - \frac{C}{(m + \mu)^2 - m_0}.$$

With this formula, if the constants are calculated so as to distribute the errors, the computed differ from the observed positions by amounts which perhaps do not exceed the probable errors of the observations at present available.

Royal College of Science, London,
March 30, 1903.

THE VARIABLE STAR 6871 *V LYRAE.*

By J. A. Parkhurst.

The variability of this star was announced by Anderson in 1895[1], this being one of many variables which are very faint for the greater part of their period, and yet have been detected by this careful observer with telescopes of two and three inches aperture. In the place just cited, magnitudes are given from the Harvard photographs from July 24, 1892, to October 6, 1894. Visual observations have been published by Yendell,[2] H. M. Parkhurst,[3] and the writer,[4] covering the maxima of each year from 1893 to 1900, inclusive. Approximate data for the minima of 1899 and 1900 are given in the last reference. Photometric magnitudes of comparison stars brighter than the twelfth magnitude have also been published by H. M. Parkhurst,[5] with which those given in this paper are in fair agreement.

INSTRUMENTS.

These are the same as given in my paper on *X Cephei*[6]—a 157 mm reflector, a 305 mm, and the 101 cm refractor of the Yerkes Observatory; the equalizing wedge photometer being attached to each of these instruments. The photograph of the field reproduced in Plate I was taken November 30, 1902, with an exposure of thirty-six minutes, from $6^h\ 24^m$ to $7^h\ 0^m$ Central Standard Time. The faintest stars shown on the plate are about 16.5 magnitude, though none fainter than ϵ, 15.80 magnitude, have been measured in this field.

POSITION OF THE VARIABLE.

The variable was connected on two nights with the comparison stars *a*, *b*, and *c*, whose *Durchmusterung* numbers will be

[1] *A. N.*, 137, 235.

[2] *Astronomical Journal*, 15, 157, 1895; 17, 27, 1896.

[3] *Ibid.*, 17, 65, 1897; 18, 100, 1897; 19, 190, 1899.

[4] *Ibid.*, 17, 102, 1897; 18, 142, 1898; Astrophysical Journal, 14, 174, 1901.

[5] *Ibid.*, 17, 65, 1897.

[6] Astrophysical Journal, 17, 48, 1903.

found in Table II. The places of these stars for 1875 were taken from the Cambridge, England, *Astronomische Gesellschaft* Catalogue. The results were as follows:

Star	*a*	*b*	*c*
R.A. 1875	$19^h\ 3^m\ 1^s.02$	$4^m\ 8^s.51$	$4^m\ 10^s.80$
Δ R.A.	$+1\ \ 9.93$	$+0\ \ 3.27$	$+0\ \ 0.30$
R.A. *V*	19 4 10.95	4 11.78	4 11.10
Mean *V* 1875	19 4 11.28		
Dec., 1875	$+29^\circ\ 36'\ 16''.2$	$42'\ 43''.9$	$17'\ 48''.7$
Δ Dec.	−8 42.4	−15 10.5	+9 43.3
V	+29 27 33.8	27 33.4	27 32.0
Mean *V* 1875	+29 27 33.1		

For convenience the following are added:

POSITION OF 6871 *V LYRAE*

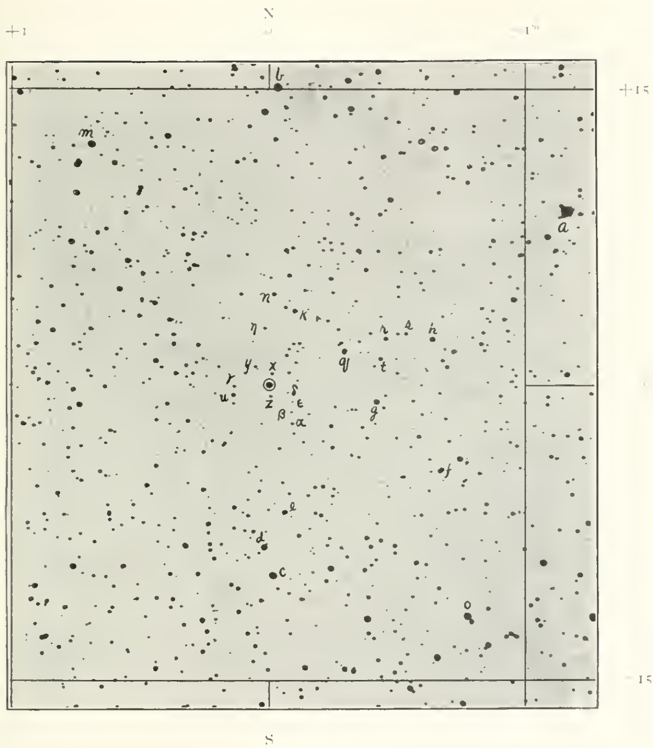
R.A. $19^h\ 3^m\ 24^s.5$, Dec. $+29^\circ\ 25'\ 42''$ (1855).
5 9.9, 29 52 (1900).

COMPARISON STARS.

The numerical data for the comparison stars are given in Tables I and II, which require little explanation. The positions of all the stars are shown on the chart. The co-ordinates from the variable for the stars brighter than twelfth magnitude were measured with the filar micrometer on the 6-inch reflector; the faint stars near the variable were measured with the 40-inch refractor, and all positions were checked from the photograph. The light scale was formed in the usual manner, from comparisons of the variable by Argelander's method; the results are given in the fourth column. The zero point of this scale, the star ϵ, is not the limit of vision of the 40-inch refractor, but simply the faintest star needed in the visual comparisons. The fifth column shows the results of the photometer measures which are given in detail in Table III.

The photometric magnitudes are based on the following standard stars:

PLATE I.



6871 *V LYRAE*

(19h 5m 9.9s; +20° 29′ 52″)

Star	B.D. No.	B.D. Mag.	1855 R.A.	1855 Dec.	Magnitudes P.D.M.	Magnitudes Harvard 24	Magnitudes Harvard 45
A	+29° 3472	6.5	19h 0m 7s.6	+29° 41′.9	6.45	6.46	6.77
B	+30 3425	7.8	1 54.4	+30 1.0	....	8.06	....
C	+30 3438	7.0	3 14.7	+30 4.4	6.94	6.86	6.90

The magnitudes in the last three columns are, respectively, from the *Potsdam Photometric Durchmusterung*, *Harvard College Observatory*, Vol. 24 (*Meridian Photometry*), and Vol. 45 (*Photometric Durchmusterung*). The magnitudes used were those of Harvard 24, as it contained all three of the stars and differed but little from the *P.D.M.* the difference in the systems being only 0.035 mag. In the sixth column of Table I are given the magnitudes of such of the stars as were not measured with the photometer. These were read off from the "Magnitude Curve," Fig. 1, in a manner to be described later.

TABLE I.

Comparison Stars for *V Lyrae*. In order of Right Ascension.

Star	Co-ordinates from *V* R. A.	Co-ordinates from *V* Dec.	Light Scale	Magnitude Measured	Magnitude From Curve	Star	Co-ordinates from *V* R. A.	Co-ordinates from *V* Dec	Light Scale	Magnitude Measured	Magnitude From Curve
	s	′ ″	steps				s	′ ″	steps		
a	−70	+8 43	....	8.29	.....	δ	−4	−0 25	1 5	15.60	.....
o	−45	−11 54	37.6	9.35	.....	*e*	−3	−6 26	29.2	.....	11.27
f	−44	−3 49	28.1	11.18	.....	*b*	−2	+15 11	44.3	8.54	.....
h	−38	+2 24	28 4	.....	11 40	*n*	−2	+4 36	26.0	12 20	.....
s	−31	+2 40	....	.....	.....	*x*	−1	+0 35	....	.....	14 8
r	−27	+2 25	19.0	.....	12.88	*c*	−1	−9 43	35.9	10.05	.....
t	−26	+1 18	....	.....	.....	*z*	−1	−0 34	8.0	14.76	.....
g	−25	−0 52	25.4	11.22	.....	η	+1	+2 54	....	13 35	.....
q	−17	+1 42	23.0	.....	12.30	*d*	+1	−8 8	31.8	.....	10.80
l	−7	+17 48	...	.....	.....	*y*	+3	+0 59	....	.....	.....
k	−5	+3 49	27.0	11.91	.....	γ	+8	−0 3	7.7	.....	14.70
ε	−6	−0 50	0.0	.....	15.80	*u*	+8	−0 29	16.4	13.08	.....
α	−5	−2 00	15.3	.....	13.52	*m*	+40	+12 00	37 0	9.81	.....
β	−5	−1 21	7 1	.....	14.78						

Table II gives the *Durchmusterung* numbers of part of the stars, also a comparison between the *Durchmusterung* magnitudes and the photometric measures of H. M. Parkhurst and the writer. There is a systematic difference of about 0.15 mag.

between the two latter, which can be explained by the fact that they are based on a different set of Meridian Photometer stars. Besides this, there is a difference of 0.60 mag. in the star *b*, which has a range of over a magnitude from the *Durchmusterung* value. The photographic magnitude of *b* is about 9.2, showing a considerable color and probable fluctuations or variation.

TABLE II.

Comparison with the *Durchmusterung* and Measures by H. M. Parkhurst.

Star	"Durchmusterung"		H. M. Parkhurst		J. A. Parkhurst
	Number	Mag.	Letter	Mag.	
a	+29° 3483	8.0			8.29
o	3486	9.2			9.35
l	3490	8.4			....
b	3492	8.1	*J*	9.14	8.54
c	3493	9.3	*IX*	9.91	10.05
d	3494	9.5	*Z*	10.70	10.80
m	+29 3498	9.2	*W*	9.67	9.81
f	....	...	*b*	10.97	11.18
h	....	...	*c*	11.02	11.40
g	....	...	*a*	10.95	11.22
e	....	...	*e*	11.28	11.27

The photometric measures of the comparison stars are given in Table III. The arrangement of this table and the methods of reduction are similar to those in my previous paper on *X Cephei*. For convenience I will repeat here that the stars used as standards are given first in each part of the table, with their magnitudes in bold-faced type. The mean of the scale readings and the mean of the magnitudes of these standard stars are taken as the zero points. For the stars whose magnitudes are sought, the difference in scale readings is converted into magnitudes by the use of the wedge constant given at the foot of each set, and added to the zero magnitude, giving the quantities in the last column. The magnitudes of the comparison stars brighter than 11.5 were measured with the 6- and 12-inch telescopes, and the mean of the results given after the set for October 5, 1902. From these as standards, the stars *n* and *k* were measured with the 12-inch, and finally the four fainter stars were measured with

the 40-inch. The set of measures made October 5, 1902, requires explanation, since a cell holding two absorption glasses was interposed in the cone of rays of the real star while measuring the standards *A*, *B*, and *C*. The absorption caused by these two glasses has been measured by standard stars and also by the "wheel photometer," the resulting value, 1.70 mag., being used in this work. The mean magnitude of the three standard stars, 7.13, becomes 8.83 when the absorption glasses are used. It will be seen that the results are in good accord with those taken without the absorption glasses.

TABLE III.

1900, August 4; 6-inch. Wedge II.

Star	Scale Readings								Mean Scale Reading	Magnitude
	First				Second					
A	11.8	10.5	10.5	8.8	12.2	9.9	10.9	11.0	10.60	6.46
B	20.0	20.9	21.2	20.1	22.2	22.0	20.9	21.5	21.12	8.06
C	15.0	14.1	14.8	14.4	14.9	13.1	14.2	13.1	14.20	6.86
a	25.2	23.6	24.2	23.9	23.6	22.8	23.8	22.0	23.64	8.21
b	25.0	25.0	25.3	25.2	24.2	24.6	24.7	25.9	24.99	8.39
m	37.2	35.1	36.1	35.7	34.1	33.9	34.6	33.6	35.04	9.70
c	34.4	36.3	37.0	35.9	37.2	38.9	37.7	37.9	37.04	9.96
o	32.9	29.2	30.9	32.6	32.4	31.9	31.9	31.9	31.72	9.26

Wedge constant, 0.130 mag.

1900, October 13; 6-inch. Wedge II; seeing fine.

C	12.2	10.0	9.5	9.8	8.8	8.2	9.75	6.86
B	17.8	18.8	19.5	17.1	17.2	17.3	17.95	8.06
a	20.1	21.1	20.9	21.0	21.2	21.0	21.04	8.39
b	23.9	23.5	24.2	22.5	24.3	22.1	23.42	8.70
m	33.5	32.6	33.0	32.3	32.2	32.6	32.70	9.91
c	35.0	34.8	34.7	33.0	34.0	35.0	34.42	10.13
o	29.1	28.3	28.8	28.0	29.9	30.1	29.03	9.43
g	42.9	45.1	43.5	....	....	....	43.83	11.36
f	42.0	43.2	43.3	....	....	....	42.50	11.18
k	48.5	50.7	50.2	....	....	....	49.80	12.13
v	36.7	39.3	37.5	....	....	....	37.83	10.38

Wedge constant, 0.130 mag.

TABLE III—*Continued.*

1902, October 5; 12-inch. Wedge V; seeing good.

C_a	12.0	13.1	11.8	12.3	13.0	12.7	12.49	6.86
B_a	17.8	18.5	18.2	21.0	20.5	19.9	19.32	8.06
A_a	9.2	9.1	9.1	10.0	9.1	8.8	9.22	6.46
g	35.0	34.4	35.7	33.0	33.0	33.3	34.07	11.07
q	41.2	42.8	42.4	43.3	42.0	41.8	42.25	11.97
k	40.7	40.0	40.9	38.5	38.3	38.9	39.55	11.68
n	43.1	44.9	44.0	41.3	40.5	40.3	42.35	11.98
b	10.6	10.3	11.0	11.2	11.8	11.2	11.02	8.54
a	8.8	9.8	8.3	8.8	8.7	7.8	8.70	8.28
v	49.9	50.2	49.8	49.1	48.7	48.0	49.29	12.75

Wedge constant, 0.110 mag.

RESULTING MAGNITUDES FROM MEASURES WITH 6- AND 12-INCH.

Star	1900 Aug. 4	1900 Oct. 13	1902 Oct. 5	Mean
a	8.21	8.39	8.28	8.29
b	8.39	8.70	8.54	8.54
o	9.26	9.43	.....	9.35
m	9.70	9.91	.....	9.81
c	9.96	10.13	.....	10.05
f	....	11.18	.....	11.18
g	....	11.36	11.07	11.22

1900, October 17; 12-inch. Wedge II; seeing fair.

Star	Scale Readings						Mean Scale Reading	Magnitude
	First			Second				
b	20.4	19.6	19.9	20.9	20.5	19.5	20.14	8.54
m	31.7	29.3	29.0	32.7	30.0	30.6	30.55	9.81
c	33.1	33.9	32.2	35.0	34.1	35.0	33.98	10.05
o	25.0	25.1	24.7	24.8	23.8	25.0	24.73	9.35
n	48.9	49.6	47.8	51.3	51.0	49.8	49.90	12.37
k	45.5	46.7	46.3	46.8	46.2	43.8	45.89	11.83
u	61.5	59.8	62.1	....	....	....	61.13	13.83
v	35.7	35.4	35.1	....	....	....	35.40	10.48

Wedge constant, 0.130 mag.

TABLE III—*Continued.*

1902, May 14; 12-inch. Wedge V; seeing good.

a	14.5	15.2	15.1	15.0	15.2	15.7	15.12	8.29
b	18.8	18.8	18.9	17.5	17.7	17.7	18.23	8.54
m	29.9	30.1	30.1	29.0	29.2	29.9	29.70	9.81
c	35.3	35.4	36.3	35.0	35.0	34.2	35.20	10.05
n	52.8	53.4	52.8	51.9	52.5	52.3	52.62	12.26
k	50.7	51.2	50.1	49.2	49.5	50.1	50.14	11.98

Wedge constant, 0.110 mag.

RESULTING MAGNITUDES FROM MEASURES WITH 6- AND 12-INCH.

Star	1900 Oct. 13	1902 Oct. 5	1900 Oct. 17	1902 May 14	Mean
n	.....	11.98	12.37	12.26	12.20
k	12.13	11.68	11.83	11.98	11.91

1900, July 20; 40-inch. Wedge II; seeing very fine.

Star	Scale Readings								Mean Scale Reading	Magnitude
	First				Second					
n	24.1	24.5	23.0	23.0	22.0	24.1	24.0	23.0	23.47	12.20
g	18.5	19.0	20.7	19.8	....	....	....	....	19.50	11.22
k	26.9	24.3	24.0	25.0	....	....	....	....	25.05	11.91
u	31.1	32.2	33.2	35.0	....	....	....	....	32.88	13.11
z	46.0	46.7	47.5	47.9	....	....	....	....	47.05	14.95
δ	57.1	52.0	52.2	51.2	54.5	....	....	....	53.40	15.69
η	36.4	34.0	35.1	34.0	....	....	....	....	34.88	13.37

Wedge constant, 0.130 mag.

1900, October 17, 40-inch. Wedge II; seeing poor.

n	32.5	32.9	34.0	31.4	28.1	30.9	31.63	12.20
k	28.0	30.0	30.9	31.0	30.0	30.9	30.13	11.91
η	41.6	41.8	40.5	39.7	39.6	40.8	41.30	13.32
δ	53.0	54.8	55.2	61.0	59.6	58.5	57.02	15.51
z	51.3	51.4	51.5	47.5	48.9	50.9	50.25	14.57
u	38.1	39.2	38.9	38.5	37.3	39.6	38.60	13.05
v	11.0	15.0	15.0	....	....	....	13.67±	9.8 ±

Wedge constant, 0.130 mag.

TABLE III—*Continued.*

RESULTING MAGNITUDES. FROM MEASURES WITH 40-INCH.

Star	1900 Oct. 17	1900 July 20	Mean
u	13.05	13.11	13.08
η	13.32	13.37	13.35
z	14.57	14.95	14.76
δ	15.51	15.69	15.60

No correction for differing zenith distance has been applied to these measures, since even in case of the most distant comparison stars, *A*, *B*, and *C*, the distance would differ by less than half a degree, giving a correction of less than 0.01 mag. at the altitude at which the measures were made. For the fainter standards the correction would be still smaller.

MAGNITUDE CURVE.

Fig. I is platted with the photometric magnitudes as abscissæ and the positions in the light-scale as ordinates, giving the

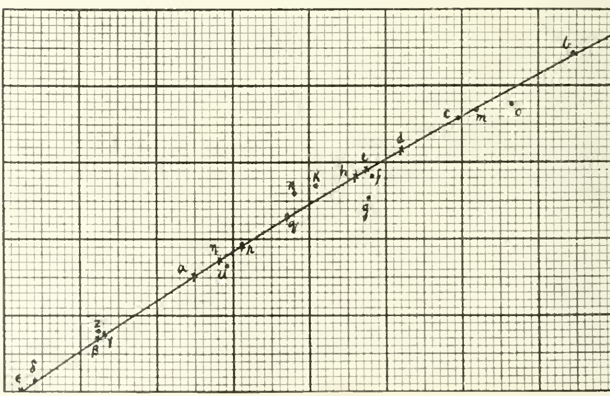


FIG. 1.

points indicated by the round dots. Through these the smooth curve was drawn and on this curve the stars not measured with the photometer were entered from their positions in the light scale, the places being indicated by crosses. The average distance between the platted points and the curve is 0.11 magnitude, the greatest residual being for the star *g*, 0.30. This depends on the measures of only two nights. Omitting this star, the mean residual is reduced to 0.09. The mean value of one step is 0.16 magnitude, and is essentially the same with the different apertures.

VISUAL OBSERVATIONS OF THE VARIABLE.

The visual comparisons by Argelander's method, and reductions to magnitudes, are given in Table IV, which should be printed as a double-page table, the comparisons on the left and the corresponding reductions on the right, For convenience the current number is repeated on the right-hand page. When the variable was invisible, the limit of vision was estimated from the faintest comparison star seen, and placed in the mean-step column, preceded by the inequality sign, which is read "fainter than" the quantity following. The limit of vision from these observations is 13.24 for the 6-inch, and 14.76 (perhaps, the record is "1900, April 4, 12-inch, power 275, *z* glimpsed) for the 12-inch.

THE LIGHT-CURVE.

In Fig. 2 the observed magnitudes are platted, showing the star's variations from 1896 to 1903. The magnitudes at maximum vary from 9.0 to 10.5, while only one minimum, that of June 4, 1900, is well enough observed to determine the limit, 15.4 magnitude. A single observation near the following minimum, July 9, 1901, at 15.0 magnitude, confirms the first, as far as it goes.

TABLE IV.

6871 V Lyrae.

Comparisons of the Variable by Argelander's Method.

No.	Date: Month	Day	Hour	Julian Day	Ocular	Aperture	Comparisons
	1896		C. S. T.	G. M. T.			
1	Sept.	22		3825	80	6	c 2 v, v 4 d
2		30	7	3833.5	40	6	c 1 v, v 4 d
3	Oct.	6	7	3839.5	40	6	c 2 v, v 3-4 d
4		13	7	3846.5	40	6	c 0-1 v, v 6 d, m 2 v
5		21	8	3854.6	40	6	v 2 c, m 1 v, o 3 v
6	Nov.	4	7	3868.5	40	6	m 4-5 v, c 2 v, v 4 d
7		12	7	3876.5	40	6	v d
					150	6	c 4 v, v 2 d, v 6 g
8		26	7	3890.5	150	6	d 1 v, v 5 e, v 1 h, v 2 g
					150	6	k 2 n, n 3 q, q 2 r
9	Dec.	9		3903	150	6	e 1 v, v g, v 3 4 n, h 1 v
10		23	6	3917.5	150	6	v 1-2 e, v g, v 3 n
	1897						
11	Jan.	9	6	3934.5	80	6	v not seen
12	May	25	9	4070.6	150	6	v not seen
13	July	1	9	4107.6	150	6	v not seen
14		26	9	4132.6	150	6	v not seen
15	Aug.	27	8	4164.6	150	6	q 2 v, v 0-1 r
16	Sept	16	7	4184.5	150	6	g 1 v, v e, v 6 k, k 3 q
17		20		4188	150	6	d 1-2 v, v 2-3 g, v 6-8 q
18		25		4193	40	6	v h, v 0-1 g, f 1 v
					150	6	v 2 g, v 1 f, d v
19	Oct.	1	7	4199.5	40	6	c 3-4 v, d 1 v, v 4 g
					150	6	c 4 v, d 1 v, v 4 g, v 3 f
20		7	7	4205.5	40	6	v f, d 1-2 v
					150	6	v 3 f, v 1-2 d, c 6 v
21		14	6	4212.5	150	6	v 4-5 f, v 2 d, c 4 v
22		23	7	4221.5	150	6	v 4 f, v d, c 5 v
23		29	6	4227.5	40	6	v 1-2 d, c 3 v
24	Nov.	3	7	4232.5	40	6	v 2 d, c 4 v
25		17	7	4246.5	150	6	c 4-5 v, v 1 d
26		20	6	4249.5	40, 150	6	d 4 v, v 1 g, v f
	1898						
27	June	25	10	4466.7	80	12	v not seen
28	July	7	10	4478.7		12	v not seen
29		20	10	4491.7		12	v not seen
30	Aug.	8	10	4510.7	80	12	v not seen
31		18	11	4520.8	80	12	v 1 u, q 10 v
					175	12	v u, r 2 v
32	Sept.	2	8	4535.6	150	6	v 3-4 f, v 3 e, d 3 v
33	Sept.	20	8	4553.6	40	6	v 1 c, m 0-1 v
34	Oct.	5	7	4568.5	40	6	v 2 m, v 4 c, b 6-7 v
35		28	7	4591.5	150	6	v 1-2 c, v m
36		31	7	4594.5	40	6	v 3 c, v 0-1 m
37	Nov.	7	6	4601.5	40	6	m 1 v, v c, v 8 c

TABLE IV.

6871 V Lyrae.

Reduction of Observations.

No.	Details in Steps	Means: Steps	Means: Mag.	Seeing	Remarks
1	33.9, 35.8	34.9	10.24	Moon, good	
2	34.9, 35.8	35.4	10.17	Moon, good	
3	33.9, 35.3	34.6	10.15	good	
4	34.4, 37.8, 35.0	35.7	10.10	Moon, fair	
5	37.9, 36.0, 34.6	36.2	10.02	good	
6	32.5, 33.9, 35.8	34.1	10.39	fair	
7	31.8 31.9, 33.8, 31.4	32.2	10.75	good, Moon	passing clouds
8	27.4 30.8, 34.2, 29.4	30.5	11.01	good	limit r
9	28.2, 25.4, 29.5, 27.4	27.6	11.52	fair, Moon	
10	30.7, 25.4, 29.0	25.8	11.81	good	
11		< 27.0	< 11.6	low, Moon	limit k
12		< 19.0	< 12.9	good	k, n, q, and r seen
13		< 19.5	< 12.9	good to fair	limit 3.4 < q
14		< 19.5	< 12.9	good to fair	limit 3.4 < q
15	21.0, 19.5	20.3	12.74	good	
16	24.4, 29.2, 33.0	28.8	11.32	fair	
17	30.3, 27.9, 30.0	29.1	11.29	good	
18	28.4, 25.9, 27.1 27.4, 29.1, 31.8	28.3	11.40	good	
19	32.4, 30.8, 29.4 31.9, 30.8, 29.4, 31.1	30.8	10.97	fair	
20	28.1, 29.3 31.1, 33.3, 29.9	30.3	11.07	Moon	
21	32.6, 32.8, 31.9	32.4	10.70	good	
22	32.1, 31.8, 30.9	31.6	10.83	good	
23	33.3, 32.9	33.1	10.57	good	
24	33.8, 31.9	33.4	10.52	good, Moon	
25	31.4, 32.8	32.1	10.76	fair to good	
26	27.8, 26.4, 28.1	27.4	11.55	good	
27		< 15	< 13.6		limit 8 < q
28		< 16	< 13.4	fine	limit t
29		< 16	< 13.4		limit 3 < s and t
30			< 14	good	limit 2 mag. < q
31	17.4, 13.0 16.4, 17.0	16.1	13.38	good	
32	31.6, 32.2, 28.8	30.9	10.95	Moon, good	
33	36.9, 36.5	36.7	9.95	fair	
34	39.0, 39.9, 37.8	38.9	9.50	good	
35	37.4, 37.0	37.2	9.81	Moon, poor	
36	38.9, 37.5	38.2	9.62	Moon, good	
37	36.0, 35.9, 37.2	37.4	9.79	good	

TABLE IV — *Continued.*

No.	Date: Month	Day	Hour	Julian Day	Ocular	Aperture	Comparisons
	1898		C. S. T.	G. M. T.			
38	Nov.	12	6	4606.5	40	6	m v, v 1 c
39		19	6	4613.5	40	6	m 3 v, c 2 v, v 1 d
					150	6	c 4 v, v 1 d
40		30	6	4624.5	40	6	m 5 v, c 5 v, v d
					150	6	c 4 v, v 1 d
41	Dec.	7	7	4631.5	150	6	c 4 v, v d, v 3 e, v g, v 6 k
42		13	6	4637.5	150	6	d 5 v, v 1 e, v 4 g
43		30	6	4654.5	150	6	g 2-3 v, v 1-2 q
	1899						
44	Jan.	8	6	4663.5	150	6	e 4 v, v f, v 2 g
45		14	6	4669.5	150	6	f 2 v ±
46	Feb.	15	17	4702.0	200	6	q 4 v, v 1-2 r
47	Mar.	22	16	4736.9		6	v not seen
48	Apr.	16	15	4761.9	150	6	v or u suspected
49	May	1	9	4776.6	200	6	v not seen
50		29	9	4804.6	200	6	v not seen'
51	June	7	9	4813.6	150	6	v not seen
52		24	9	4830.6	150	6	v not seen
53		26	9	4832.6	150	6	v not seen
54	July	6	10	4842.7	150	6	v not seen
55		11	9	4847.6	200	6	v not seen
56		29	9	4865.6	150	6	v not seen
57	Aug.	9	9	4876.6	150	6	r 1-2 v, v 1-2 u
58		12	9	4879.6	150	6	r 2 v, v 1 u
59		22	8	4889.6	150	6	q 2-3 v, v 0-1 r
60		30	9	4897.6	150	6	q 0-1 v, v 4 r
61	Sept.	4	8	4902.6		6	g 2 v, v 3 q
62		11	8	4909.6	150	6	v 2 g, d 2-3 v
63		26	8	4924.6	40	6	v 2 c, v 3-4 m, o 4 v
64	Oct.	6	7	4934.5	40	6	b 4 v, v 4 m, v 3-4 c, v 1 x, o 1 v
65		21	7	4949.5	40	6	b 1-2 v, o 1 v, v 5 m, v 4 x
66		28	7	4956.5	40	6	b 4-5 v, o 0-1 v, v 2 m
67	Nov.	4	6	4963.5	40	6	o 2-3 v, v 2 m
68		15	6	4974.5	40	6	v m, v 1 c, o 4-5 v
69		22	6	4981.5	40	6	o 5 v, m 3 v, c 1-2 v, v 3 d
70		27	7	4986.5	40	6	o 6-8 v, c 3-4 v, m 3 v, v d
					150	6	c 2-3 v, v 2-3 d
71		5	6	4994.5	40	6	c 3 v, v 2 d
					150	6	c v, v 3 d
72	Dec.	19	6	5008.5	150	6	d 2 v, v 4 e, v 2 g
	1900						
73	Jan.	1	6	5021.5	150	6	g 1 v, v 1 k, v 3 q
74	Feb.	22	17	5074.0	350	40	q 5-6 v, v 3 u
75	Mar.	22	15	5101.9	350	40	v u
76	Apr.	4	15	5114.9	275	12	v 4 u ±
77		6	16	5116.9	350	40	u 6 v, v 3-4 z, zx
78	May	2	15	5142.6	350	40	x 4 v, z 3 v, v 2 δ, v 4 ε
79		11	13	5151.8	350	40	z 5 v, v 3 δ
80		24	10	5164.7	275	12	v not seen

TABLE IV — *Continued.*

No.	Details in Steps	Means: Steps	Means: Mag.	Seeing	Remarks
38	37.0, 36.9	37.0	9.85	good	
39	34.0, 33.9, 32.8 33.9, 32.8	33.5	10.50	Moon, good	
40	32.0, 30.9, 31.8 31.9, 32.8	30.2	11.09	good	
41	31.9, 31.8, 32.2, 25.4, 33.0	30.9	10.96	good	
42	26.8, 30.2, 29.4	28.8	11.30	good	
43	22.9, 24.5	23.7	12.19	good	
44	25.2, 28.1, 27.4	26.9	11.62	good	
45		26	11.8	difficult	
46	19.0, 20.5	19.8	12.81	good	
47		< 19	13.0		limit *r*
48		< 23	12.3		limit *q*
49		< 23	12.3	low, good	
50		< 19	13.0	good, low	
51		< 19	13.0	good, low	
52		< 19	13.0	fair	
53		< 19	13.0	good	
54		< 19	13.0	good	
55		< 16	13.4	good	
56		< 19	13.0	good	
57	17.5, 17.9	17.7	13.15	good	
58	17.0, 17.4	17.2	13.03	good	limit *u*
59	20.5, 19.5	20.0	12.80	fair, Moon	limit *v*
60	22.5, 23.0	22.8	12.31	fair	limit *r*
61	23.4, 26.0	24.7	12.00		limit 1 < *u*
62	27.4, 28.3	27.9	11.48	Moon, good	
63	37.9, 40.5, 33.6	37.7	9.73	good	
64	40.3, 41.0, 39.4, 36.6, 40.1	39.5	9.37	good	
65	42.8, 36.6, 42.0, 43.1	41.1	9.07	good	
66	39.8, 37.1, 39.0	38.6	9.55	good	
67	35.1, 39.0, 40.9	38.3	9.61	good	
68	37.0, 36.9, 33.1	35.7	10.12	good	
69	32.6, 34.0, 34.4, 34.8	33.9	10.42	good	
70	30.6, 32.4, 34.0, 31.8 33.4, 34.3	32.8	10.61		
71	32.9, 33.8 35.9, 34.8	34.4	10.33	good	
72	29.8, 33.2, 27.4	30.1	11.08	good	
73	24.4, 28.0, 26.0	26.1	11.78	good	
74	17.5, 19.4	18.5	13.00	Moon, poor	
75		16.4	13.34	Moon, fair	
76		12.4	13.97	good	*z* glimpsed
77	10.4, 11.5	11.1	14.18	haze	
78	3.7, 5.0, 3.5, 4.0	4.1	15.18	good	
79	3.0, 4.5	3.8	15.21	fair, Moon	
80		< 11	14.2	fair to good	limit 5 < *u*

TABLE IV—*Continued.*

No.	Date				Ocular	Aperture	Comparisions
	Month	Day	Hour	Julian Day			
	1900		C. S. T.	G. M. T.			
81	May	29	12	5169.8	237	40	u 5–6 z, z 5 v, v δ, v 4 ε
82	June	6	10	5177.7	350	40	v not seen
83		28	10	5199.7	237	40	z 4–5 v, v 5 ε, v 3 δ
84	July	20	10	5221.7	237	40	z 1 v, x 2–3 v, v 5 δ
85	Aug.	2	9	5234.6	460	40	v not seen
86		16	9	5248.6	350	40	z 1 v, v 1 x, v 4 ε
					350	40	z v, v 4 δ, v 6–8 ε
87		29	12	5261.8	460	40	u 6 v, α 4 v, v 4 z, v 3 x
88	Oct.	4	9	5297.6	237	40	v 5–6 k, v 3 g, v 3 e, d 1 v
89		10	7	5303.5	150	6	v 4 e, c 4 v
90		13	7	5306.5		6	
91		17	8			12	
92		26	6	5319.5	150	6	v 6 k, v 3 g, v 4 e, c 2 v
93	Nov.	13	6	5337.5	40	6	m 4–5 v, c 3 v, v 5 f, v 6 g
94		29	7	5353.5	40	6	m 5 v, c 5 v, v d, v 2 g
95	Dec.	11	6	5365.5	40	6	m 8 v, d 4 v, v 2 g
					150	6	v 1 g, v 8–10 k
96		29	8	5383.6	150	6	g 3 v, v 2–3 k
	1901						
97	July	9	9	5575.6	350	40	z 5 v, v x
98	Nov.	12		5701	350	40	v > n or k
	1902						
99	Mar.	5	14	5814.8	237	40	u 2 v, v α, v 6–7 z, g 4 e
100		28	15	5837.9	237	40	z 2–3 v
101	May	14	13	5884.8		12	v < n or k
102	Oct.	5	10	6028.7		12	
						12	v 4–5 u
103	Nov.	30	7	6084.5		24	
104	Dec.	26	7	6110.5	237	40	v d ±
	1903						
105	Mar.	27	16	6201.9	237	40	n 6–8 v, v 5 η, v 3–4 u
106	Apr.	3	16	6208.9	237	40	u 1–2 v

In Chandler's *Third Catalogue* the provisional elements of maximum from observations of 1893–1895, are given as—

$$\text{Maximum} = 1893 \text{ August } 24 \text{ (J. D. } 2412700) + 377\ E.$$

These elements fairly represent the above maxima, but the agreement is somewhat improved by placing the zero epoch thirteen days earlier, giving the elements—

$$\text{Maximum} = 1893 \text{ August } 11 \text{ (J. D. } 2412687) + 377\ E.$$

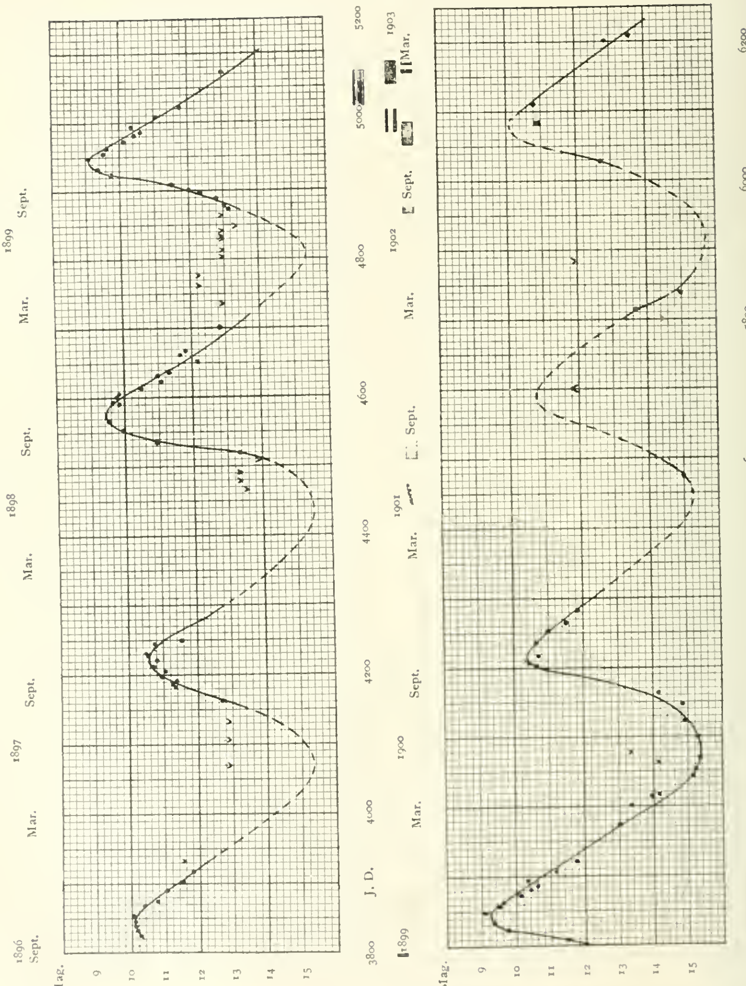
The agreement with these elements is shown by the following table:

TABLE IV — *Continued*

	Details in Steps	Means: Steps	Means: Mag.	Seeing	Remarks
81	3.0, 1.5, 4.0	2.8	15.37	fair to good	
82		<16	13.4	haze	limit *u*
83	3.5, 4.0, 4.5	4.0	15.20	good	
84	7.0, 5.2, 6.5	6.2	14.91	fine	
85					
86	7.0, 8.7, 4.0 8.0, 5.5, 7.0	6.7	14.86	fair better	
87	10.4, 11.3, 12.0, 10.7	11.1	14.17	good	
88	32.5, 28.4, 32.2, 30.8	31.0	10.94		*x* and *z* seen
89	33.2, 31.9	32.6	10.68	good	
90			10.45	good	photom., 6-in.
91			10.31	good	photom., 12-in.
92	33.0, 28.4, 33.2, 33.9	32.1	10.75	good	
93	32.5, 32.9, 33.1, 31.4	32.5	10.69	good	
94	32.0, 30.9, 31.8, 27.4	30.5	11.00	Moon	
95	29.0, 27.9, 27.4 26.4 (36.0)	27.7	11.52	good	
96	22.4, 29.0	25.7	11.83	Moon	
97	3.0, 7.7	5.4	15.00	fair	*v* & *x* near limit
98		<26	<11.8		
99	14.4, 14.5	14.5	13.68	fair	
100		5.5	15.00	Moon	
101					*v* is < *n* or *k*
102		20.9	<12 12.76		photometer
103			10.9		photograph
104		31.8	10.08	good	
105	19.0, 19.9, 19.9	19.6	12.87	good	
106		14.9	13.6	clouds	

Maxima: Epoch	Cal.	Obs.		O-C.	Minima: Epoch	Cal.	Obs.		O-C.
	J. D.	J. D.	Cal.			J. D.	J. D.	Cal.	
3	3818	3845	1896, Oct. 12	−27	3	3677	....	....	...
4	4195	4220	1897, Oct. 22	−25	4	4044	4070	1897, May 25	−26
5	4572	4575	1898, Oct. 12	− 3	5	4421	4433	1898, May 23	12
6	4949	4940	1899, Oct. 12	− 9	6	4798	4808	1899, June 2	−10
7	5326	5315	1900, Oct. 22	− 9	7	5175	5175	1900, June 4	0
8	5703	....	....	....	8	5552	5545	1901, June 9	− 7
9	6080	6080	1902, Nov. 26	0	9	5929	....	....	....

Fig. 2.



Light Curve of *V Lyrae*.

The large residuals of epochs 3 and 4 require mention. The observations began too late to fix maximum 3 with precision. It doubtless took place about twenty days earlier; in fact, the calculated date, J. D. 3818, is the mean of the dates given by Yendell and H. M. Parkhurst (Max. A). For epoch 4 the curve at maximum is not of the usual shape, the difference accounting for something like twenty days, and in the right direction to improve the agreement with the ephemeris. The residuals for the remaining maxima are within the errors of observations.

I wish to acknowledge the assistance rendered, as in the work on *X Cephei*, to Miss Bloodgood, and Messrs. Ellerman and Sullivan.

YERKES OBSERVATORY,
April 1903

PECULIARITIES AND CHANGES OF FRAUNHOFER LINES INTERPRETED AS CONSEQUENCES OF ANOMALOUS DISPERSION OF SUNLIGHT IN THE CORONA.[1]

By W. H. Julius.

Attention has been drawn to several variable peculiarities of Fraunhofer lines, mainly by Jewell's investigations on the coincidence of solar and metallic lines.[2] Here we do not mean the irregularities occurring in the spectrum of spots or of faculæ, which relate to disturbances in comparatively small parts of the Sun, but abnormalities shown by average sunlight, as observed when the slit is illuminated by a long strip of an imperfectly focused solar image. In that case, according to Doppler's principle, we may, of course, expect displacements of the lines in consequence of the Sun's rotation, the rotation of the Earth, and the change in the distance between Sun and Earth caused by the eccentricity of the Earth's orbit. But even when all these influences have been allowed for, some irregularities still remain.

Indeed, Jewell has observed that while some Fraunhofer lines exactly coincide with the emission lines in the arc spectrum of elements, others do not, and that the displacements are unequal both for lines of different elements and for the various lines of one and the same element. Moreover, the shifting of certain lines on one set of photographic plates was sometimes found to be different from that on a set of plates taken at another time. With several lines the intensity also appeared to be variable.

Jewell explains these phenomena on certain hypotheses as to

[1] Communicated to the Royal Academy of Sciences, Amsterdam, at the meeting of February 28, 1903.

[2] L. E. Jewell, "The Coincidence of Solar and Metallic Lines: A Study of the Appearance of Lines in the Spectra of the Electric Arc and the Sun," Astrophysical Journal, 3, 89–113, 1896; "Spectroscopic Notes: Absolute Wave-Lengths, Spectroscopic Determinations of Motions in the Line of Sight, and Other Related Subjects," *ibid.*, 11, 234–240, 1900.

density, pressure, and temperature of the absorbing and emitting gases in the different layers of the solar atmosphere, and by variable ascending and descending velocities of matter.

HALE'S ABNORMAL SOLAR SPECTRUM.

Much greater irregularities than those mentioned are found in an "abnormal" solar spectrum, lately described by G. E. Hale.[1]

This highly remarkable spectrum was accidentally photographed as long ago as February, 1894, in a series of exposures, made with the sole intention of investigating the peculiarities of the grating. Only a few months later it was discovered that a very extraordinary phenomenon had been photographed. Hale hesitated to publish this accidental discovery. Copies of the plate were sent to several spectroscopists for examination, with the request that an explanation referring the phenomenon to some origin other than solar be supplied, if possible. As no such explanation was forthcoming, the spectra were very carefully measured and described.

On one and the same plate twelve exposures had been successfully made in the third-order spectrum of a plane grating. A solar image 51 mm. in diameter was so adjusted that the image of a spot fell exactly on the slit. The length of the slit (6.5 mm) corresponded to about one-eighth of the Sun's diameter.

The first exposures show the normal spectrum without any considerable changes. Then came the disturbance which culminated in the eighth spectrum, and in the following four decreased rapidly. Hale gives reproductions of four spectra, each of them extending from λ 3812 to λ 4132. No. 1 was taken before the disturbance occurred; No. 2 is the most abnormal spectrum; No. 3 is called by Hale the "intermediate" spectrum; it was obtained a few moments after the abnormal one; No. 4 shows once more the normal solar spectrum, as it was photographed at another time on another plate. Nos. 1, 2, and 3

[1] "Solar Research at the Yerkes Observatory," ASTROPHYSICAL JOURNAL, 16, 211-233, 1902.

show a dark band throughout the whole spectrum, corresponding to the Sun-spot which had been focused on the slit.

The most prominent features of the abnormal spectrum are:

1. The band due to the spot appears much fainter than in the spectra photographed before and after the disturbance.

2. In the case of several Fraunhofer lines the intensity or the width is *greatly diminished*. This is most conspicuous with the broad, dark calcium bands H and K and with the hydrogen line $H\delta$, these being almost totally absent in the abnormal spectrum.

3. Other Fraunhofer lines, on the contrary, appear *uncommonly strengthened*.

4. Many lines are more or less displaced.

The same peculiarities are noticed, though generally in a lesser degree, in the intermediate spectrum, so that the latter, in fact, forms a link between the abnormal and the normal spectrum.

This marvelously complicated disturbance was not confined to light coming from a comparatively small part of the solar disk, for instance from the immediate surroundings of a spot; on the contrary, it extended almost equally over the whole width of the spectrum and was therefore nearly the same for all the light which came from a very great area of the Sun.

The moments of the twelve exposures and the exact date had not been recorded; but there was sufficient evidence that the whole process of the disturbance lasted only a very short time. Hale calls the phenomenon "a remarkable disturbance of the reversing layer." But is it not almost impossible to imagine a rather thin layer in the solar atmosphere undergoing suddenly and simultaneously over a great part of the Sun a change so great as to make its absorbing and radiating power in some parts of the spectrum for a while nearly unrecognizable?

It occurred to me, therefore, that the origin of the phenomenon should be looked for somewhere on the path of the light between the Sun and the Earth. If on this path there be media causing anomalous dispersion, the beam must show an altered composition.

As I formerly indicated,[1] the properties of the chromospheric light may be derived from the supposition that this light has been separated from the photospheric light by anomalous dispersion. According to this hypothesis the spectrum of the chromosphere informs us which are the kinds of light that may follow rather strongly curved paths in the solar atmosphere. So the idea suggested itself that the same waves might play a striking part in Hale's abnormal spectrum.

In order to investigate the question as impartially as possible, I marked (before consulting Hale's table or a table of chromospheric lines) on the reproductions of the spectra in the ASTROPHYSICAL JOURNAL a number of lines which struck me as being *weakened* in the abnormal spectrum. By means of George Higgs' photographic atlas of the normal solar spectrum the wave-lengths of the selected lines were easily read. They are to be found in the first column of Table I.

The second, third, and fourth columns show the intensities of these lines in the normal, the intermediate, and the abnormal spectrum as given by Hale (for the normal spectrum from Rowland's tables, for the other two from estimates by Mr. Adams). Hale remarks that the intensities of the lines were estimated independently for the two disturbed spectra.[2] The fifth column indicates the intensities of corresponding chromospheric lines as found by Lockyer in the spectrum secured at Visiadrug[3] during the 1898 eclipse; the sixth column shows the absorbing substances.

Table II has been prepared in a similar way; here we find the lines, which on the reproduction appeared to be *strengthened* in the abnormal spectrum.

[1] ASTROPHYSICAL JOURNAL, **12**, 185–200; **15**, 28–37; *Physikalische Zeitschrift*, **4**, 85–90; 132–136.

[2] In selecting the lines that appeared weakened in the abnormal spectrum, I of course compared the three spectra together. That is why in my table some lines occur whose intensities as estimated by Mr. Adams are not comparatively low in the abnormal spectrum.

[3] LOCKYER, CHRISHOLM-BATTEN, AND PEDLER, "Total Eclipse of the Sun, January 22, 1898.—Observations at Visiadrug," *Phil. Trans.*, A, **197**, 151–227, 1901.

TABLE I.

LINES WHOSE INTENSITY IS LESS IN THE ABNORMAL THAN IN THE NORMAL SPECTRUM.

Wave-Length	Intensity				Elements	Remarks
	Normal (Rowland)	Intermediate (Hale)	Abnormal (Hale)	Chromosphere (Lockyer)		
3871.4				4	*C*	Not mentioned in Hale's list, but distinctly weakened in the abnormal spectrum on reproduction.
3872.6				4	*Fe*	
3874.09	4	9	..	2(?)	*Fe*	
3878.47	22	25	..	3–3	*Fe, Fe*	λ = 3878.15 and λ = 3878.72. Hale mentions *Fe, Mn.*
Hζ 3889.05	?	15	..	8	*H*	
3895.80	7	12	..	3	*Fe*	
3899.30	5	4	..	2	*V?*	
3903.09	10	12	..	2–3	*Fe*	
3905.66	12	20	..	2	*Cr, Si*	
3906.70	14	..	4	2	*Fe*	
3913.63	9	7	..	6	*Ti*	
3914.49	7	8	5*		*Ti*	* These intensities are very probably estimated too high when compared with the numbers in the second column. *Cf.* note 2 on p. 53.
3916.54	3	..	4*	3	*V*	
3920.41	10	10*	10*	3	*Fe*	
3923.05	12	12*	12*	3	*Fe*	
K 3933.82				10	*Ca*	
3944.16	15	15*	12*	5	*Al*	
3948.91	13	15	..	3	*Fe*	
3950.10	5	..	2	3	*Fe*	
3953.02	17	15	..		*Fe*, etc.	
3958.35	5	8	..	4	*Ti*	
3961.67	20	20	..	6	*Al*	
H 3968.63	(700)	7	7	10	*Ca*	
Hε 3970.18	7	8	..	10	*H*	
3977.89	6	8	..	2	*Fe*	
3986.90	6	8	..		....	
3998.78	4	4	4*	4	*Ti*	
4012.50	5	4	5*	5–6	*Ti*, etc.	
4033.22	7	12	3	3–4	*Mn, Fe*	
4034.64	6	10	..	3–4	*Mn, Fe*	
4045.98	30	30	5	7	*Fe*	
4063.76	20	20	..	6–7	*Fe*	
4071.91	15	15	15*	6	*Fe*	
4077.88	8	10	7*	10	*Sr*	
Hδ 4102.00	40	7	..	10	*H*	

The result is very striking. *Weakened lines correspond to chromospheric lines almost without exception; most of the strengthened lines, on the other hand, are not to be found in the spectrum of the chromosphere.*

Lockyer gives the strength of the chromospheric lines on a scale such that 10 indicates the strongest and 1 the faintest lines. If we take into account that in his list the greater part of

TABLE II.

LINES WHOSE INTENSITY IS GREATER IN THE ABNORMAL THAN IN THE NORMAL SPECTRUM.

Wave-Length	Intensity				Elements	Remarks
	Normal (Rowland)	Intermediate (Hale)	Abnormal (Hale)	Chromosphere (Lockyer)		
3921.86	4	..	20		*Zr, Mn*	
3927.77	..	..	25		?	
3930.45	8	15	28	3-4	*Fe*	
3937.39	..	..	10		?	
3940.25	..	7	12		?	
3950.50	2	..	13		*Y*	
3962.29	3	..	11		*Fe* ?	
3973.77	6	..	15	2 (?)	*NiZrFeCa*	
3981.92	4	13	30	6*	*Ti, Fe*	* In Humphreys' table of chromospheric lines (1901 eclipse) this line does *not* occur.
3992.97	3	4	10		*V, Cr*	
3996.80	..	..	9		?	
4013.90	8	12	15		*Ti, Fe*	
4014.67	5	9	20		*Fe*	
4023.38	..	..	10		?	
4033.77	2	3	15		*Mn*	
4040.79	3	6	20	4	*Fe*	
4044.09	5	20	15		*Fe*	

the lines bear the numbers 1 and 2, our table shows us that by merely observing the abnormal solar spectrum we have been able to pick out *strong* chromospheric lines. This cannot be chance. Undoubtedly both phenomena — the weakening of Fraunhofer lines in the abnormal spectrum and the origin of the chromospheric spectrum — are to be explained in close relationship with each other.

The *strengthening* of lines in the abnormal spectrum does not, on the contrary, seem to be so directly connected with the composition of the chromospheric spectrum.

If our view be correct that the chromospheric light has been separated by strong ray-curving from the "white" light emitted by deeper layers, those special radiations must, as a rule, show reduced intensity in the spectrum of the Sun's disk.[1] Fraun-

[1] It might be thought that the rays forming the chromospheric light need to be absent only from the spectrum of the *edge*, but not from that of the central portions of the Sun's disk. By a simple consideration following from a glance at Fig. 4 of my paper read in February, 1900 (ASTROPHYSICAL JOURNAL, 12. 191), we see, however, that the chromospheric light visible to us may very well, in part, have its origin even

hofer lines corresponding to chromospheric lines will therefore have a more or less darkened background in the ordinary solar spectrum. The rate of darkening at various distances from the center of an absorption line is, of course, connected with the shape of the dispersion curve near that line; whereas the average shading depends also (1) on the quantity of matter causing anomalous dispersion and (2) on the slopes and directions of the density gradients in the gases through which the light is transmitted, viz., on the Sun's "activity."[1]

We distinguish, therefore, a twofold origin of the dark lines in the solar spectrum, viz.: real *absorption* of those waves which exactly correspond to the periods of the media and *dispersion* of the strongly deviated[2] neighboring light.

The dispersion will be especially evident where extraordinary differences in the density of the medium occur; in this way the widening of most of the Fraunhofer lines in the spectra of spots may be accounted for.

Dispersed light has not, of course, vanished; the absence of certain rays in the spectrum of a spot is counterbalanced by the increased intensity of the same radiations in the light coming from the neighboring faculæ. Thus the distribution of the density in the solar gases may locally be such that a limited part of the disk seems to emit a considerable amount of rays with abnormally high or abnormally low refractive indices. In the spectrum of this part not only will the Fraunhofer lines appear

in points of the Sun which lie opposite to the Earth's direction. The chromospheric light reaching the Earth may proceed from *any* point of Schmidt's "critical sphere." For the greater part it is likely to come from the back half of the Sun. But then the half facing us furnishes the chromospheric light which travels to other regions of the universe, and this light, of course, is wanting in the spectrum of the disk. There is some reason for supposing that, on an average, more chromospheric light is sent forth in directions making great angles with the Sun's equator than to the equatorial regions, including the Earth's orbit.

[1] The possible influence of the general or regular ray-curving (on Schmidt's principle) on the appearance of the spectral lines has in the present paper been left out of consideration. If we were able to observe or to calculate the radii of the "critical spheres" for radiations undergoing anomalous refraction, it would be possible to estimate that influence; but as yet sufficient data are wanting.

[2] ASTROPHYSICAL JOURNAL, 12, 191, 1900.

narrower and fainter than usually, but we may even meet with lines contrasting *brightly* with their surroundings. These bright lines will not coincide with the corresponding absorption lines; their average wave-length will in general be greater or smaller than that of the absorbed light, for, according to the accidental distribution of the density, we shall find either the rays with high or those with low refractive indices most prominent in the beam.

The above considerations suggest an explanation of Hale's abnormal spectrum. In fact, the lines showing especially faint in this spectrum were exactly those which cause strong anomalous dispersion — witness the chromospheric spectrum. With H, K, $H\delta$, and some iron lines it is very evident that the abnormal faintness refers mainly to the broad dark shadings of the lines, *i. e.*, those parts whose darkness in the normal spectrum we attributed, not to absorption, but to dispersion. Moreover, the dark band due to the spot has nearly disappeared. This means that waves, which in normal circumstances are wanting in the spot spectrum on account of their strong dispersion, at the time of the disturbance had been gathered again into the beam, reaching the instrument. How all this may happen will become evident as soon as we shall be able to establish a plausible cause by which, within an angular space great enough to include a considerable part of the solar disk, the *strongly dispersed rays might be gathered again.*

It is not necessary to introduce a new hypothesis for the purpose. The same idea about the Sun's constitution which enabled us to explain the properties of the chromosphere and the prominences[1] furnishes us once more with the required data. Indeed, if (according to Schmidt's theory) the Sun is an unlimited mass of gas, surfaces of discontinuity must exist similar to those whose general feature has been determined by Emden[2] for a sharply outlined radiating and rotating Sun. These surfaces must extend to the remotest parts of the gaseous body — a conclusion in excellent harmony with the visible structure of the corona. For along the surfaces of discontinuity waves and whirls are formed; the core-lines of the vortices nearly coincide

[1] *Physikalische Zeitschrift*, 4, 85-90. [2] ASTROPHYSICAL JOURNAL, 15, 38-59, 1901.

with the generatrices of the surfaces of revolution, and in these cores the density is a minimum. This may account for the streaky appearance shown more or less distinctly in all good photographs and drawings of the corona.

This particular appearance may have another cause, however, but for what follows this is immaterial. We only assume that the density of the coronal matter varies in such a way as to correspond to the striped structure visible at the time of a total eclipse of the Sun.

A coronal streamer which at a given moment runs exactly in the direction of the Earth may be very roughly compared, then, to a bundle of glass tubes through which we are looking lengthwise. Such a structure will gather and conduct rays of various directions, entering it at one end. This takes place also if the parts with the greater and those with the smaller optical density do not alternate abruptly, but gradually.

In Fig. 1 the optical density of the matter may be represented by the compactness of the streaking. A ray for which the medium has a large positive refraction constant would, for instance, follow the path AA', curving round the denser parts of the structure; a ray BB', for which the medium possesses a large negative refraction constant, would move in a similar way through the more rarefied regions. On the other hand, the light CC' for which the constant exactly equals zero is not influenced by the fluctuations of the density; and if for some kind of light the refraction constant is very nearly zero, the ray would have to travel a long way almost parallel to the structure before its curving would be perceptible.

Now the corona sometimes shows exceedingly long, pointed streamers. We only have to suppose *that the Earth was exactly in the direction of such a streamer at the moment the abnormal spectrum was photographed;* then all the irregularities observed in this spectrum become clear. Light under normal circumstances absent from the solar spectrum through strong dispersion has been collected by the coronal streamer; hence the *weakening* of the Fraunhofer lines, especially, also, of those in the spectrum of the spot. As the abnormalities were caused by a peculiar

distribution of matter in the vast regions of the corona lying between the source of light and the Earth (and not by disturbances in a relatively thin "reversing layer"), they could appear in the same way over a great part of the Sun's disk. The *rarity* of the phenomenon is the result of the slight chance we have to take a photograph at the very moment on which an uncommonly long coronal streamer is projected exactly on the part of the Sun's disk illuminating the slit; finally, the *short duration* is a consequence of the Earth's orbital motion, and probably of the rotation of the corona.

Fig. 1.

As we have mentioned before, *no* chromospheric lines correspond, in general, to those lines appearing extraordinarily *strong* in the abnormal spectrum. How are we to account for the strengthening of these lines?

We might be tempted to think of absorption in the corona; for if it be true that a streamer was turned toward the Earth, the rays had to go an uncommonly long way through an absorbing medium. But on closer examination this idea is less probable.

The particles of the extremely rarefied coronal gases will hardly influence each other; their periods will, therefore, be almost absolutely constant, so as to cause very sharp, narrow absorption lines. Thus it is difficult to understand how an absorption line already present in the normal solar spectrum might be strengthened by the absorbing power of the corona. Further in studying Hale's table, we observe that many lines which are strong in the abnormal spectrum show a much smaller intensity in the intermediate spectrum (taken only a few moments later); while the reverse happens as well, viz., that lines are strong in the intermediate and very weak in the abnormal spectrum. This hardly fits in with the absorption hypothesis. Some lines showing this peculiarity are given in Table III:

TABLE III.

LINES WHOSE INTENSITY IS VERY DIFFERENT IN THE INTERMEDIATE AND THE ABNORMAL SPECTRUM.

Wave-Length	Intensity				Elements	Remarks
	Normal (Rowland)	Intermediate (Hale)	Abnormal (Hale)	Chromosphere (Lockyer)		
3905.66	12	20	. .	2	*Cr, Si*	
3905.81	21	. .	20		*Si*	
3921.71	9	14	. .		*Ti, La, Zr, Mn*	
3921.87	4	. .	20		*Zr, Mn*	
3950.33	. .	10	. .		?	
3950.51	2	. .	13		*Y*	
3972.30	2	12	. .		*Ni*	
3972.61	2	. .	12		?	
4005.86	3	25	5		?	
4057.39	4	. .	15	1–2	*Co, Fe*	
4957.66	7	10	. .		?	

In the chromospheric spectrum corresponding lines seem to be wanting (at λ 3905.66 and λ 4057.39 the faint chromospheric line may possibly belong to another element than the abnormally strengthened absorption line).

To arrive at a more satisfactory explanation of the strengthening phenomenon, we suppose that these absorption lines do indeed cause anomalous dispersion of neighboring waves, but in a very slight degree. Then, the refractive indices of the neighboring waves differing but little from unity, the direction of those rays will be perceptibly changed only after they have traveled a very long way through the corona and almost parallel to its structure lines. Whereas the strongly refracted rays, entering the coronal streamer in various directions, were obliged to follow the structure lines, curving about them, and so in a sense were concentrated on the Earth, it may happen with the very slightly curved rays we are now considering that they have been bent, for instance, only once over the whole length of the streamer and continue their way in a direction not meeting the observing station. The divergence of a beam consisting of these rays will have increased, the intensity diminished. Thus the resultant spreading of neighboring light causes the absorption

line to appear somewhat widened, and therefore strengthened. But obviously it must be possible, too, that after a short time, under the influence of another part of the corona, circumstances assist that slightly curved light to reach the observer. In that case the absorption line is weak again. (Similar alternations, of course, also occur with the more strongly refracted rays, and that in quicker succession; but this does not alter the fact of their *average* intensity appearing increased as long as the structure lines of the coronal streamer are turned toward the spectroscope. For a detailed discussion of this case see the note at the end of this paper.)

In both abnormal spectra a number of absorption lines are more or less displaced. Perhaps this is partly due to motion in the line of sight, but after the foregoing it will not be necessary to explain in detail that anomalous dispersion also can account for this phenomenon. Dissymmetric form of the dispersion curve as well as a peculiar distribution of the density of the coronal matter may unequally affect the intensity of the light on both sides of the absorption line, and thus bring about a seeming displacement of the line.

CERTAIN PECULIARITIES OF LINES IN THE NORMAL SOLAR SPECTRUM.

If we have been right in connecting the uncommonly great abnormalities in Hale's spectrum with a very particular position of the Earth with respect to the corona, it is to be expected that similar irregularities, though in less degree, will continually be found, as the sunlight always reaches us through the corona.

According to Jewell's above-mentioned investigations, this supposition proves to be well founded. Many solar lines have varying intensities and positions, so that Jewell deems them unfit for standards for very accurate determinations of wave-lengths. And these are for the greater part the most prominent lines of the spectrum, especially the shaded ones.[1]

Jewell emphasizes the fact that all distinctly shaded lines in the solar spectrum show to a greater or less degree the following

[1] ASTROPHYSICAL JOURNAL, 11, 236, 1900.

typical feature:[1] With a broad, shaded, moderately dark background a much darker central absorption line contrasts rather sharply (Fig. 2). Besides, the absorption curve often shows depressions close to the central line, as in Fig. 3, sometimes symmetrical, sometimes dissymmetrical. Jewell affirms that this is not an optical delusion, due to contrast, but a real phenomenon. He assumes, therefore, that the broad absorption band is produced in the lower portions of the solar atmosphere and under a great range of pressure; that in higher levels radiation prevails again, producing a rather wide emission line; and that finally in the highest parts, where the pressure is very much less, the sharp absorption line is produced. The position of this central absorption line with respect to the emission line is usually unsymmetrical, which is conspicuous in the case of H and K. The central line itself also varies somewhat in width upon different plates, and its maximum of intensity is not always in the middle of the line. The displacement of this central line in H and K varies in magnitude, but, so far as has been observed, always toward the red with respect to the emission line and the corresponding metallic line (in the arc). Jewell concludes that the absorbing calcium vapor descends all over the solar surface with a velocity sometimes amounting to about seventy-five miles per minute.

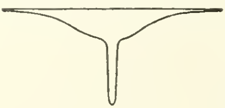


Fig. 2

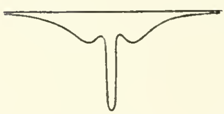


Fig. 3.

Upon the same plates showing strong dissymmetry in H and K the shaded lines of other elements (*Fe*, *Al*, *Mg*, *Si*) have been examined. The strongest iron lines and one aluminum line showed displacements of the same character as that observed in the case of H and K, but to a much smaller degree, and sometimes toward the violet, sometimes toward the red. Certain

[1] "Certain Peculiarities in the Appearance of Lines in the Solar Spectrum and Their Interpretation," *Ibid.*, 3, 1896.

shaded lines of *Mg* and *Si*, on the contrary, showed no evidence of a displacement, nor did the iron lines without considerable shading, the faint calcium line at λ 3949,056, and many other lines.

If we admit no other explanation of line-shifting and widening besides those based on Doppler's principle and on the effect of pressure and temperature, we arrive at very strange conclusions with regard to the condition of the elements in the solar atmosphere. Not less surprising, as noticed by Jewell,[1] is the small amount of the absorption in the shaded parts of the lines, when we consider the enormous depth of the solar atmosphere and the high pressure which must exist in the absorbing layers for them to produce a broad absorption band.

By making various suppositions concerning the condition of the gases in the solar atmosphere, Jewell succeeds in finding an interpretation of most of these astonishing facts. But it must be granted that his explanations include a greater number of arbitrary and mutually independent hypotheses than is the case with our explanations, founded as they are on selective ray-curving and readily deduced from that principle for each separate phenomenon, without introducing new suppositions.

Only the dark central lines of the Fraunhofer lines are to be ascribed, in our theory, to real absorption. Their shaded background of varying intensity we consider as an effect of anomalous dispersion of the not absorbed neighboring waves. This selective scattering will be strongest in those places where the density-gradients are relatively steep, viz., in whirls in the deeper regions of the gaseous body. But some of the widely dispersed rays may be gathered by the corona owing to its "tubular" structure, and be conducted along its greater or smaller streamers. This will especially apply to the most strongly refracted waves, whose position in the spectrum is very close to the real absorption lines ; thus pseudo-emission lines are produced in about the middle of the pseudo-absorption bands.[2]

[1] ASTROPHYSICAL JOURNAL, 3, 106, 1896.

[2] A most remarkable fact is that the shading of H, K, the iron line λ 3720.086 and of some other strong shaded lines is sometimes partially broken up in a series of faint

Probably Hale's abnormal spectrum has shown us a case where these seeming emission bands acquired an uncommon extent. We may therefore expect that a systematical investigation of solar spectra, photographed at different times, will afford all kinds of intermediate cases.

It would be desirable, for the moments when the photographs are taken, to know the form and position of the coronal streamers directed toward the Earth. At all events the actual phase of the Sun-spot period, with which the shape of the corona seems to be connected, should be taken into consideration ; and perhaps the simultaneous observation of the photospheric reticulation discovered by Janssen may procure some evidence concerning the position of coronal streamers, and thus contribute to our knowledge of their influence on the Fraunhofer spectrum.

UTRECHT, April 7, 1903.

nebulous lines, symmetrically situated about the central line (JEWELL, ASTROPHYSICAL Journal, 3, 108, 1896). It might have been predicted by our theory that we should meet with this phenomenon. Let us consider a beam of light of an exactly defined wave-length belonging to the shaded background of an absorption line. This beam leaves the deeper layers of the Sun with a certain divergence. As it passes along a "tube" of the corona, its divergence will alternately diminish and increase, and on reaching the Earth it shows in the spectrum an intensity depending on the divergence (or, perhaps, convergence) with which it has left the last traces of the corona. For a beam of light whose wave-length is only slightly nearer to that of the absorption line, the medium will have a considerably greater refraction-constant, so that the rays of this beam on their way through the corona may make part of a bend more than the former ones. The beam may therefore arrive with a quite different degree of divergence, and, consequently, of intensity. Thus, proceeding toward the absorption line from either side, we easily see that we must meet with a periodically changing intensity. Rays corresponding to the middle of one of the fringes so formed will have made one full bend more or less than the rays belonging to the middle of the next fringes.

If this interpretation be correct, the width and the number of fringes visible must prove to be variable. So far as I know, the observations made on this point are not numerous. I trust that the proposed views may serve to further the investigation of this interesting phenomenon.

THE WAVE-LENGTHS OF THE SILICON LINES λ4128 AND λ4131 AND OF THE CARBON LINE λ4267.

By J. Hartmann.

Supplementing my recent determination[1] of the wave-length of the magnesium line λ4481, I should like to communicate my measurements on three other lines which, like the magnesium line, occur as well-measurable lines in stellar spectra of the first type, although hitherto their very diffuse character has not permitted an accurate laboratory determination of their wave lengths.

The lines at λ4128 and λ4131 are among the strongest in the spark spectrum of silicon. I will here mention only the following of the earlier determinations of their wave-lengths:

Eder and Valenta (*Denkschriften d. Wiener Akad.*,[2] 60, 260, 1893)	4128.5	4131.5
Lockyer (*Proc. R. S.*, 61, 443, 1897)	4128.6	4131.4
Exner and Haschek (Astrophysical Journal, 12, 49, 1900)	4128.1	4131.0

It may readily be seen from the great difference in the above values how uncertain the wave-lengths of these lines have been hitherto on account of their diffuseness. Like the magnesium line, however, they may be obtained as perfectly sharp lines if the pressure of the luminous vapor is greatly reduced. In collaboration with Dr. Eberhard I have accordingly made with Spectrograph III a number of photographs of the spectrum of Geissler tubes containing silicon tetrafluoride under low pressure. Basing the determinations upon Kayser's wave-length of the three iron lines at λ4118.709, 4132.217,[3] and 4144.033, I obtained the following wave-length of the silicon lines:

Plate III: 528, 530	4128.205	4131.038
531	.201	.036
532	.207	.042
533	.203	.042
Mean	4128.204	4131.040

[1] *Physikalische Zeitschrift*, 4, 427, 1903.

[2] In the original the wave-length is given by mistake as 4126.5.

[3] Communicated by Professor Kayser in a letter.

The line at 4267 appears very strong and diffuse in the spark spectrum of carbon. The most important earlier determinations of its wave-length are as follows:

Angström and Thalén (*Nova Act. Soc. Upsala*, 1875) - - -	4266.0
Hartley and Adeney (*Phil. Trans.*, 1884)- - - - - -	4266.3
Eder and Valenta (*Denkschriften d. Wiener Akad.*, 1893) - -	4267.5
Deslandres (*Comptes rendus*, 1895) - - - - - - -	4267.0
Exner and Haschek (*Sitzungsberichte d. Wiener Akad.*, 1897) -	4267.10

This wave-length was accordingly hitherto quite inaccurate.

The line may be obtained sharp, if powerful spark discharges (with condenser and spark-gap) are passed through a Geissler tube containing a carbon compound at low pressure. In this way, using a cyanogen tube, I made the following two determinations of the wave-length of this line some time ago.

Plate III:	495 - - - - -	4267.304
	496 - - - -	.298
	Mean - - -	4267.301

This value is also referred to Kayser's standards.

Even in a vacuum the line always remains rather broad, and it gives the impression of duplicity, the components not being resolvable with the dispersion employed. The determination of the wave-length is therefore somewhat uncertain.

POTSDAM,
April 13, 1903.

SOME MISCELLANEOUS RADIAL VELOCITY DETERMINATIONS WITH THE BRUCE SPECTROGRAPH.

By WALTER S. ADAMS.

THE RADIAL VELOCITIES OF THE TWO COMPONENTS OF 61 *CYGNI.*

THE recent adaptation of the Bruce spectrograph for use with one prism has made possible a determination of the radial velocities of the two stars composing 61 *Cygni* which is of interest because of its important bearing upon the question of the physical connection and relative parallax of this celebrated pair. The spectrograms obtained are as follows:

61^1 *CYGNI.*

Plate	Date	No. of Lines	Velocity
B 387	1902, August 11	17	−63 km
DB^1 25	1903, May 17	13	61
DB^1 36	June 6	17	63
		Mean	−62 km

62^2 *CYGNI.*

Plate	Date	No. of Lines	Velocity
C 13	1903, January 9	9	−63 km
DB^1 10	May 7	13	65
DB^1 32	June 5	16	61
		Mean	−63 km

Plate B 387 was taken with the regular three-prism train and the 24-inch camera; plate C 13, with the same prism train and a 10.5-inch camera; the remainder of the plates, with a single prism and the 24-inch camera.

The degree of accuracy attained is probably somewhat higher in the case of 61^1. The error of the mean should, however, in neither case exceed about 3 km. A value of −54 km was obtained for 61^1 *Cygni* by Bélopolsky at Pulkowa in 1895, based

upon the measurement of five lines on each of two plates. This constitutes the only previously published determination for either of the stars.

The agreement of the radial velocities of the two components, when considered in connection with their similar proper motions, would seem to indicate that the stars are unquestionably physically connected. Their real motion in reference to the Sun, under the assumption of a parallax of 0″.4 and a proper motion of 5″.2, would be about 80 km, or in space, when corrected for the solar motion, about 64 km.

THE RADIAL VELOCITY OF ϵ *URSAE MAJORIS.*

The bright star ϵ *Ursae Majoris* was included in the list of stars whose radial velocities were determined by Vogel and Scheiner at Potsdam in 1889–90. The mean of their values is −30.4 km for the epoch 1889.39. A plate obtained with the Bruce spectrograph in April 1902 gave a value of −8 km. While the spectrum of this star is of the Ia2 type of Vogel's classification with faint and broad metallic lines whose accurate measurement is difficult, a discrepancy of this amount is too large to be accounted for in such a way, and is not encountered among other stars of the Potsdam list having similar spectra. Accordingly the star was placed upon the observing list, and the following plates were obtained:

Plate	Date	No. of Lines	Velocity
B 339	1902, April 30	10	−8 km
B 344	May 14	8	8
B 364	June 20	9	8
A 357	July 23	7	8
B 479	Dec. 31	12	11
B 496	1903, March 24	16	10
A 422	April 2	13	10
A 431	April 8	13	10
A 440	April 16	12	11
		Mean	−9.4 km

The accordance of these measures is entirely satisfactory for a star with this type of spectrum, and it would appear that no appreciable change has taken place in the star's radial velocity

during the interval of a year covered by the observations. The difference as compared with the Potsdam results consequently is not accounted for, but it seems quite possible that we may here be dealing with a spectroscopic binary of much longer period than any met with hitherto.

THE VARIABLE VELOCITY OF β *SCORPII* IN THE LINE OF SIGHT.

Four spectrograms give the following values of the radial velocity of this bright star:

Plate	Date	No. of Lines	Velocity
B 327	1902, April 16	7	− 9 km
A 439	1903, April 16	4	+19
DB[1] 29	June 5	5	−99
DB[1] 42	June 12	4	−97

The spectrum is of the *Orion* type, but all the lines are exceedingly broad, and the measures are uncertain to the extent of several kilometers. The use of low dispersion in photographing the spectrum of this star has proved of decided advantage, the gain in narrowness of the lines more than counteracting the effect of the reduced scale.

THE VARIABLE VELOCITY OF ϵ *HERCULIS* IN THE LINE OF SIGHT.

The spectrum of this star appears to be composite, and its variations will be made the subject of further investigation. Three spectrograms furnish the following velocity for the star which gives the stronger lines, the spectrum being of the Ia2 type:

Plate	Date	No. of Lines	Velocity
A 449	1903, April 30	4	−58 km
DB[1] 17	May 7	4	−43
DB[1] 33	June 6	8	−22

These measures are to be regarded as preliminary and may be changed considerably in a further discussion of the star's motion.

YERKES OBSERVATORY,
June 15, 1903.

Minor Contributions and Notes.

A PHOTOGRAPHIC MAP OF THE ENTIRE SKY.[1]

THE collection of photographs at the Harvard College Observatory contains, in addition to the plates taken with the larger instruments, numerous photographs taken with two small anastigmatic lenses, each having an aperture of one inch, and a focal length of about thirteen inches. A region of more than thirty degrees square is covered by a single eight by ten inch plate. With exposures of one hour, stars as faint as the twelfth magnitude are, in some cases, obtained. Owing to the scale of the plates, identification of the individual stars would become difficult if, by using longer exposures, the number of stars were increased. One of these lenses is mounted at Cambridge, and is used principally for the northern stars. The other is similarly used for the southern stars at Arequipa. At each station two sets of photographs have been taken, the first having centers in declinations 0°, 30°, 60°, and 90°, and the second in declinations 15°, 45°, and 75°, the centers of the second set coinciding as nearly as possible with the corners of the first. An attempt is made to cover all parts of the sky, not too near the Sun, at least twice each month, once with each set. These photographs have proved unexpectedly useful here for determining the past as well as the present changes in light of variable stars, new stars, and similar objects. (See *Circular* No. 69, and elsewhere.) Of course, the small scale diminishes their value for measuring positions, although the minuteness of the images in part compensates for this difficulty.

The amount of useful material contained in these plates is so great that we are able to extract but a small portion of it, although an appropriation from the Carnegie Institution has, this year, permitted a great increase to be made. Various plans have been considered for placing copies of the photographs thus collected within the reach of astronomers. It was at first proposed to print a series of engravings on the same scale as the charts of the *Durchmusterung*. Numerous difficulties presented themselves, especially if an attempt was made to engrave the parallels and meridians upon the charts. The defects

[1] *Harvard College Observatory Circular* No. 71.

introduced by paper and ink are very troublesome, and are likely to differ in different copies. The variation in intensity of images which can be represented on paper is small compared with that on glass, and, finally, the expense would be large. The advantages of glass negatives are very great, especially to one accustomed to use them. They can be reproduced by contact printing so as to give results but slightly inferior to the original. A single contact print, forming a positive, with bright stars on a dark background, although more nearly resembling the sky itself, does not prove convenient in actual use. It cannot be superposed upon another positive, nor upon a paper map. A double contact print, however, furnishes a negative which is for some purposes nearly as useful as the original. Measurements of position or intensity of the images can be made, the cost is not large, and is nearly proportional to the number of copies furnished. A set of fifty-five of these prints on glass, covering the sky from the north to the south pole, has accordingly been prepared, and is described in Table I. A current number for designating the plate, and the approximate right ascension and declination of the center, are given in the first three columns. The designation of the original negative is given in the fourth column. It consists of the letters indicating the series, and the number of the plate in that series. The plates taken with the anastigmatic lens at Cambridge are indicated by the letters AC, those at Arequipa by AM. The date, the Greenwich Mean Time, and the length of exposure, expressed in minutes, are given in the next three columns. Remarks on some of the plates follow the table. The position of any particular object is indicated by two numbers placed in brackets. The first gives the distance of the object, in millimeters, from the left-hand edge of the exposed portion of the plate. The second number gives the corresponding distance from the lower edge of the exposed portion.

In the use of these plates certain suggestions may be made, especially for the benefit of those unaccustomed to astronomical photographs. To compare them with the sky, they should be examined with the glass side toward the eye. Objects of interest may then be conveniently marked on this side of the plate with a pen. The label is on the southern end of each plate, excepting in the cases of Nos. 1 and 54, which contain the north and south poles, respectively. Conspicuous configurations, like those of *Orion*, *Ursa Major*, and *Scorpius*, are easily recognized by inspection, aided when necessary by a small atlas. For the faint stars, a comparison may be made with

TABLE I.

CATALOGUE OF PLATES.

No.	R. A.	Dec.	Negative	Date	G. M. T.	Ex.
	h. m.			y. m. d.	h. m.	m.
1	0 00	+90	AC 2161	1902, January 4	22 43	39
2	0 00	+60	AC 1943	1901, November 2	17 51	63
3	3 00	+60	....	... .	.. ..	..
4	6 00	+60	....	....	.. ..	..
5	9 00	+60	AC 3625	1903, May 13	13 16	59
6	12 00	+60	AC 3466	1903, March 31	18 42	71
7	15 00	+60	AC 3620	1903, May 12	14 51	58
8	18 00	+60	AC 3629	1903, May 13	18 14	70
9	21 00	+60	AC 3630	1903, May 13	19 25	60
10	0 00	+30	AC 2152	1902, January 4	13 06	57
11	2 00	+30	AC 124	1898, December 16	13 09	62
12	4 00	+30	AC 2155	1902, January 4	16 16	59
13	6 00	+30	AC 3469	1903, April 1	12 48	75
14	8 00	+30	AC 3363	1903, February 26	16 16	59
15	10 00	+30	AC 2160	1902, January 4	21 55	56
16	12 00	+30	AC 3332	1903, February 20	21 04	67
17	14 00	+30	AC 1105	1900, December 28	22 04	68
18	16 00	+30	AC 252	1899, March 20	20 19	64
19	18 00	+30	AC 1013	1900, September 24	12 12	56
20	20 00	+30	AC 3353	1903, February 23	22 03	62
21	22 00	+30	AC 1014	1900, September 24	13 16	69
22	0 00	0	AM 1569	1902, September 25	15 32	58
23	2 00	0	AM 151	1899, September 14	18 45	60
24	4 00	0	AC 1093	1900, December 27	13 27	71
25	6 00	0	AC 2156	1902, January 4	17 22	73
26	8 00	0	AC 513	1899, November 9	22 01	64
27	10 00	0	AC 2518	1902, May 2	14 19	65
28	12 00	0	AM 1431	1902, July 8	12 09	65
29	14 00	0	AC 2547	1902, May 13	16 35	73
30	16 00	0	AM 1420	1902, July 3	13 15	60
31	18 00	0	AM 1436	1902, July 9	14 18	60
32	20 00	0	AM 1439	1902, July 9	17 30	60
33	22 00	0	AM 1441	1902, July 9	19 39	60
34	0 00	−30	AM 1427	1902, July 3	20 53	60
35	2 00	−30	AM 1451	1902, July 10	20 56	61
36	4 00	−30	AM 1802	1903, January 16	15 55	68
37	6 00	−30	AM 1785	1903, January 3	17 33	60
38	8 00	−30	AM 1798	1903, January 15	16 35	60
39	10 00	−30	AM 1419	1902, July 3	12 01	61
40	12 00	−30	AM 1443	1902, July 10	12 07	67
41	14 00	−30	AM 1444	1902, July 10	13 15	60
42	16 00	−30	AM 1452	1902, July 11	14 39	60
43	18 00	−30	AM 1461	1902, July 12	14 37	60
44	20 00	−30	AM 1463	1902, July 12	17 26	61
45	22 00	−30	AM 1440	1902, July 9	18 36	61
46	0 00	−60	AM 703	1900, November 1	13 37	61
47	3 00	−60	AM 1782	1903, January 3	14 12	64
48	6 00	−60	AM 712	1900, November 10	18 33	60
49	9 00	−60	AM 1247	1902, May 15	12 43	60
50	12 00	−60	AM 459	1900, May 8	15 10	72
51	15 00	−60	AM 1469	1902, July 14	14 13	60
52	18 00	−60	AM 809	1901, May 27	17 48	61
53	21 00	−60	AM 1464	1902, July 12	18 32	60
54	0 00	−85	AM 626	1900, September 3	17 11	60
55	14 00	−75	AM 1389	1902, June 25	15 13	60

the charts of the *Durchmusterung*, which are on nearly four times the scale. The plates may be examined with a reading glass or a two-inch positive eyepiece. The latter will, in almost all cases, serve to distinguish defects from actual star images. It will be noticed that images of bright stars are surrounded by sixteen rays, due to the iris diaphragm of the lens.

REMARKS.

1. The position of the North Pole is [95, 104]. [97, 111], the Pole Star. The circular form and absence of diffraction rays show that [110, 118] is a defect.

2. See No. 10.

3. Owing to a change in the scheme of work, photographs have not been obtained in the position of this and of the following region. These omissions will be supplied in a few weeks, or as soon as their positions permit photographs to be obtained.

4. See No. 3.

6. [25, 89], ζ *Ursae Majoris;* also on No. 7 [172, 85].

7. See No. 6.

10. [48, 190], Nebula in *Andromeda;* also on No. 2 [47, 11].

12. [117, 69], *Pleiades*, showing also the nebulosity surrounding them. Star l and perhaps m of the sequence given in *Harvard Annals*, XVIII, p. 153, appears on this plate. [131, 185], *Nova Persei*, No. 2. Nos. 54, 58, and 64 of Hagen's Catalogue are readily seen.

13. [80, 58, *Neptune*. [39, 107], *Nova Geminorum*. Stars t and w of *Circular* No. 70 (*Hagen* 73 and 77) appear on this plate. Numerous clusters are also shown where it is crossed by the Milky Way. [82, 71], the cluster *N. G. C.* 2129.

14. [40, 48], *Praesepe*.

17. [115, 92], the cluster *Messier* 3, N. G. C. 5272.

18. [51, 125], *Messier* 13, *N. G. C.* 6205, the cluster in *Hercules*.

19. [46, 163], the double stars ϵ and 5, *Lyrae;* also on Plate 20 [180, 169]. [31, 124], *Messier* 57, *N. G. C.* 6720. The Ring Nebula in *Lyra;* also on Plate 20 [175, 127].

20. See No. 19.

25. [134, 80], *N. G. C.* 1976, the great Nebula in *Orion*, [126, 91], ζ *Orionis* showing the faint nebulosity, *N. G. C.* 2024, north following it. [106, 105], α *Orionis* The red color of this star is well shown by its faintness on the photograph. [40, 161], the cluster *Messier* 14, *N. G. C.* 2287. [31, 3], *Sirius*. The circle around this, and other very bright stars, is due to light reflected from the back of the plate.

30. [167, 116], the cluster *Messier* 5, *N. G. C.* 5904.

37. [74, 83], *Pallas*.

40. The fogging on the northern edge is due to the Moon.

42. [68, 119], α *Scorpii*. The red color of this star is well shown by its faintness on the photograph.

43. [166, 141], *Uranus*. [126, 89], the cluster *Messier* 7, *N. G. C.* 6475. [109, 75], the cluster *Messier* 6, *N. G. C.* 6405. [99, 144], the Trifid Nebula, N. G. C. 6514. See *Annals* XXVI, p. 204, and Plate III. [96, 136], *N. G. C.* 6523.

44. [127, 136], *Vesta*. [113, 149], *Saturn*.

45. [169, 169], *Jupiter*. Satellite III is seen about half a millimeter to the left of it

46. [87, 25], the cluster 47 *Tucanae, N. G. C.* 104. [76, 17], the Small Magellanic Cloud. See *Annals* XXVI, p. 205, and Plate IV. [24, 102], *Achernar* α *Eridani*. Next to *Sirius*, this is the brightest star in the sky; also on No. 47 [167, 102].

48. [115, 40], the Large Magellanic Cloud. See *Annals*, XXVI, p. 206, and Plate IV. [77, 137], *Canopus*, α *Carinae*. [13, 73], the cluster *N. G. C.* 2516; also on No. 49 [145, 86].

49. Several fine clusters appear on this plate, which are better shown on Plate 50.

50. [182, 49], the cluster *N. G. C.* 3114; also on Plate 49 [56, 94]. [153, 64], η *Carinae* or η *Argus*; also on Plate 49 [24, 88]. This is one of the most remarkable regions in the sky. See *Annals* XXVI, p. 206, and Plates V and VII. [138, 74], *N. G. C.* 3523; also on Plate 49, [6, 86]. [113, 61], *N. G. C.* 3766. The Southern Cross is well shown on this plate. See also *Annals* XXVI, p. 203, and Plate II. [85, 79], δ *Crucis*. [79, 52], α *Crucis*. [72, 88], γ *Crucis*. [62, 71], β *Crucis*. The faintness of γ *Crucis* is due to its red color. This renders the Cross less conspicuous on the photograph than to the eye. [58, 66], the cluster κ *Crucis, N. G. C.* 4755. [7, 50], β *Centauri*. [6, 134], the cluster ω *Centauri, N. G. C.* 5139, the finest globular cluster in the sky.

51. [144, 90], β *Centauri*; also on No. 55 [99, 193]. [118, 91], α *Centauri*; also on No. 55 [72, 188]. So far as known this is the nearest star in the sky. [42, 124], the cluster *N. G. C.* 6067.

54. As the latitude of the Arequipa station is $-16^\circ 22'$, a good photograph could not be obtained if the declination of the center of the plate was at -90°, Accordingly, this plate has been taken in R. A. $0^h. 0^m$, December $-8^\circ.36$. Plate 55, taken from the other set of photographs, has been added, to cover the region which would otherwise be omitted. The position of the South Pole is [92, 61]; also on No. 55 [96, 15]. [96, 64], σ *Octantis*; also on No. 55 [91, 14]. [84, 166], 47 *Tucanae*. [73, 157], the Small Magellanic Cloud.

55. See Nos. 51 and 54.

The cost of these plates will prevent the wide, gratuitous distribution of them that is made of the *Annals*. In any case, this would not be advisable, since many would thus be placed where little or no use would be made of them. Copies of this set of photographs, consisting of fifty-five glass negatives, each eight by ten inches, will be supplied for $15; selected sets of ten plates, $3. This is less than the actual cost, the balance being paid from the Advancement of Astronomical Science Fund of the Harvard Observatory. The privilege of increasing the price later is reserved. If the demand justifies it, copies of the second set of plates, whose centers are near the corners of these, will be issued later.

EDWARD C. PICKERING.

MAY 19, 1903.

Reviews

The Theory of Optics. By PAUL DRUDE. Translated from the German by C. RIBORG MANN and ROBERT A. MILLIKAN, of the University of Chicago. New York: Longmans, Green & Co., 1902.

IT is a satisfaction to note that there has appeared a translation of this work, which received such instant recognition at the hands of physicists the world over upon its appearance in Germany. The translators deserve much praise for the conscientious and accurate manner in which they have accomplished their task.

As to the subject-matter: Professor Drude has produced a work of the greatest value—distinctly modern and up to date, logical and concise, yet clear. The keynote of the work is sounded by the author in his preface: "My purpose is attained if these pages strengthen the reader in the view that optics is not an old and worn-out branch of physics, but that in it also there pulses a new life whose further nourishing must be inviting to everyone." (The English here is not a fair sample of the work of the translators.)

In detail we may note the following:

The mathematical treatment of the formation of images is very thorough. A brief practical survey of optical instruments (telescope, microscope, photographic lens systems, etc.) is given. The subject of photography in natural colors is merely touched upon.

One of the most valuable and remarkable parts of the book is that which deals with diffraction problems. Starting with Huyghen's elementary light-wave principle, Fresnel's modification of it, and Kirchhoff's further development (simplified by Voigt), the reader is led logically through an exhaustive mathematical discussion of important diffraction phenomena. This is accompanied by a brief treatment of grating, prism, and echelon.

Under polarization phenomena Wiener's stationary light-wave experiment is mentioned and emphasized, and rightly so, for from it the important conclusion follows inevitably that the direction of the light vector is perpendicular to the plane of polarization.

Attention is called to the first brilliant success of the electromagnetic theory of light, that "the velocity of light in ether is equal to the ratio of the electromagnetic to the electrostatic units."

In discussing the subject of dispersion in the magnetic rotation of the plane of polarization, the bearing of the phenomenon known as the

Hall effect is shown, and the Ampère-Weber theory of molecular currents is modified so as to give the correct results. From this discussion we are then led directly to Voigt's theory of the Zeeman effect. The statement is made that the physical significance of Voigt's explanation of anomalous Zeeman effects has not yet been shown.

The "ion hypothesis leads to some new dispersion formulæ for the natural and magnetic rotation of the plane of polarization."

A comprehensive treatment of light problems with reference to bodies in motion leads to the aberration of light and to Michelson's experiment with the refractometer oriented in two positions with respect to the rotation of the earth. This latter is followed by the rather startling explanatory proposition advanced by Lorentz and by Fitzgerald, that the length of a solid body may depend upon its absolute motion in space.

Part III, "Radiation," covering in the original German but fifty-seven pages, and in the translation sixty, is a fine piece of work. The subject treated might well demand an entirely separate book; yet Professor Drude's discussion is clear and logical.

The electromagnetic theory is adopted as fulfilling most completely the test of experiment; and as Professor Michelson says in his preface to the English edition, "No complete development of the electromagnetic theory in all its bearings, and no comprehensive discussion of the relation between the laws of radiation and the principles of theorenodynamics have yet been attempted in any general text in English."

Throughout the whole treatise we discern a strong tendency to view nature as a mechanism. The discussion and interpretation of equations resulting from mathematical reasoning are both able and clear. Descriptively, the book is fully on a par with Preston's *Theory of Light* and mathematically more valuable, as well as more lucid and attractive, than Basset's *Treatise on Physical Optics*.

The general criticism may be made that the book is unbalanced, too little space being given to some subjects, too much to others. This, however, is quite a pardonable fault, especially as the author makes "no claim to such completeness as is aimed at in Mascart's excellent treatise, or in Winkelmann's *Handbuch*," and writes: "For the sake of brevity I have passed over many interesting and important fields of optical investigation."

Professor Drude's *Theory of Optics* should be assigned a most prominent place in the library of every modern physicist—a place which it should hold for many a year despite the rapid progress of the science. N. A. K.

THE

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ON MEASUREMENTS OF WAVE-LENGTH WITH THE CONCAVE GRATING OBJECTIVE SPECTROSCOPE.

By F. L. O. WADSWORTH.

In a recent paper[1] I have investigated the question of the aberration which is introduced by the use of the concave grating as a direct objective spectroscope. It was found that for all points not exactly on the axis of the grating the aberration was different in nature, and, in most cases, very much greater in amount than that produced by using the grating in the manner proposed and adopted by Rowland. The question of the effect of this increased unsymmetrical aberration on the size of the field of good definition and on the accuracy of measurement of wave-lengths with the instrument were very briefly discussed in the paper to which reference has been made and in a second paper, "On the Optical Conditions Required to Secure Maximum Accuracy of Measurement in the Use of the Telescope and Spectroscope," published in preceding numbers of this JOURNAL.[2]

[1] "On the Aberration of a Concave Grating when Used as an Objective Spectroscope," *Phil. Mag.*, (6) **6**, 119–156, July 1903.

[2] **16**, 267–299; **17**, 1–19, 100–133, 163; See particularly pp. 285–288 (note errata on p. 287 as corrected in **17**, 163).

The theory and method of using the concave grating as an objective spectroscope was first published by the writer in 1896.[1] Since then it has been used in this way by a number of investigators, among whom may be mentioned Poor, Mitchell, Frost, Mohler and Daniel, and Jewell and Humphreys. It was used by the writer in 1900 at Union Springs, Alabama, in photographing the flash spectrum and coronal rings during the total solar eclipse of May 28.[2] The unsatisfactory results yielded by the preliminary examination of the plates then obtained led me to the more detailed investigation of the aberration of the instrument; and the conclusions reached indicate that both as a spectrograph and as a spectrometer the concave grating objective spectroscope is far below the standard of the objective prism and the plane grating objective spectroscope. In the case of our own plates the errors of displacement of the lines are so large and the definition at the edges of the field so imperfect that I have not considered the results of wave-length measurement that might be obtained from them worthy of comparison with those determined by other parties with the slit, objective prism, and plane grating objective spectrographs.[3] This bad definition and lack of accuracy is in part due, in our case, to the very large angular aperture employed (with the view of securing great rapidity), and in part to the method of mounting adopted (also with a specific object). This, as I have since found by the mathematical investigation, is less favorable than another form would have been.

Other observers who have used the instrument in eclipse work also report unfavorably[4] as to the performance of the instrument, but ascribe the poor results either to the use of improper plates or to defects in the focusing. While these causes may have operated to contribute to the difficulties, they

[1] Astrophysical Journal, 3, 54–60, January 1896.

[2] *Report of the Director of the Allegheny Observatory for 1900*, pp. 23–24 (9). Also "Account of Allegheny Observatory Eclipse Expedition," S. M. Kintner, *Western University Courant*, 15, 266–270.

[3] See also footnote on p. 87.

[4] See for example Frost, Astrophysical Journal, 12, 85, 307; Mohler and Daniel, *ibid.*, 12, 361 and Humphreys, *ibid.*, 16, 313.

are not, as it now appears, the chief sources of error. The latter are to be found in the inherent optical defects of the instrument itself.

The general expression for the aberration in the primary plane of a concave grating when used as an objective spectroscope is, as has been already found,[1]

$$\begin{aligned} Z &= -\frac{\rho}{2}\sin^3\beta\sin\theta\cos i\,\frac{(\cos i+\cos\theta)}{\cos^2\theta} \\ &\quad+\frac{\rho}{8}\sin^4\beta\,\frac{\cos i+\cos\theta}{\cos^2\theta}\left[\cos^2 i-(\cos\theta+2\cos i)^2\tan^2\theta\right] \qquad (1) \\ &= -U+S, \end{aligned}$$

where ρ is the radius of curvature of the grating surface, β is the semi-angular aperture measured at the center of curvature, and i and θ are the angles of incidence and diffraction respectively. Both β and θ are measured positively to the right from the normal to the grating (viewed from the center of curvature), while i is measured positively to the left of this axis. The nature and numerical value of the aberration depends both on the dimensions and optical constants of the grating and on the form of mounting adopted for the instrument. As described in a previous paper, there are four general types of mounting that may be employed, and these have been designated respectively as A–B, C–D, E–F, and G–H.

In the solar eclipse work with which we are now dealing only two of these types have been used; *i.e.*, the A–B type by Frost, Mohler and Daniel, and Jewell and Humphreys, and the G–H type by the writer. The dimensions and other optical constants of the four gratings and the positions in which they were used are given in Table I. The aberrations at the centers and the edges of the photographic fields, computed from the data in Table I by the aid of (1), are tabulated in Table II.

In order to determine the effect of a given amount of aberration, Z, on the form and position of the spectral image at different parts of the field we must find how the distribution in intensity in the diffraction pattern at the focal plane is affected by the unsymmetrical constituent U and the symmetrical constit-

[1] *Phil. Mag.*, 6, 124, July 1903.

TABLE I.

Observer	Grating	Constants of Grating: ρ	a	$\beta - \frac{a}{2\rho}$	n	$N=nb$	r In Tenth meters	ϵ	Position of Use: Mounting	i	θ_0	Field: λ_1 to λ_2	κ: θ_1 to θ_2
		cm	cm			about	about	about					
Frost, Wadesboro, 1900	F.	150	4.4	0.0147	5684	25,000	0.17 0.20	0.01	A-B	15° 10′	0°0′	4200 4600 4950	+(1° 18′) to −(1° 9′)
Mohler and Daniel, 1900	M.	305	10.0	.0164	5684	57,000	0.08 0.10	0.006	A-B	16°	0° 0′	4300 to 5880	+(1° 45′) to −(3° 25′)
Wadsworth, Union Spgs., 1900	W.	175.3	13.2	.0377	1421	18,750	0.10 0.11	0.007	G H	12° 50′	12° 50′	5880 3d 000 6270 3d	+(28° 15′) to −(2° 35′)
Jewell and Humphreys, Fort de Kock, 1901	J.*	915	10.0 2.0	$\beta_1 = .0109$ $\beta_2 = .0022$	5684	68,200	0.045 0.075	0.003 0.005	A-B	14° 50′	0°0′	3100 4500 5200	+(4° 34′) to −(2° 17′)

TABLE II. Aberrations.

Grating	$\sin^3\beta$	$\frac{1}{2}\rho \sin^3$	$\frac{1}{8}\rho \sin^4\beta$	At Center of Field. $\lambda_0\ \theta_0$: Sym. S. 10^{-6}	Sym. S. λ	Unsym. U. 10^{-6}	Unsym. U. λ	Unsym. U. $d\lambda$	At Right Edge. $\lambda_1\ \kappa_1$: Sym. S. 10^{-6}	Sym. S. λ	Unsym. U. 10^{-6}	Unsym. U. λ	Unsym. U. $d\lambda$	At Left Edge. $\lambda_2\ \kappa_2$: Sym. S. 10^{-6}	Sym. S. λ	Unsym. U. 10^{-6}	Unsym. U. λ	Unsym. U. $d\lambda$
F.	3.18×10^{-6}	238×10^{-6}	0.88×10^{-6}	1.6	.03	....	....	....	1.6	0.03	10.2	0.24	.04	1.6	0.03	9.02	0.18	.04
M.	4.41×10^{-6}	672×10^{-6}	2.75×10^{-6}	4.98	.09	....	....	....	4.9	.09	38.6	.90	.07	4.8	.09	75.6	1.3	.13
W.	53.6×10^{-6}	4698×10^{-6}	44.3×10^{-6}	44.3	.80	2087	37	?	144	2.5	5180	88.0	?	82.6	1.2	408.0	6.8	?
*J β_1	1.29×10^{-6}	590×10^{-6}	1.6×10^{-6}															
J β_2	0.01×10^{-6}	5×10^{-6}	0.00×10^{-6}	2.94	.07	....	....	....	2.78	.09	89.6	2.9	.15	2.92	.06	44.7	0.9	.09

TABLE III.

$\frac{a}{a_0}$	I^2	I_u^2, $U=\frac{\lambda}{4}$	$U=\frac{\lambda}{2}$		$U=\frac{3}{4}\lambda$		$U=\lambda$		$U=\frac{7}{4}\lambda$		$U=2\lambda$	
			$\frac{a}{a_0}$	I_u^2	$\frac{a}{a_0}$	I_u^2	$\frac{a}{a_0}$	I_u^2	$\frac{a}{a_0}$	I_u^2	$\frac{a}{a_0}$	I_u^2
−2.0	.0000	.0086	−2.52	.0030	−2.88	.0016	−3.16	.0001	−3.84	.0004	−4.0	.0000
−1.9	.0027	.0081	−2.40	.0026	−2.74	.0009	−3.00	.0000	−3.65	.0001	−3.8	.0001
−1.8	.0108	.0061	−2.27	.0014	−2.59	.0002	−2.84	.0006	−3.46	.0000	−3.6	.0002
−1.7	.0229	.0035	−2.14	.0002	−2.45	.0000	−2.68	.0015	−3.26	.0004	−3.4	.0002
−1.6	.0358	.0012	−2.02	.0001	−2.30	.0005	−2.53	.0020	−3.07	.0008	−3.2	.0000
−1.5	.0450	.0000	−1.89	.0016	−2.16	.0010	−2.37	.0019	−2.88	.0010	−3.0	.0001
−1.4	.0468	.0004	−1.76	.0045	−2.02	.0010	−2.21	.0012	−2.69	.0008	−2.8	.0006
−1.3	.0392	.0018	−1.64	.0081	−1.87	.0004	−2.06	.0005	−2.50	.0004	−2.6	.0014
−1.2	.0243	.0032	−1.51	.0110	−1.73	.0000	−1.90	.0001	−2.30	.0001	−2.4	.0018
−1.1	.0080	.0033	−1.38	.0123	−1.58	.0011	−1.74	.0000	−2.11	.0000	−2.2	.0018
−1.0	.0000	.0018	−1.26	.0125	−1.44	.0049	−1.58	.0001	−1.92	.0002	−2.0	.0015
−0.9	.0119	.0001	−1.14	.0112	−1.30	.0119	−1.42	.0006	−1.73	.0013	−1.8	.0014
−0.8	.0547	.0017	−1.01	.0101	−1.15	.0212	−1.26	.0034	−1.54	.0042	−1.6	.0020
−0.7	.1353	.0125	−0.88	.0099	−1.01	.0313	−1.10	.0112	−1.34	.0101	−1.4	.0041
−0.6	.2545	.0399	−0.76	.0120	−0.86	.0408	−0.95	.0234	−1.15	.0164	−1.2	.0092
−0.5	.4053	.0911	−0.63	.0181	−0.72	.0493	−0.79	.0428	−0.96	.0287	−1.0	.0189
−0.4	.5754	.1714	−0.51	.0323	−0.58	.0579	−0.63	.0672	−0.77	.0388	−0.8	.0337
−0.3	.7368	.2813	−0.38	.0605	−0.43	.0688	−0.48	.0934	−0.58	.0489	−0.6	.0522
−0.2	.8751	.4159	−0.25	.1103	−0.29	.0856	−0.32	.1194	−0.38	.0605	−0.4	.0719
−0.1	.9675	.5640	−0.12	.1886	−0.14	.1128	−0.16	.1440	−0.19	.0764	−0.2	.0910
−0.0	1.0000	.7093	−0.00	.2976	−0.00	.1553	−0.00	.1688	−0.00	.1002	−0.0	.1097
+0.1	.9675	.8326	+0.12	.4312	+0.14	.2178	+0.16	.1966	+0.19	.1348	+0.2	.1302
+0.2	.8751	.9158	+0.25	.5731	+0.29	.3013	+0.32	.2304	+0.38	.1804	+0.4	.1551
+0.3	.7368	.9450	+0.38	.6983	+0.43	.3998	+0.48	.2722	+0.58	.2320	+0.6	.1860
+0.4	.5754	.9137	+0.51	.7790	+0.58	.4984	+0.63	.3205	+0.77	.2784	+0.8	.2208
+0.5	.4053	.8246	+0.63	.7919	+0.72	.5733	+0.79	.3682	+0.96	.3056	+1.0	.2519
+0.6	.2545	.6901	+0.76	.7278	+0.86	.5994	+0.95	.4029	+1.15	.3022	+1.2	.2679
+0.7	.1353	.5287	+0.88	.5951	+1.01	.5592	+1.10	.4077	+1.34	.2653	+1.4	.2569
+0.8	.0547	.3629	+1.01	.4206	+1.15	.4528	+1.26	.3695	+1.54	.2032	+1.6	.2147
+0.9	.0119	.2144	+1.14	.2422	+1.30	.3027	+1.42	.2863	+1.73	.1316	+1.8	.1490
+1.0	.0000	.0999	+1.26	.0983	+1.44	.1498	+1.58	.1748	+1.92	.0673	+2.0	.0785
+1.1	.0080	.0287	+1.38	.0161	+1.58	.0386	+1.74	.0686	+2.11	.0222	+2.2	.0244
+1.2	.0243	.0009	+1.51	.0029	+1.73	.0000	+1.90	.0061	+2.30	.0015	+2.4	.0007
+1.3	.0392	.0089	+1.64	.0457	+1.87	.0379	+2.06	.0135	+2.50	.0000	+2.6	.0090
+1.4	.0468	.0395	+1.76	.1159	+2.02	.1262	+2.21	.0864	+2.69	.0281	+2.8	.0305
+1.5	.0450	.0776	+1.89	.1807	+2.16	.2198	+2.37	.1913	+2.88	.0656	+3.0	.0776
+1.6	.0358	.1095	+2.02	.2138	+2.30	.2749	+2.53	.2764	+3.07	.1080	+3.2	.1095
+1.7	.0229	.1261	+2.14	.2046	+2.45	.2674	+2.68	.3088	+3.26	.1424	+3.4	.1258
+1.8	.0108	.1239	+2.27	.1592	+2.59	.2030	+2.84	.2467	+3.46	.1538	+3.6	.1215
+1.9	.0027	.1052	+2.40	.0968	+2.74	.1128	+3.00	.1473	+3.65	.1323	+3.8	.0968
+2.0	.0000	.0762	+2.52	.0402	+2.88	.0358	+3.16	.0505	+3.84	.0822	+4.0	.0580

uent S respectively. The problem has been investigated in part by Lord Rayleigh for small values of U and S., *i.e.*, values not exceeding 0.5λ.[1] The aberrations with which we have to deal in the above cases range from 0.03λ for S with the F grating, to

[1] *Phil. Mag.*, (5) 8, 404.

88λ for U with the W grating. Disregarding the very large values for this latter instrument, we have still to consider the effect of unsymmetrical aberrations amounting to as much as 2.9λ for the J grating. I have therefore extended Rayleigh's results for the effect of an unsymmetrical aberration to values as large as $U=2\lambda$. The values of I^2_u for $U \cong \frac{1}{4}\lambda$, $\frac{1}{2}\lambda$, $\frac{3}{4}\lambda$, 1λ, $1\frac{3}{4}\lambda$, and 2λ for a rectangular aperture are tabulated in columns 3, 5, 7, 9, 11, and 13, respectively, of Table III. For the sake of comparison the value of I^2_0 for $U=0$ (no aberration), is given in column 2 of the same table. The abscissae a for each curve are expressed as usual in terms of a_0, the angular distance from the center, o, of the geometrical image to the first minimum, m_0, of the normal diffraction pattern I^2 (column 2). Owing to the manner in which the integral involved was originally evaluated by Airy,[1] the tabulated values of I^2_u correspond to different values of a for each value of U.[2]

The curves expressing the relation between I^2_u and $\frac{a}{a_0}$ for these different values of U (0 to 2λ) are plotted in Fig. 1.

From an examination of the values of the table, or better still, of the above curves, we see that the relation between U, the extreme unsymmetrical wave front aberration, and the angular displacement $o-o'$ of the point of maximum intensity of the resulting distorted image is almost linear, and that the amount of such displacement is

$$o-o'=\Delta\gamma=0.3\ a_0\frac{\frac{U}{\lambda}}{4}\ , \tag{2}$$

or, expressed in linear measure at the focal plane,

$$\Delta\xi=1.2\frac{u}{b}U\ . \tag{3}$$

Hence, substituting the value of U from (1), we obtain

$$\Delta\xi=0.6\frac{\rho}{\beta}u\sin^3\beta\frac{\sin\theta}{\cos^2\theta}(\cos i+\cos\theta)\cos i\ , \tag{4}$$

where u is the focal length of the grating.

[1] See *Cambridge Phil. Trans.*, **6**, 402, 1838.

[2] See RAYLEIGH, *Phil. Mag.*, (5) **8**, 405, 406.

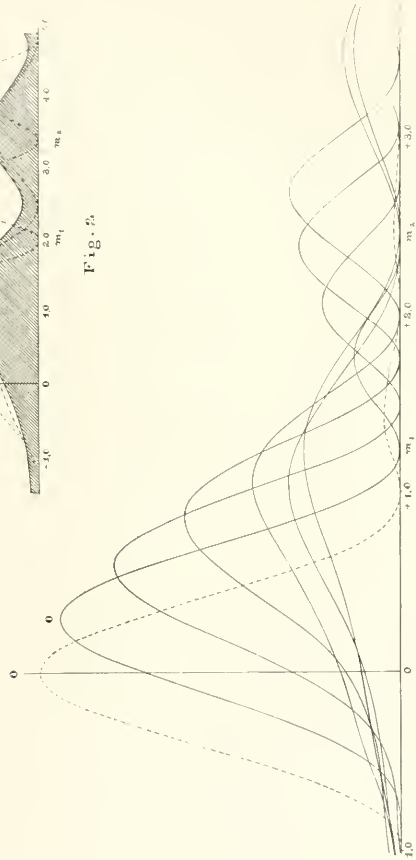


Fig. 1

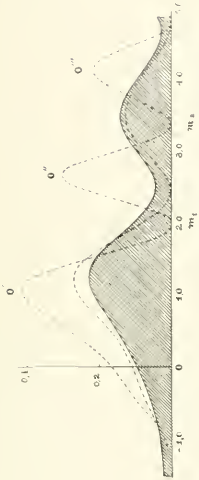


Fig. 2

For the purpose of determining the effect of this displacement on the measurement of wave-lengths it is convenient to express $\Delta\xi$ in terms of $\Delta\lambda$. From the general equation of the grating, we have

$$N\lambda = b(\sin i - \sin\theta) \tag{5}$$

and

$$\begin{aligned} d\lambda &= -\frac{b}{N}\cos\theta\, d\theta \\ &= -\frac{b}{N}\cos\theta\frac{dx}{u}\ . \end{aligned} \tag{6}$$

If the cross-wires of the measuring micrometer are always set on the point of maximum intensity, o', of the unsymmetrical image, the error in the wave-length measurement due to the aberration of the concave grating spectroscope will therefore be

$$d\lambda = -1.2\frac{U}{N}\cos\theta\ . \tag{7}$$

These values have been computed for the values of U given by (1) for the four instruments already considered and are found in the columns headed $d\lambda$ of Table II.

The maximum errors of displacement due to aberration are therefore 0.04 tenth-meters in the case of Frost's plates; 0.13 tenth-meters in the plates obtained by Mohler and Daniel; 0.15 tenth-meters in the case of those obtained by Jewell and Humphreys; and so large as to be indeterminate at both the ends and center of the plates obtained by the writer. Comparing these errors of displacement with the quantities ϵ of Table I, which represent the limiting metrological power of the instruments under the best conditions of use, we see that the effect of aberration is to displace the spectral images by amounts which, in the case of gratings F, M, and J, are respectively, about four, fourteen to nineteen, and twenty to one hundred and fifty times as great as the metrological errors of setting on the lines. In the case of grating W, $d\lambda$ is, as already stated, so large as to be indeterminate.

The above considerations are based on the assumption that the setting of the micrometer wire in the measurements for determination of wave-length is affected only by the displace-

ment of the maximum of intensity in the central band of the diffraction pattern. For small values of U this assumption will be very nearly correct, but for large values the setting will also be affected by the depression of the principal maximum, o', and the very decided augmentation of the lateral bands, o'' and o'''. The effect of the symmetrical aberration S will also tend to depress still further the principal maximum, o', and to obliterate the minima, m_1, m_2, m_3, by a re-distribution of the relative intensities between the points, m_1o', m_2o'', m_3o''', etc.

In the case of the M and J gratings the extreme values of U are 1.3λ and 2.9λ, respectively. At the same points in the field the corresponding values of S are about 0.1λ. The effect of this amount of symmetrical aberration is to depress the central maximum about 5 per cent. and correspondingly augment the lateral bands. Considering this, and taking into account also the effect of mechanical vibrations, small errors of focusing, etc. all of which tend in the same direction, we shall obtain for these large values of U a distribution in intensity in the spectral image similar to that represented in the shaded curve in Fig. 2. In setting the micrometer wire on such an image as this, the tendency would undoubtedly be to place it considerably farther to the right than the point o', which marks the location of the principal maximum. The error of setting in these cases would therefore considerably exceed the quantity $\Delta\xi$ as given by (3).

In cases like that of the W grating, where the aberration, both symmetrical and unsymmetrical, amounts to a very much larger number of wave-lengths, the central and lateral bands are all blended into a broad indefinite blur on which it is impossible to set the micrometer wire with any pretensions to accuracy. At the points very near the axis of this grating ($\theta = 0$) there is a narrow strip over which the definition is about equal to that at the extreme edges of the field of the M and J gratings; but even here, as we have already seen, the aberrations are from five to twelve times greater than the limit imposed by considerations of "good definition," and from twenty to one hundred and fifty times the limit imposed by the conditions of maximum accuracy. It is for these reasons that the plates taken at Union Springs

have been, as already stated, rejected as unsuitable for accurate measurement.[1]

On account of the small angular and linear apertures of F and J[2] the definition of these gratings is good not only at the center of the plate, but over a fair extent of field. The plates obtained with these instruments have been measured and the wave-lengths of the lines so determined published.[3] Mohler and Daniel have also published a short list of wave-length measurements obtained with grating M.[4]

I have been interested in comparing the apparent accuracy attained in these measurements with that to be expected from the preceding considerations. The probable uncertainties in the wave-length determinations as assigned by the observers themselves are about 0.05 tenth-meters at the center and about 0.2

[1] If the mathematical investigation of the aberration of the concave grating objective spectroscope had been undertaken before, instead of after, these eclipse observations had been made, much disappointment on my own part at least would have been avoided. No excuse can be given for such lack of preliminary examination save that of an unavoidable delay that occurred in organizing and financing the expedition, and the lack of sufficient assistance. The grating had been hastily examined beforehand to test its definition near the axis at the principal focus, and this had been found to be very good. In these tests, however, I had no collimator of sufficiently large aperture to cover the entire surface of the grating, and the semi-angular aperture β was in consequence of this only about six-tenths as large as that used in the actual eclipse work. The aberration on the axis of the grating, was, under these circumstances, only about one-eighth as great as with full aperture, and therefore considerably less than would injuriously affect definition. The effect of increasing the *angular aperture* and the *field* was not considered as carefully as it should have been, but this was partly in consequence of the results which Mitchell had published a short time previously (ASTROPHYSICAL JOURNAL, **10**, 29). As I have already pointed out in my previous paper (*Philosophical Magazine*, July 1903) these results, although obtained with a grating of even larger angular aperture than that here used, should not have been accepted as conclusive. The whole experience is a good illustration of the danger of adopting a new type of instrument without sufficient preliminary examination of its optical defects and limitations in the line of work which it is desired to carry out.

[2] In the case of J the small angular aperture β_2 was due to a part of the ruled surface being defective. In the light of subsequent developments this must be regarded as a rather fortunate accident. Had it not been for this, the aberration at the edges of the field would have been even larger than it was.

[3] FROST, ASTROPHYSICAL JOURNAL, **12**, 342–344; HUMPHREYS, *ibid.*, **15**, 318–325, Table II.

[4] MOHLER and DANIEL, *ibid.*, **12**, 363.

tenth-meters at the edges of the plates by Frost (pp. 325, 326);[1] and about 1 tenth-meter by Mohler and Daniel (p. 363). In both cases these errors or uncertainties, whichever they may be called, are considerably greater than the theoretical errors of displacement $d\lambda$. At first sight this would indicate that the effect of bad definition due to both the terms U and S is more important than that of the actual displacement $d\lambda$ produced by U alone; and this would probably be the case were it not for the fact that the errors of measurement due to the first cause are likely to be either positive or negative, while those due to the second cause are negative alone. To detect any effect of the latter nature we must examine the residuals, $\lambda_{O.G}-\lambda_R$, for systematic and periodic changes in sign and magnitude. In doing this we are met at the outset with a difficulty which is introduced by the method of reduction and interpolation employed. The values of $\lambda_{O.G}$ from the objective grating plates have been determined not as absolute values, but as relative values, by identifying certain lines on the *O.-G.* plates with lines on the Rowland map, and assuming that the average residuals, $\lambda_{O.G}-\lambda_R$, are zero for these lines and that the spectrum is *normal* between them. This process is a perfectly legitimate and accurate one on plates taken with long focus concave gratings mounted in the manner originally designated by Rowland, and for which there is *no* unsymmetrical aberration. It cannot, however, be adopted without question for the *O.-G.* plates, where conditions both as to curvature of field and aberration are so different.

From equation (5) we have for any point θ in the field of a grating

$$\lambda = \lambda_0 - \frac{b}{N}\sin\theta \ , \qquad (8)$$

where λ_0 is the wave-length of the line which falls on the axis of the grating, $\theta = 0$. When there is an aberrational displacement, $d\lambda$, of the position of the geometrical image, the line of wave-

[1] In this connection it seems desirable to call attention to the fact that Professor Frost himself recognized the inferiority of the wave-length measurements made with the objective grating to those made with the objective prism train, and by far the greater part of his wave-lengths was determined from plates obtained with the latter instrument.

length λ will not fall at the point θ indicated by (8) but at a point

$$\theta_m = \theta + \Delta\gamma \ ,$$

and therefore

$$\theta = \theta_m - 1.2\frac{U}{b} \ , \tag{9}$$

as indicated in (2) and (3). From (1) we obtain for U in terms of θ_m (neglecting powers higher than the fourth) the value

$$U = \frac{\rho}{2}\sin^3\beta\left[a_1 + \left(\frac{5}{6}a_1 - \frac{1}{2}\cos i\right)\theta_m^2\right]\theta_m \ , \tag{10}$$

where

$$a_1 = (1 + \cos i)\cos i \ , \tag{11}$$

as defined and tabulated (for different values of i) in my previous paper (Table I).[1]

From (8), (9), and (10) we finally obtain

$$\lambda - \lambda_0 = \Delta\lambda = -\frac{b}{N}\sin\left\{1 - 0.3\sin^2\beta\left[a_1 + \left(\frac{5}{6}a_1 - \frac{1}{2}\cos i\right)\theta_m^2\right]\right\}\theta_m \ . \tag{12}$$

Let us put for convenience

$$\begin{aligned} 0.3\,a_1 &= a_2 \\ 0.3\left(\frac{5}{6}a_1 - \frac{1}{2}\cos i\right) &= a_3 \end{aligned} \ . \tag{14}$$

Then expressing (12) in terms of θ we have to the same degree of approximation as before

$$\Delta\lambda = -\frac{b}{N}\left\{(1 - a_2\sin^2\beta)\,\theta_m - \left[\frac{1}{6} - \sin^2\beta\left(\frac{a_2}{2} - a_3\right)\right]\theta_m^3\right\} \ . \tag{15}$$

The values of a_2 and a_3 which appear in (14) and (15) have, for convenience, been computed for the same values of i used in the table for a and a_1 above referred to. The results are tabulated in Table IV.

TABLE IV.

i	a_1	a_2 $(=0.3\,a_1)$	a_3	i	a_1	a_2 $(=0.3\,a_1)$	a_3
0	2.000	0.600	0.350	50°	1.056	0.317	0.168
5°	1.988	.596	.348	55°	.902	.271	.139
10°	1.954	.586	.340	60°	.750	.225	.113
15	1.899	.570	.330	65	.601	.180	.087
20	1.823	.547	.315	70	.459	.138	.064
25°	1.728	.518	.296	75°	.326	.098	.043
30	1.616	.485	.274	80°	.203	.061	.025
35	1.490	.447	.249	85	.095	.029	.011
40	1.353	.406	.223	90°	.000	.000	.000
45	1.207	.362	.196				

[1] *Phil. Mag.*, (6) **6**, 130.

When the plate is bent to the focal curve

$$u = \frac{\rho}{1 + \cos i} \cos\theta \qquad (17)$$

it has a curvature of $\frac{1}{\rho}$. If it is then straightened and measured on a linear dividing engine we have for the relation between the linear run ΔS of a screw and the angle θ_m

$$\theta_m = \frac{\Delta S}{u_0} = \Delta S \cdot \frac{1 + \cos i}{\rho}. \qquad (18)$$

From (8), (15) and (18) we finally obtain

$$\Delta\lambda = -\frac{b}{N}(1 - a_2 \sin^2\beta)\frac{\Delta S}{u} + \left[\frac{1}{6} - \sin^2\beta\left(\frac{a_2}{2} - a\right)\right]\frac{N^2}{b^2}(\Delta\lambda)^3 \qquad (19)$$
$$= A\Delta S + x_c.$$

If the spectra are both photographed and measured on a flat plate the relation between the run of the micrometer screw and the angle θ_m will be

$$\theta_m = \tan g^{-1}\frac{\Delta S}{u_0}$$
$$= \frac{\Delta S}{u_0} - \frac{1}{3}\theta_m^3.$$

Substituting this value in (15) and reducing as before, we obtain

$$\Delta\lambda = -\frac{b}{N}(1 - a_2 \sin^2\beta)\frac{\Delta S}{u_0} + \left[\frac{1}{2} - \sin^2\beta\left(\frac{5}{6}a_2 - a_1\right)\right]\frac{N^2}{b^2}(\Delta\lambda)^3 \qquad (20)$$
$$= A\Delta S + x_f.$$

The amount from which the spectrum departs from normal is therefore about three times as great when photographed on flat plates as when photographed on curved plates.

In the case of Frost's measurements, in which the plates used were flat, the extreme value of $\Delta\lambda$ (see Table I) was about 400 tenth-meters. The value of $\frac{N}{b} = n$ for the F grating is 5684×10^{-8}. Hence the value of x_f, the second term of (20), which expresses the correction required to a "normal" spectrum is about

0.1 tenth-meter.

The corrections at the intermediate points, $\Delta\lambda = 100$, 200 and 300 tenth-meters, are respectively

For $\Delta\lambda = 100$ tenth-meters, $x = 0.002$
For $\Delta\lambda = 200$ tenth-meters, $x = 0.013$
For $\Delta\lambda = 300$ tenth-meters, $x = 0.043$
For $\Delta\lambda = 400$ tenth-meters, $x = 0.103$.

Plotting these values as abscissae and ordinates respectively we obtain the curve in Fig. 3. This expresses graphically the corrections that must be applied to the measurements on the assumption that the spectrum is normal over the entire field.

In the actual reduction of the plate the necessary corrections were somewhat reduced by dividing the plate into three sections; the first extending from $\lambda 4200$ to 4383, the second extending from $\lambda 4383$ to 4572, and the third from $\lambda 4572$ to 4950. The spectrum was assumed normal over each of these three sections. As the constant A of the micrometer screw was determined separately for each section, the corrections between these points would be the differences between the ordinates to the curve, a, b, c, o, d, e, and the straight lines, ab, bc, and cd. The maximum correction in section I (ab) would therefore be about 0.015 tenth-meters, in section II (bc) not more than 0.004 tenth-meters, and in section III (cd) about 0.03 tenth-meters.

In section I the correction is just a little larger than the limit of metrological power ϵ (see Table I), and in section II it may be entirely disregarded; in section III, however, it is about three times as large as ϵ. In this last section the correction is sufficiently large to make its neglect felt in the sign of the residual errors. In order to determine whether this were true I formed a table of residuals, $f(\lambda) =$ Rowland — Frost, for all lines whose identification was assumed to be reasonably certain. These residuals are plotted in Fig. 4 and the sums taken for each interval of 100 tenth-meters (from 4200 to 4300, from 4300 to 4400, etc.). The latter quantities are indicated by the circled crosses ⊗ of the figure.

These averaged residual differences correspond in distribution and sign to those which would be produced by neglect of the correction x for sections I and II, but are larger in magnitude than would be expected. For section III they also correspond if the constant of the micrometer run was determined by settings

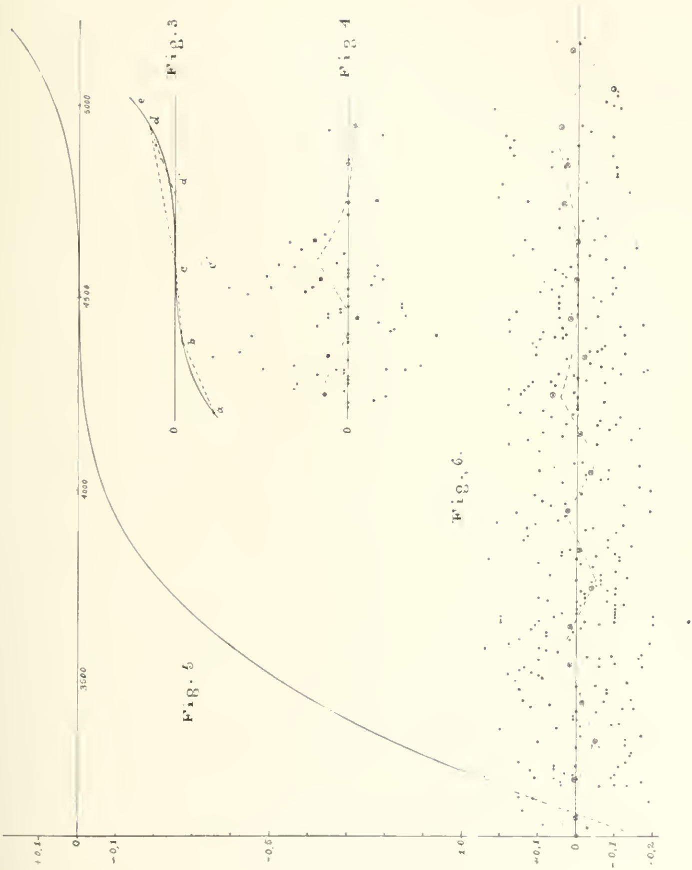


Fig. 3
Fig. 4
Fig. 5
Fig. 6.
5000
4500
4000
3500
+0,1
0
-0,1
-0,5
1,0
-0,2

on lines near d and d', or was assumed to be the same as in section I. In such case the line dc would be parallel or nearly so to the line ab as at dc', and the corrections for x would be the differences of the ordinates to dc', and the curve c, o, d, c. These are nearly the same, not only in sign but also in magnitude, as those actually found. The processes employed in the work of reduction of the plates are not stated in sufficient detail by Professor Frost to enable us to determine whether or not this explanation is the correct one, but the periodic character of the averaged residuals, $\Sigma f(\lambda)$, is quite striking and the great preponderance of residuals of positive sign is an evidence of some systematic error of the kind indicated.

In the case of Humphreys' measurements the plates were curved. The maximum values of $\Delta\lambda$ (see Table I) measured are

$$+\Delta\lambda = 700$$
$$-\Delta\lambda = 1400 .$$

The values of x_c (from 19) are

For $\Delta\lambda = 200$ tenth-meters, $x_c = 0.004$
For $\Delta\lambda = 400$ tenth-meters, $x_c = 0.034$
For $\Delta\lambda = 600$ tenth-meters, $x_c = 0.117$
For $\Delta\lambda = 700$ tenth-meters, $x_c = 0.185$
For $\Delta\lambda = 800$ tenth-meters, $x_c = 0.276$
For $\Delta\lambda = 900$ tenth-meters, $x_c = 0.394$
For $\Delta\lambda = 1000$ tenth-meters, $x_c = 0.538$
For $\Delta\lambda = 1100$ tenth-meters, $x_c = 0.716$
For $\Delta\lambda = 1200$ tenth-meters, $x_c = 0.930$
For $\Delta\lambda = 1300$ tenth-meters, $x_c = 1.182$
For $\Delta\lambda = 1400$ tenth-meters, $x_c = 1.476$

In this case the correction required for curvature at the violet end of the spectrum is nearly five hundred times the limiting metrological power, ϵ.

The relation between $\Delta\lambda$ and x is plotted in Fig. 5. The residuals $h(\lambda) =$ Rowland $-$ Humphreys, and their sums over each interval of 100 tenth-meters, have been found in the same way as for Frost's measurements and are plotted on the same scale in Fig. 6.

In this case also there is a marked correspondence between x and the residuals $\Sigma h(\lambda)$. Near the center of the plate where the spectrum is nearly normal and x is small, the average residuals, $\Sigma h(\lambda)$, are also small and lie very nearly in a straight line parallel to the λ axis. As we go from this point toward the violet the residuals become distinctly periodic in character. This would be explained if the wave-lengths in this part of the spectrum were determined by extrapolation from standard reference lines at about 3250, 3500, 3700, 3850, 4000 and 4150. In Humphreys' paper the method of reduction is not described, and here again it is not possible to satisfactorily compare the actual results of computation and measurement.

EFFECT OF ABERRATION ON THE APPEARANCE OF THE LINES.

For small values of θ, *i.e.*, near the center of the plate, the value of U is small and the value of S is relatively large. In this portion of the field the lines will therefore be simply broadened without being sensibly displaced or rendered assymmetrical. As we go away from the center the value of S decreases and the value of U increases. The effect of this is not only to displace the apparent center of intensity of the central image but to increase very greatly the relative intensities of the lateral diffraction fringes on the right hand, *i.e.*, the violet side, of the central image.

An examination of Fig. 2 shows that for large values of U the first lateral fringe on the right becomes very nearly as bright as the central image, and is much narrower and sharper. This image was actually selected by Humphreys as the one corresponding most nearly to the true geometrical image. The multiplication of lateral images at the edges of the field is not, as Humphreys assumes, an evidence of imperfect focusing, but of *good* focusing. Under favorable conditions it is possible to observe not only the principal (central) component of the diffraction image and the first two right hand fringes (forming an apparent tripling of the line) but several more lateral fringes still further to the right.

ALLEGHENY OBSERVATORY,
March 1903.

THE FLUORESCENCE AND ABSORPTION SPECTRA OF SODIUM VAPOR.

By R. W. Wood and J. H. Moore.

THE FLUORESCENCE OF SODIUM VAPOR.

The green fluorescence which sodium vapor exhibits when illuminated with an intense beam of white light, was first observed and studied by Wiedemann and Schmidt.[1] The method which they employed was to heat the metal in an exhausted glass bulb, concentrating a beam of sunlight on the vapor by means of a lens. A cone of green light is seen where the intense beam enters the mass of metallic vapor. Examined with the spectroscope, this light was found to consist of a band in the red, a narrower band nearly in the position of the D lines, and a broad green band which appeared to be broken up into channels or bands. The wave-lengths of some of these bands were roughly determined, but the authors do not appear to have determined exactly the relation which they bore to the absorption bands which appear in the same part of the spectrum.

Inasmuch as we have at the present time no very satisfactory theory of fluorescence, and as practically all quantitative work has been done with solutions, it seemed worth while to make a careful study of the relation between the fluorescent light emitted by sodium vapor and the light absorbed by the vapor under the same conditions. The chief points of interest which have been brought out by these investigations are the establishment of the fact that the fluorescent spectrum coincides exactly with the absorption spectrum, band for band and line for line, and a determination of the relation existing between the wave-length of the light which provokes the fluorescence and the nature of the fluorescent spectrum. The somewhat remarkable fact has been ascertained that the D line absorption

[1] *Wied. Ann.*, 57, 447, 1896.

is in no way connected with the production of the fluorescence, though the absorption at this point of the spectrum is much more intense than at any other.

By means of improved apparatus we have not only been able to photograph the fluorescent spectrum by means of a concave grating, but have been able to observe by means of a grating the fluorescent spectrum when provoked by approximately monochromatic light obtained with the Fuess monochromatic illuminator. The results of the work throw a great deal of light on the mechanics of the sodium molecule, and will doubtles prove of considerable value in the development of the theory of fluorescence.

APPARATUS EMPLOYED.

The fluorescence as observed in exhausted glass bulbs is never very intense, and the experiments can be continued only for a few moments owing to the speedy corrosion of the glass surface. Moreover, it is not possible to make use of very dense vapor, the generation of which requires a high temperature, owing to its action on glass. The form of tube which was employed by one of us in a previous investigation on the subject[1] enables vapor of great density to be obtained, but owing to the rapid distillation to colder parts of the tube, the experiment cannot be continued long enough for satisfactory photographic records. It is important not only to have a very dense vapor, but also to have the vapor confined within a small region and sharply bounded, in order that the light may not be weakened by absorption before it reaches the denser portions. To meet these requirements a new form of tube was designed and constructed which gave admirable results. With it a fluorescence ten or fifteen times as bright as anything that can be secured with glass bulbs was obtained and maintained continuously for five or six hours without recharging the tube. This tube we have had in action for fully forty hours, and it is only just beginning to show signs of leakage around the brazed joints, due to the action of sodium at a red heat on the silver, with which the joints were brazed. The tube can be very easily constructed, and when

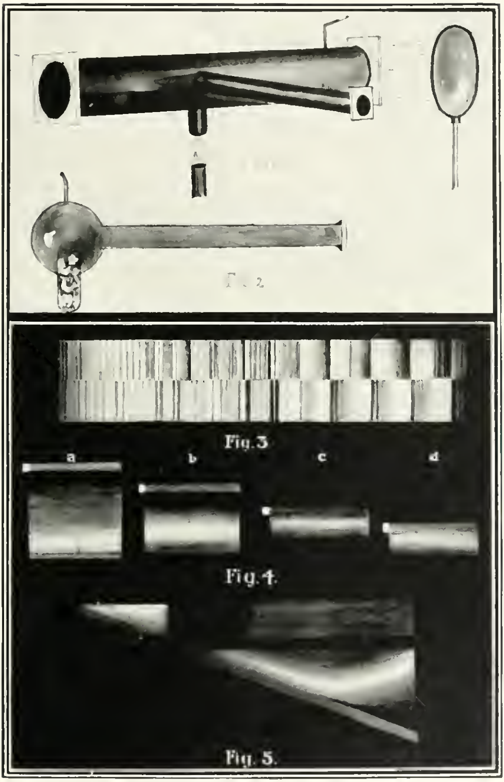
[1] R. W. WOOD, *Proc. R. S.*, 69, 157; also *Phil. Mag.*, (6) 3, 141, 1902.

once charged can be used over and over again without any preparation whatever, for the exhibition of this most beautiful example of fluorescence. Its general form is shown in Plate II, Fig. 1. The large tube was a piece of three-inch (sixteen gauge) Shelby seamless steel tubing, two feet in length. A ten-inch piece of thin three-fourths inch steel tubing was brazed with silver into the large tube at the center, making a right angle with it. (Fig. 2, is a cross section of tube.)

Immediately below this tube, and at a right angles to both, a small iron crucible was brazed into the larger tube, projecting into its interior to such a distance that its rim was visible through the side tube. This crucible was made by boring out a three-fourths-inch iron cylinder. A small piece of brass tubing was also brazed into the large tube near one end, through which the whole could be exhausted. The crucible was filled with lumps of sodium, and the ends of the large tube closed with plate glass, carefully cemented on with sealing wax. The side tube was then closed in the same manner, and the whole exhausted to a pressure of about a millimeter, by means of a mercurial pump. The tube was then sealed off from the pump, a small piece of glass tubing having been cemented into the brass tube and drawn down into a capillary.

The tube was now mounted in a horizontal position and a large Bunsen burner placed beneath the crucible, which was soon raised to a red heat. The dense sodium vapor poured out of the mouth of the crucible and gradually condensed on the cooler portions of the tube. Sunlight from a heliostat was sent down the large tube, a lens placed close to the glass window forming an image of the Sun immediately above the neck of the crucible. On looking down the side tube an intensely brilliant green cone of light was seen, many times brighter than anything that has ever been obtained with glass bulbs. The floating specks of oxide which appear when the tube is first heated, and which shine with a dazzling white light, soon disappear and leave the fluorescence entirely uncontaminated. It is apparent that with this arrangement light enters at once into the densest vapor without suffering previous loss by absorption in vapor of less density.

PLATE II.



Red
Yellow
D_1 D_2

Green blue

Blue

Red Orange Yellow Green Gr.-Blue Blue

Moreover, the fluorescent light passes down the observation tube without having to traverse more than a very thin layer of the vapor, a matter of considerable importance, as we wish to examine the fluorescent light unmodified by subsequent absorption. The large amount of sodium which could be stored in the crucible enables us to deliver a dense stream of vapor in front of the observation tube for a very long time, which is absolutely essential if photographic records are to be obtained.

SPECTRUM OF THE FLUORESCENT LIGHT.

The spectrum of the fluorescent light was first examined with a two-prism Steinheil spectroscope. The spectrum consisted of a red band and a green band, the latter appearing distinctly fluted. No trace of any bright line or band at or near the position of the D lines has ever been seen in any of our experiments. Its presence in the spectrum described by Wiedemann and Schmidt, and also by one of us, may possible have been due to the fact that in both of these cases the vapor was contained in a glass bulb heated by a Bunsen burner. This flame, colored by the sodium of the glass, may have been responsible for the appearance of a bright line in the place mentioned, a matter which can be very easily settled by repeating the experiments with the bulbs.

The marked resemblance which the fluted spectrum bears to the absorption at once suggests that it may be due to the fact that the fluorescent light has to pass through a certain amount of vapor before reaching the spectroscope; in other words, that it does not belong to the fluorescent spectrum at all, but is the result of absorption. To determine whether or not this was the case, an absorption comparison spectrum was formed by throwing some of the light which had passed through the tube into the instrument by means of a couple of mirrors and a small right-angled prism. It was at once apparent that the bright lines and bands of the fluorescent spectrum were exactly out of step with those of the absorption spectrum. To secure a fixed record of this fact, a camera was attached to the spectroscope and the two spectra photographed. The spectrogram confirmed the

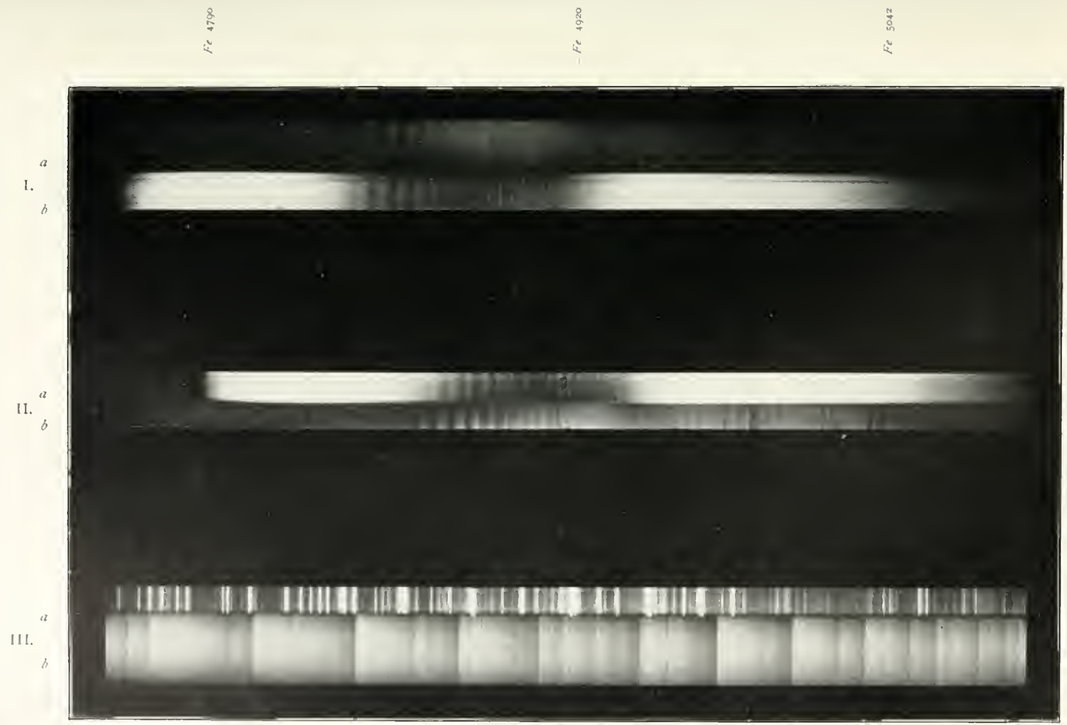
visual observations in every respect, but the dispersion was too small to make the pictures very satisfactory.

A Rowland concave grating with 15,000 lines to the inch, of one meter radius, was then used in place of the spectroscope, and some excellent photographs were obtained with an exposure of less than an hour. The fluorescent spectrum was found to extend from wave-length 5340 to wave-length 4600 in the green and blue region. All of the photographs show in addition a faint solar spectrum extending from the end of the fluorescent spectrum down to the H and K lines. This is due to a small amount of white light which is scattered by occasional specks of oxide, or perhaps reflected from the wall of the tube. So far as we have been able to determine, the fluorescent spectrum is not contaminated with solar lines, since it is located in a less actinic region, and the scattered light is not of sufficient intensity to leave any appreciable record in this region.

These photographs brought out the remarkable fact that the fluorescent spectrum is the exact complement of the absorption spectrum. The two spectra were photographed in contact on the same film, and either one might have been a contact print taken from the other. In the upper spectrum, for example, there were two broad light bands with a fine dark line down the center, while in the lower spectrum occurred two broad dark bands with bright lines down the center. This same thing was true for all of the irregularities of shading in the very complicated fluted spectrum. A number of these photographs are reproduced in Plate III, Figs. 2 and 3. As most of the fine detail will doubtless be lost in the process of reproduction, a very careful drawing of the two spectra has been prepared from the original negative, which is reproduced with the direct records. (Plate II, Fig. 3. Absorption spectrum above, fluorescent spectrum below.)

The fluorescent spectrum in the green-blue region may be divided into three groups of bands: (1) those from λ 5340 to λ 5080, consisting of six broad hazy bands, which correspond in position to the fluted bands of the absorption spectrum; higher dispersion would doubtless resolve them into lines; (2) those

PLATE III.



I (a). Absorption spectrum of sodium vapor. (Region from λ 4750 to λ 5100.)
(b). Corresponding comparison spectrum of iron.
II (a). Fluorescent spectrum. (The solar spectrum showing at the left half of the plate is due to diffuse light.)
(b). Absorption spectrum, taken with same grating.
III (a). Absorption spectrum, (b) fluorescent spectrum, showing their complementary character.

from λ 5080 to λ 4865, a group of irregularly fluted bands, the heads of which point toward the red; *i. e.*, they are strongest on the less refrangible side and shade off on the side of the shorter wave-lengths; (3) those from λ 4865 to λ 4600 which appear, under the dispersion used, as uniform narrow bands. The spectrum is the exact complement of the absorption spectrum taken with the same instrument, and further work with the concave grating of fourteen feet radius will show whether all of the very minute and narrow lines in the flutings of the latter spectrum are present in the former. We feel sure that the spectrum can be photographed with the large grating without difficulty.

Visual observations have shown that the red fluorescence is also fluted, though the flutings are only to be seen when the vapor is very dense and the illumination very intense. It has not yet been definitely proven whether this fluting belongs properly to the fluorescent spectrum or is the result of absorption. A new form of tube has been designed which, it is believed, will give a much denser vapor and make the settlement of this question possible.

The complementary nature of the photographs of the fluorescent and absorption spectra might lead one to suppose that the wave-length absorbed was re-emitted without change of wave-length. To test whether or not this was the case, it was necessary to illuminate the vapor with monochromatic light.

The Fuess monochromatic illuminator, which enables one to cut out a region of any width from a very intense solar spectrum and focus this light at a definite point in space, was arranged so as to send a cone of violet light down the tube, the rays meeting just above the mouth of the crucible. No fluorescence was detected. On gradually increasing the wave-length of the light, by turning the graduated screw which rotated the prisms of the instrument, it was found that the fluorescence appeared when wave-length 4600 was reached. On looking into the end of the large tube a very beautiful phenomenon was seen. The cone of deep-blue monochromatic light was distinctly visible, owing to

traces of oxide floating about, while at the point where the rays met in the dense sodium vapor there appeared a brilliant spot of *green* fluorescent light. As the wave-length was further increased, this spot increased in brilliancy, still remaining green, passed through a maximum, and then gradually faded away, disappearing entirely when the illuminating light became yellow. The vapor remained dark until the wave-length of the light exceeded that of the D lines, when the red fluorescence gradually developed, passing through a maximum in the same manner and then fading away.

The fluorescence of the vapor produced in this way was much less brilliant than in the former experiment, since the total radiant energy thrown into the vapor was very much less than when direct sunlight was employed; still the light sent down the lateral tube was intense enough to give a spectrum when the Steinheil spectroscope was directed down the tube. This spectrum was very feeble, however, and it was only with great difficulty that the changes could be followed which took place when the wave-length of the illuminating beam was changed. When blue light was employed, the complete green fluorescent spectrum seemed to be present with a maximum intensity at the yellow end. As the wave-length of the light was increased, the point of maximum intensity in the fluorescent spectrum moved toward the blue.

The important bearing which the phenomenon has on the theory of fluorescence made a more careful study of the relation between the wave-lengths of the absorbed and emitted light very desirable. To accomplish this, the arrangement of the apparatus was changed in the following manner: the monochromatic illuminator was rotated through a right angle, so that its emitting slit was horizontal, instead of vertical. The dense sodium vapor above the mouth of the crucible was now illuminated with a thin horizontal sheet of monochromatic light (the image of the horizontal slit of the illuminator projected in space by a lens). This arrangement, of course, limits the fluorescence to a thin horizontal layer of vapor, which, when viewed through the lateral tube, appears as a narrow line of bright green light. By viewing the

sheet of vapor edgewise we get a much more intense light, and since its width is small, we can dispense with a spectroscope entirely, simply viewing it through a prism or grating.

A small transmission grating of 14,480 lines to the inch, which gives a first-order spectrum nearly as bright as a 60° flint prism, was mounted in front of the lateral tube, and the line of fluorescent light viewed through it, the head being covered with a black cloth to cut out all extraneous light. It was found that by slightly inclining the large tube to the illuminating beam, it was possible to illuminate a minute projection on the inside edge of the lateral tube with the monochromatic light which caused the fluorescence; in other words, the near end of the horizontal image of the slit was brought upon this projection. The line of fluorescent light was thus tipped at one end with a small point of light similar to the light which produced the fluorescence. The spectrum of this small illuminated spot, which was of course a narrow band, fell alongside of the fluorescent spectrum, enabling a comparison to be made at once. Any exception to Stokes's law would make itself evident as an extension of the fluorescent spectrum on the more refrangible side of the position occupied by the small spot of illuminating light.

Some very remarkable and significant results were obtained with this disposition of the apparatus. Starting with violet illumination, there appeared through the grating only the small comparison spot of light which moved along as the wave-length was increased. As soon as wave-length 4600 was reached, the fluorescent spectrum appeared. Its appearance is indicated in Plate II, Fig. 4*a*. A strip of blue fluoresent light continuous with the reference spot showed that light of the same wave-length as the absorbed light was being emitted by the vapor. A region of darkness occurred on the less refrangible side (usual sense of the term), and then came a broad green band with a maximum of intensity on the end toward the yellow. Then came another gap extending considerably above the D lines, followed by a *very faint* trace of the red fluorescent band (not shown). On gradually increasing the wave-length of the light the following changes were noted. The spot of reference light, accompanied

by its fluorescent prolongation, moved up the spectrum, pushing the dark region before it, while the point of maximum fluorescence in the wide green band moved down the spectrum to meet the advancing spot. These changes are represented in Fig. 4 *b*, *c*, *d*, the point of maximum fluorescence nearly coinciding in position with the reference spot in *d*. On still further increasing the wave-length, the fluorescence became very faint, and appeared to extend a little farther down the spectrum than the position occupied by the small spot of light. This means an exception to Stokes's law when the wave-length of the illuminating light (green) has the maximum value which still yields fluorescence.

In Plate II, Fig. 5, an attempt has been made to represent these changes in the conventional way.[1] Everything shown in this diagram is, however, due to fluorescence, the deviated continuous spectrum of the light exciting fluorescence having been omitted. It will be seen that there is an emission of fluorescent light of the same wave-length as that of the exciting light, which gives the diagram an appearance not unlike the diagrams where the deviated continuous spectrum is shown. To make this diagram exhibit the changes just described, one has only to move a narrow vertical slit across it from right to left.

It was impossible to tell whether flutings were present in any portion of the fluorescent spectrum or not, owing to the comparatively wide source, and small dispersion employed. There is no reason for believing that they are not, but it does not seem safe to assume that the spectra are identical when the fluorescence is produced by white light and monochromatic blue light. The extent of the spectrum, and the distribution of the intensity in it, have been shown to be different in the two cases, and the flutings, say in the green, which are present when white light is used, may be absent when the fluorescence is produced by light of much shorter wave-length. To test this the spectrum will have to be photographed when the tube is illuminated with monochromatic blue light. To accomplish this with the present apparatus would probably require an exposure of six or eight

[1] MULLER-POUILLET, *Lehrbuch der Physik*, II, 1, 368.

hours. More favorable conditions are expected with a new form of tube now under construction, and it seems probable that we shall eventually learn a good deal about the dynamics of the sodium molecule. The results already obtained apparently prove that the light of longer wave-length emitted when the vapor is fluorescing is not the result of damping of the vibration, as assumed in Lommel's theory, but an emission resulting from the fact that the ions of longer free period are set in vibration either by the waves of shorter period or by the ions of short period which are thrown into vibration by these waves. It is not worth while to speculate about this, however, until the fluorescent spectrum produced by monochromatic blue light has been photographed.

In general, the fluorescence of a substance has its maximum intensity when the wave-length of the light is that of the light most strongly absorbed. Sodium vapor is an exception to this rule, for the D line absorption, which is far more intense than the fluted absorption, has nothing to do with the fluorescence. On Lommel's theory of fluorescence, the absence of any lateral emission of light by an absorbing medium is explained in one of two ways: either the absorption is of waves of different period (an octave below, for example) from the free period of the ions, or else the damping factor is so large that the emitted light lies in the infra-red region. In the case of sodium vapor, neither explanation is sufficient to account for the absence of fluorescence when the wave-length of the exciting light is that of the D lines, for, since Kirchhoff's law is obeyed in this case, the absorbed waves and emitted waves have the same period, and the absorption is a resonance phenomenon; moreover, the damping factor must be very small, since we have interference with large path-difference in the case of sodium light. The non-luminous vapor employed in these experiments may, of course, be in a different state from the vapor in a sodium flame, but even if this be the case, it does not seem justifiable to assume a large amount of damping, for this should cause a displacement of the absorption lines with reference to the position which they occupy in the case of absorption by a sodium flame.

It is worthy of note that lines corresponding in position to the fluted bands are absent in the emission spectrum of sodium vapor, except perhaps in the case of the temperature emission studied by Evershed, which does not appear to have been studied under high dispersion.

There seems to be no way of explaining the emission of the green light when the vapor is illuminated with blue light. It cannot be the result of the damping of the ions, whose free period is that of the blue light, for it is in all probability a discontinuous spectrum complementary to the fluted absorption spectrum.

It will be interesting to see whether the absorption of the vapor is directly affected by the circumstance that it is fluorescing at the same time. This was found to be the case in some experiments made by Burke upon uranium glass. It is also important to determine in what way the absorption and fluorescence are influenced by pressure. These matters will be investigated in the near future.

The statement that, when the wave-length of the exciting light is that of the D lines, no fluorescence is produced, requires modification. Strictly speaking, this is not true, though it is almost certain that the D line absorption is in no way responsible. This is due to the fact that the beam from the monochromatic illuminator is not strictly monochromatic, being in fact a band varying from ten to twenty Ångström units in width. As we shall show in the part of this paper dealing with the fluted absorption of the vapor, the fine lines can be traced up to the very edges of the broad band produced by the widening of the D lines when the vapor is very dense. It is unquestionably the absorption at these lines which gives rise to the very feeble reddish fluorescence which can be seen when the light furnished by the monchromatic illuminator is symmetrical about the D lines. Strictly monochromatic light of the wave-lengths of D_1 and D_2, no matter how intense, we feel sure would produce no fluorescence, unless the flutings actually cross this region, which is very likely the case. The only light which produces no fluorescence is green light in the vicinity of $\lambda 5530$ and the violet

below λ 4600, which, as we have shown, is all that is transmitted by the vapor when it is very dense.

THE ABSORPTION SPECTRUM OF SODIUM VAPOR.

The fluted absorption spectrum of sodium vapor was first observed and studied by Roscoe and Schuster in 1874. Subsequent investigations were made by Liveing and Dewar in connection with their work on the reversal of the lines of metallic vapors, and also by one of the writers.

Previous experiments by one of us having shown that it was impossible to secure photographs of the fluted spectrum with the concave grating, using the arc as a source of light, that were not contaminated by bright lines from the vapor of the lamp, it was necessary to find a source of light of great intensity and having a continuous spectrum. After experimenting with various sources of light, we finally adopted the Nernst lamp, which was found to fulfil the conditions specified.

In the first series of experiments the sodium was vaporized in an atmosphere of hydrogen, generated by electrolysis, and dried by passage over calcium chloride and phosphorus pentoxide. In order to remove traces of oxygen which caused the tube to smoke, the gas was finally passed over red-hot copper gauze. The metal was heated in tubes of thin steel, the ends of which were closed with plate glass. In the later experiments the tubes were exhausted with a mercury pump, and the metal volatilized in a vacuum. The latter method was found to be most satisfactory. The spectra were found to be identical in the two cases.

The tubes were either heated by means of Bunsen burners, or by a coil of No. 20 iron wire, insulated from the tube by a thin layer of asbestos board. The vapor is more uniform when the tube is heated electrically, for if the upper side of the tube is colder than the lower, a non-homogeneous medium results, the density being greatest along the floor of the tube. A Bunsen burner is better, however, for some experiments, where a very dense vapor is required. With it the metal does not distil so rapidly to the colder parts of the tube, and, with careful reg-

ulation of the flow of gas to the burner, it was used very successfully in many of the experiments.

The absorption spectrum was photographed with a fourteen-foot concave grating in the first order, the time of exposure varying from twenty minutes to an hour, according to the density of the vapor. The second-order spectrum was observed from time to time during the exposure, in order to keep the density of the vapor properly regulated.

It was found that the best results were obtained with a slit-width of 0.065 mm. The width of the slit for the iron comparison spectrum was 0.030 mm. In order to eliminate any errors due to changing the width of the slit, photographs were taken of both spectra with a slit-width of 0.030 mm, and the measurements made with the two sets of plates compared. The plates were measured on the dividing engine of the laboratory, and the measurements can be considered accurate to within 0.05 of an Ångström unit. The ultra-violet region was explored in the same manner, using a tube closed with quartz plates. No traces of any flutings were found, but the lines of the principal series were strongly reversed.

The fluted absorption spectrum makes its first appearance when the D lines are two or three Ångström units in width. It begins as nine bands, the heads of which point toward the violet end of the spectrum, *i. e.*, in each band the absorption decreases from the head toward the longer wave-lengths. The heads of the bands as they first appear are:

I.	4783.35	VI.	4932.97
II.	4809.92	VII.	4962.96
III.	4837.72	VIII.	5001.94
IV.	4865.60	IX.	5040.71
V.	4894.94		

There is also evidence of absorption lines in the first seven bands. These lines, however, do not come out strong at this stage, but appear as a slight shading on the bright background. In Band I can be seen the line $\lambda 4793.10$; in Band II, line $\lambda 4820.72$; in Band IX, $\lambda 5053.50$, and in Band VII, $\lambda 4979.28$, which appear as fainter heads of small bands into which the

others break up. Bands V and VI are about the center of the region of absorption.

A very slight increase in the density of the vapor is sufficient to bring out the heads of the bands as strong dark lines, as well as the fainter heads in the bands themselves. The bands are now seen to be made up of an immense number of lines, some of which are broad and some exceedingly fine, and in some of the bands there is an appearance as if two series were superposed.

At this time the region of absorption extends from about $\lambda 4600$ to $\lambda 5200$, a series of three bands, the more refrangible edges of which are broken up into smaller bands, appearing between $\lambda 5079$ and $\lambda 5200$, and a series of twelve bands, each of which is divided into smaller bands coming into view between $\lambda 4780$ and $\lambda 4600$. The wave-lengths of the principal lines in the fluted spectrum have been carefully measured by comparing them with standard lines in the iron spectrum.

A fluted absorption spectrum makes its appearance in the red and orange portion, when the density of the vapor is such as to give the green-blue flutings at their best. This spectrum has not been photographed and measured at the present, owing to the difficulty of getting plates sufficiently sensitive to the red. The Erythro plates, which were used by one of us in securing photographs of this region with a fourteen-foot grating some years ago, are no longer on the market, and the few experiments which we have made in sensitizing our own plates have been only partially successful. This region will be studied as soon as suitable plates can be obtained.

As the density of the vapor is further increased, the greenish-blue region disappears entirely, and the fine lines can be traced nearly up to wave-length 5400 from the blue end of the spectrum, and from the red end they are seen to fill the spectrum quite up to the broad D line absorption, which is now forty Ångström units in width. On crossing this broad black band the fine lines make their appearance again, which makes it seem probable that the red fluted spectrum crosses the region occupied by the D lines. A further increase in the density results in blotting

out the red, orange, and yellow completely, leaving only a rather narrow green region, the center of which is at wave-length 5530, and the violet below $\lambda 4600$. The color of the transmitted light is now a very deep violet. The fine lines can be pushed into the green band from its opposite edges, and traces of them have been found all through it. Finally a very black broad band appears at the center of the green strip, which in a spectroscope of small dispersion appears as a rather narrow line. The center of this band is at wave-length 5530, approximately. We have not been able to photograph it with the grating, owing to the feebleness of the light, and the difficulty of keeping the vapor at this great density for a sufficient length of time; but its position was determined visually with a grating of about eight feet radius. This band has been seen by previous observers (Liveing and Dewar) and is described as persisting until the last traces of the fluted spectrum disappeared, as the vapor cooled off. We cannot understand this, for we have found it only when the vapor had the maximum density attainable, namely, when the blast-lamp played directly against the bottom of the tube, raising it to a bright red heat.

In order to see if an increase in the length of the absorbing column of the vapor produced the same effect as increasing the density of a short column, a steel tube five feet in length was used. In this were placed eight lumps of sodium of the same size, at intervals of about six inches. Eight Bunsen burners were regulated so that when one of them was placed under a lump of sodium the D lines were strongly reversed, but no trace of the fluted spectrum appeared. On adding burner after burner, exactly the same sequence of events were observed as in the previous experiments, where the density of the vapor was increased by raising the temperature of the tube. Though this was to be expected, it seemed worth while to try the experiment.

We have measured the wave-lengths of about 460 of the strongest lines between wave-lengths 4616 and 5738, which are given in the following tables :

WAVE-LENGTHS OF THE PRINCIPAL LINES AND BANDS IN THE ABSORPTION SPECTRUM.

GROUP A. λ 4600–5200.

Wave-Length	Character	Wave-Length	Character	Wave-Length	Character	Wave-Length	Character
4616.42	f	4815.84	8 b	4858.05	7 fine	4889.84	8
52.04	f	16.49	6 fine	58.75	8	90.75	7
52.18	b.h.	17.14	7 fine	59.67	7	91.52	9
59.60	b.h.	17.64	8 fine	60.41	8	92.43	7 fine
84.69	fine	18.57	8 b	60.99	fine	93.28	10 b
92.89	7	19.64	8	61.52	fine	94.13	6 fine
4704.36	1	20.72	7	61.78	fine	95.00	H 0.7
07.09	7	21.88	8	62.21	9		wide
09.72	1	23.15	7 fine	63.02	fine	96.06	4
18.29	7	23.82	8	63.78	6 b.h.	97.02	8
21.33	8	24.46	8 fine	64.45	fine	97.92	6
24.42	8	25.81	7 fine	64.88	7 fine	98.96	7 b.h.
28.82	8	26.58	8 fine	65.67	II broad	99.99	6
32.77	6	27.27	7 fine		0.4 wide	4900.94	9
35.78	6 b	27.99	8 fine	66.64	7	01.86	8
36.67	8	28.66	9 fine	67.44	7	02.64	4 fine
40.95	8 fine	30.28	8 b.h.	68.53	7	04.96	6
47.56	8 fine	32.83	8	69.54	8	05.79	7 fine
58.55	7 b.h.	33.47	9 b	70.01	10 b.h. Hs	06.43	4
51.96	8.5	35.36	fine	70.44	7 fine	07.16	7 b.h.
53.83	8.5	36.10	fine	71.33	8 b	08.22	9
58.68	9	36.51	7	72.05	fine	08.84	fine
60.85	7 fine	36.93	7	72.59	fine	09.47	9
61.86	7 fine	37.71	H *broad head*	73.01	fine	10.12	fine
63.05	8 fine		0.5 wide	73.50	fine	10.73	8
64.98	9 b	38.92	9 b.h.	73.93	fine	11.41	6
66.88	9 fine Hs	39.88	8 b.h.	74.46	fine	12.88	7
66.99	6 fine Hs	40.86	9	74.93	8	13.57	7
70.45	7 b	41.60	9	75.55	fine	14.26	8
71.47	7	42.40	8 b	75.95	fine	15.09	fine
77.63	8 fine Hs	43.14	9 fine	76.55	7	15.90	9
81.43	10	44.01	9 fine	77.11	8	16.62	6 fine
82.32	8 b	44.92	9	77.77	7 fine	17.52	7 fine
83.35	H 10 b	45.83	8 fine	78.31	8	18.27	7
89.83	9 fine	46.84	10	78.93	8 fine	19.10	4
90.84	9 fine	47.98	7 fine	79.52	9	19.99	8 fine
93.00	10 fine Hs	49.12	7	80.16	8.5	21.66	6 b.h.
95.30	10 b	49.75	fine	80.80	9.5	22.52	6
99.30	10 fine	50.39	9	81.51	7 fine	23.40	2
4800.74	8 fine	51.00	fine	82.22	10	24.33	7
02.86	8 fine	51.61	9	82.93	7 fine	25.28	7
04.62	10	52.27	fine	83.63	10	27.11	faint
05.77	9	52.89	9	84.33	7 fine	28.14	7
06.46	6 fine	53.65	fine	85.17	8	29.13	7
07.20	9 b	54.35	9	85.88	8	31.22	7 fine
07.90	7 b	55.06	fine	86.59	8	32.20	7 fine
09.93	H 8 b.h.	55.74	9	87.42	7 fine	33.03	H 0.7
10.68	10	56.54	7 fine	88.18	9		wide
13.68	7 fine	57.32	7 b.h.	89.06	7	33.70	fine

WAVE-LENGTHS OF THE PRINCIPAL LINES AND BANDS IN THE ABSORPTION SPECTRUM — *Continued.*

GROUP A. λ 4600–5200.

Wave-Length	Character	Wave-Length	Character	Wave-Length	Character	Wave-Length	Character
4934.27	7	4970.74	8 fine	5002.55	7 fine	5034.36	fine
35.22	9	71.21	fine	06.31	fine	35.38	7 b
36.04	7	71.68	8 fine	06.90	7 fine	36.33	5 b
37.00	fine	72.09	fine	07.62	7 fine	37.34	8 fine
37.61	7 b.h.	72.61	8 fine	08.38	fine	38.17	fine
38.80	6	73.16	fine	08.78	7 fine	39.52	fine
39.54	7	73.70	7 fine	09.26	fine	40.02	fine
40.26	fine	74.28	fine	09.76	7 fine	40.94	H. b. 0.9
40.74	fine	74.40	8	10.24	7 fine		wide
41.29	fine	75.41	fine	10.68	7 fine	42.58	7
42.23	6 fine	75.98	8	11.26	7 fine	43.44	fine
43.16	7	76.61	7	11.77	7 fine	44.93	fine
43.82	fine	77.17	9	12.36	7 fine	44.59	fine
44.37	8	77.85	fine	12.83	7 fine	48.81	6
44.97	7	78.45	8	13.38	7 fine	53.39	6 Hs
45.53	fine	79.20	4	13.99	7 fine	53.49	10 Hs
46.10	7	79.91	6	14.50	7 fine	54.46	8 b.h.
46.72	7	80.56	7	15.13	7 fine	56.96	8 b.h.
47.49	7 fine	81.26	8	15.77	7 fine	58.34	7
48.07	7 fine	82.00	6	16.47	7 fine	58.97	7
49.24	8	82.65	7	17.06	7 fine	61.07	7
50.08	8	84.08	8	17.77	7 fine	62.79	7
50.83	fine	86.52	6	18.38	7 fine	63.38	8
51.50	8	87.18	6 fine	19.05	7 fine	64.11	fine
52.23	2 fine	88.19	6	19.68	6 fine	64.84	h
52.97	2 fine	88.98	10 b	20.42	8	66.69	7
54.39	10 b	90.90	5	21.14	6 fine	67.37	7
56.86	2	91.52	fine	21.89	8	68.29	b
57.70	2 fine	92.56	9 b	22.61	6	69.08	6 b
59.43	8	93.34	fine	23.37	7	69.82	7 b
60.15	fine	94.18	5 b.h.	24.18	6 fine	80.38	H 10 fine
61.06	6	95.15	4 fine	24.99	7	87.37	H 7 b
62.13	fine	96.11	fine	25.77	5	95.89	9 H
63.02	H	96.53	fine	26.33	7	5118.78	Hs b
63.80	f.h.	97.29	3 b	27.37	fine	26.84	b
64.88	10 b.h.	98.05	8 b	28.21	8	33.80	Hs b
66.81	fine	99.25	8	28.98	7 fine	41.27	Hs b
67.37	fine	5000.09	6 b	29.99	8		
68.79	8 b.h.	00.86	6	30.73	7 fine		
69.31	fine	01.84	H 0.7 wide	31.76	8		
69.77	7 fine		wide	32.53	6		
70.23	fine	02.17	fine	33.47	5		

Group C. λ5300–5700.

The lines of this group are all of about the same intensity and very faint. Only the strong lines of intensity 8 or greater are marked.

Wave-Length	Character	Wave-Length	Character	Wave-Length	Character	Wave-Length	Character
5269.38		5409.78		5572.53		5651.94	broad and
75.34		14.96		74.82			strong 8
79.58		16.52		75.90		52.80	strong
5305.68		18.90		76.81		54.27	strong
11.47		25.42		81.86		55.83	
17.65		26.20		83.07		59.97	strong
25.64		32.68		84.59		64.12	
30.61		33.63		93.45	strong 8	65.63	broad
34.85		39.25		94.87	strong 8	71.25	
37.70		40.33		5602.04	broad	72.68	strong 8
43.66		41.36		03.34	broad	79.03	strong 8
46.47		48.20		07.60		83.01	strong 8
49.67		52.94		12.31		84.30	
52.44		58.09		13.78		89.70	
58.36		63.11		19.09	strong 8	97.87	strong and
65.31		82.21		20.56	strong 8		broad 9
66.09		96.99		26.31	strong 8	5706.08	strong and
70.28		5502.48		28.13	strong 8		broad 9
71.23		23.48		32.79		10.84	strong 8
78.19		28.58		36.90		12.48	
82.12		30.44		38.02		13.56	
83.38		33.16		41.18		14.72	
84.33		35.89		42.26		17.23	
90.46		40.03		43.55	broad	18.05	
91.41		59.52		45.37	broad	20.82	
97.77		62.85		46.92	broad	34.78	
98.46		64.28		48.78		36.89	
5402.61		65.70		49.60		38.32	
03.90							

Johns Hopkins University,
Baltimore, June 1903.

THE SPECTRUM OF HYDROGEN.

By Louis A. Parsons.

Ångström[1] was the first to observe the hydrogen spectrum. In 1853, upon examining the light from an electric spark in hydrogen at atmospheric pressure, he noticed "an intense line in C not clearly limited, and two maxima in F and G; and a third maximum in *h* was observed later." Plücker and Hittorf[2] were the first to employ glass tubes with metallic electrodes and provided with capillary portions, and to study the spectra of gases at low pressure which were rendered luminous by an electric discharge. They found that the spectrum of some gases they examined was different when they employed a condenser in the discharge circuit of their induction coil, from that produced by the induction coil alone, and concluded that the former was due to higher temperature resulting from the condenser discharge. They obtained by means of the condenser discharge in hydrogen the three bright lines which they designated as $H\alpha$, $H\beta$, and $H\gamma$. These lines they found expanded as the pressure of the gas was increased, and at high pressures the spectrum was transformed into a continuous one with $H\alpha$ as a broad band rising from it. Without the condenser they observed another spectrum of hydrogen, "consisting of very many bright fine lines, especially in the neighborhood of the sodium line. This spectrum," they observed, "may be seen simultaneously with the three characteristic lines $H\alpha$, $H\beta$, and $H\gamma$; but at increased temperature, when these lines begin to expand, it entirely disappears." With water vapor alone they obtained the lines $H\alpha$, $H\beta$, and $H\gamma$.

There are now generally recognized two spectra of hydrogen: (1) the "elementary" or "four-line" spectrum, and (2) the many-line spectrum. The former includes the lines $H\alpha$, $H\beta$, $H\gamma$, and $H\delta$ in the visible portion, the last of which was soon added to the three given by Plücker and Hittorf, and nine lines in the ultra-violet. The many-line spectrum has also been designated

[1] *Phil. Mag.*, (4) **42**, 395, 1871. [2] *Phil. Trans.*, **155**, 1, 1865.

as the "compound spectrum" or the "secondary spectrum." The work of various observers has shown that the two spectra may exist together, and that in general as the lines of the elementary spectrum broaden the compound spectrum disappears. The question was early raised as to what caused the broadening of the lines, and what determined the presence of one spectrum or the other. Plücker and Hittorf, Wüllner,[1] and others[2] believed that the broadening of the lines and the change in the four-line spectrum was due to increased temperature. Others believed the effect was due to pressure,[3] and still others that the intensity of the electric discharge was the controlling factor.[4]

Trowbridge and Richards in 1897[5] found that a continuous discharge from their high potential storage battery of 5,000 cells, obtained by attaching the terminals directly to the electrodes of the vacuum tube with a high resistance in series, always gave a white glow in the capillary and the many-line spectrum of hydrogen, while the discharge from a condenser of considerable capacity gave the four-line spectrum, the color of the discharge being deep red. The introduction of self-induction, or of resistance to damp the oscillations, changed it back to the many-line spectrum. They believed that one spectrum was characteristic of the oscillatory and the other of the non-oscillatory discharge, the character of the electric discharge being the determining factor.

Further work, in which he employed his enlarged battery of 20,000 cells, however, led Trowbridge to abandon this idea[6] and to come to the conclusion that the many-line spectrum is the true spectrum of hydrogen, while the so-called four-line spectrum is due to water vapor; the existence of the four lines in the solar spectrum being, as he believed, due to the presence of water vapor in the Sun. His reasons for the conclusion were: (1) With powerful condenser discharges in apparently dry hydrogen he always obtained the four-line spectrum, and essentially the same

[1] *Phil. Mag.*, (4) **37**, 405, 1869. [2] FIEVEZ, *Comptes Rendus*, **92**, 521, 1881.

[3] SCHUSTER, *Brit. Assoc. Rep.*, **39**, 1873.

[4] STEARN and LEE, *Phil. Mag.*, (4) **46**, 406, 1873.

[5] *Phil. Mag.*, (5) **43**, 135, 1897. [6] *Ibid.*, (5) **50**, 338, 1900.

spectrum whether the tube was filled with nitrogen, air, or hydrogen, notwithstanding the greatest care taken in drying the tubes. He believed it was impossible to so thoroughly dry glass tubes as to remove all water vapor, and that the water on the glass is torn off by the powerful condenser discharges. (2) The four-line spectrum was easily obtained in all cases where water vapor was known to be present. (3) After a partial melting of the aluminium electrodes it was difficult or impossible to get the many-line spectrum (owing to the formation of compounds), but the four-line spectrum was obtained as easily as before with the powerful condenser discharges.

In a later paper Trowbridge[1] described some further experiments on high potential discharges through hydrogen. In one experiment, by using heavy copper electrodes he obtained by means of a continuous current a deposit of bright copper around the kathode and of oxidized copper at the anode, "the dissociation of the water vapor thus showing an electrolytic action closely analogous to that of a voltaic cell." He repeats his conviction "that the so-called line spectrum of hydrogen cannot be considered apart from the spectrum of water vapor," and is also "led to the conclusion that a certain amount of water vapor is essential to all electrical discharges through gases." He says: "Just as aqueous vapor seems to play an important rôle in most chemical reactions, so, it seems to me, its presence in rarified gases enables dissociation to take place which determines the strength and character of the electrical discharge;" and in another place: "The passage of electricity through a gas depends on the dissociation of the hydrogen and oxygen, by means of which change in the distribution of energy the gases are made luminous." While still holding that the four-line spectrum, as distinguished from the many-line one, is that of water vapor, he here advances to the idea that water vapor is essential to all electric conduction, in that it makes possible the passage of the current. During the passage of the current there may be produced the characteristic spectrum of the gas and also the spectrum of water vapor. In a recent article Trow-

[1] *Amer. Jour. Sci.*, 12, 310, 1901.

bridge speaks of another spectrum of water produced by condenser discharges of great intensity giving a brilliant white light, which spectrum is continuous throughout the visible portion and consists of bands in the ultra-violet. These "spectra of water vapor" he believes are spectra arising from dissociation of water molecules.

J. J. Thomson in 1895[1] observed what appeared to be electrolysis of hydrogen gas in a vacuum tube, or the separation of positive and negative hydrogen ions. Using a tube with a metallic diaphragm, and allowing the current to run in one direction for some time, he obtained the *Hα* line strong on the positive side of the plate and *Hβ* faint, while the reverse was true on the negative side. His interpretation was that on the negative-side there was an excess of positively charged hydrogen ions, and on the positive side of negatively charged hydrogen ions,

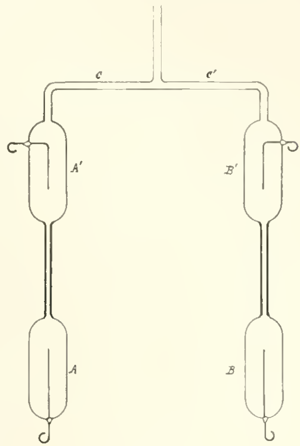


FIG. 1.

the former having a natural vibration giving *Hβ* and the latter one corresponding to *Hα*. He also obtained by passing the current in one direction for a time through a mixture of hydrogen and chlorine, the hydrogen lines bright near the kathode and the chlorine lines weak or absent, and the reverse at the anode. This experiment was called in question by Morris-Airey[2] working in Kayser's laboratory, who believes that these effects were due merely to temperature differences. He employed two tubes, connected as shown in the figure, running a continuous current through a mixture of chlorine and hydrogen between electrodes *A* and *B* until the chlorine lines practically disappeared and the hydrogen lines became very bright at the kathode, and then

[1] *Nature*, 52, 451, 1895. [2] *Phil. Mag.*, (5) 49. 307, 1900.

he immediately sealed off the tubes at c and c'. Afterwards when they were used as separate tubes they gave identical spectra.

In a number of the *Philosophical Magazine* about a year ago, Lewis[1] discussed "the rôle of water vapor in gaseous conduction" suggested by Trowbridge's paper. He says: "It seems that the complete removal of water vapor from vacuum tubes is perhaps possible, and still the hydrogen or other gases contained therein may transmit the current and give their characteristic spectra." Deslandres, Wiedemann, and Lewis himself all found that by heating sodium in a tube containing hydrogen, although the hydrogen lines were weakened when the sodium vapor was dense, they came out with their full intensity when the sodium was cool. Using hydrogen with the same treatment, however, Deslandres succeded in getting rid of the water vapor lines. Lewis put pure sodium in a tube filled with hydrogen, and after heating it frequently to drive off impurities, he filled it with pure hydrogen, and then formed a "sodium dew" over the capillary and walls of the tube. Whenever it was heated the hydrogen lines were weak, but came out quite strong when the tube was cooled, and were just as strong after the tube had been used continuously for two weeks. When the tube was broken there were found large patches of sodium whose metallic surface was not tarnished, which led Lewis to observe that whatever other impurities might have been there, it is difficult to see how any free water vapor or oxygen could have been present." He concludes that "it seems at least doubtful whether water vapor is necessary to gaseous conduction."

Wilsing[2] recently made some interesting experiments on the spectrum of hydrogen. Making the usual observation that in general the introduction of a spark gap changed the many-line to the four-line spectrum and broadened the lines, he remarks that it cannot be an electrical resonance effect due to the oscillatory discharge, for the effect was not changed by varying the

[1] *Phil. Mag.*, (6) **3**, 512, 1902.

[2] "Investigations on the Spectrum of *Nova Aurigae*," *Pub. d. Astrophys. Observ. zu Potsdam*, **12**, 77–102, 1900.

period of the oscillation; and he comes to the conclusion that it is due to high temperature resulting from the condenser discharge.

A large number of the ultra-violet lines of the elementary spectra have been found as bright lines in the chromosphere of the Sun, but, on looking for their reversals, among the Fraunhofer lines of the solar spectrum, those of shorter wave-length $H\delta$ (H line) do not appear. The possibility of certain conditions of temperature or pressure in the photosphere being responsible for this suggests itself. Campbell[1] found that in the spectra of certain stars, the ultra-violet hydrogen lines are dark, while those of greater length are bright, there being a regular gradation between the two; and Kayser[2] calls attention to the fact that this is in accordance with the deductions of Kirchhoff's laws, assuming that they apply to this radiation.

Schumann[3] on examining the spectra of hydrogen from an end-on tube found, on varying the pressure, what appeared to be a reversal and a displacement of $H\beta$ or separation into components. Trowbridge[4] recently obtained a reversal of hydrogen lines in employing his intense condenser discharges, which he attributed to a selective reversibility of the silver salts of the photographic plate, and concluded that it was unsafe to draw conclusions from reversals as to the temperature, pressure or any other condition of the source, and considered this to have a very important bearing on astrophysical problems. Wood,[5] however, obtained effects entirely analogous to those of Trowbridge by subjecting a photographic plate to two light radiations, one of short duration and great intensity, followed by one of relatively great duration and feeble intensity (the Clayden effect); and he also found no selective reversibility in the silver salts, and concluded this could not have any bearing on astrophysical problems.

It is because of the bearing on astrophysical problems, the question of the physical condition and constitution of the heavenly bodies, that the determination of the exact conditions governing

[1] Astrophysical Journal, 2, 177, 1895.

[2] *Ibid.*, 4, 313, 1901.

[3] *Astronomy and Astro-Physics*, 12, 159, 1893.

[4] *Phil. Mag.*, (6) 4, 156, 1902.

[5] *Phil. Mag.*, (6) 4, 606, 1902.

the changes in the hydrogen spectrum become of great importance. There are certain differences in the spectra of hydrogen as obtained in the laboratory and in the spectrum of the Sun and of the stars. Wright and Campbell,[1] from measurements of hydrogen lines in the spectrum of *o Ceti*, came to the conclusion that there was an error of one-tenth of an Ångström unit in the value of the wave-length of $H\delta$ given in Rowland's tables of the solar spectrum. The same conclusion has been reached by Evershed from measurements of the flash spectrum obtained in India during a total eclipse of the Sun. Jewell,[2] after carefully going over Rowland's measurements again, however, concluded that there was no error in the original calculation, the wave-length of $H\delta$ as a dark line in the Sun being 4102.00, while the value obtained by Evershed and by Wright and Campbell for $H\delta$ as a bright line in the Sun and stars (4101.85) agreed with the value obtained in vacuum tubes, which last also agreed with the place as given by Balmer's formula ($H\epsilon$ in the Sun also is not in accord with Balmer's formula).

Not all of the lines of the four-line spectrum of hydrogen appear in the dark-line spectrum of the Sun. Only $H\alpha$, $H\beta$, $H\gamma$, and $H\delta$ are distinct; those of shorter wave-lengths are of a different character—broad, hazy, and indistinct; and those of shortest wave-length are not distinguishable at all. This also is at variance with the four-line spectrum as ordinarily observed in the laboratory.

The elementary, or four-line, spectrum, according to the classification of spectra founded on the numerical relations and character of the lines, is not the principal but the first subordinate series. In 1897 Pickering[3] discovered another series of hydrogen lines in the spectrum of the star *ζ Puppis*, which according to Kayser[4] and Rydberg[5] constitute a second subordinate series following a different law from the first series, but ending at the

[1] W. H. WRIGHT, ASTROPHYSICAL JOURNAL, 9, 50, 1899.

[2] ASTROPHYSICAL JOURNAL, 9, 211, 1899.

[3] *Harvard College Observatory Circular* No. 12; ASTROPHYSICAL JOURNAL, 5, 92, 1897, also 13, 230, 1901.

[4] ASTROPHYSICAL JOURNAL, 5, 95, 243, 1897. [5] *Ibid.*, 6, 233, 1897.

same point in the spectrum. Rydberg also calculated the wave-lengths of the principal series of hydrogen, beginning at the same point in the spectrum as the second subordinate series. The second subordinate series, though observed in some stars, has never been obtained in the laboratory, and the principal series has never been observed at all.

The spectrum of hydrogen, then, is not by any means invariable; there are differences in the spectrum as observed in the Sun, in the stars, and in vacuum tubes in the laboratory; and in the latter there are observed changes due to different conditions of the discharge. Because of the bearing of the latter on the astrophysical problems involved, it becomes of great importance to accurately determine the causes of the changes we are able to bring about in the laboratory so as to have a guide in interpreting the particular variations in the spectra of the heavenly bodies. What, we are led to inquire, is the cause determining the occurrence of the four-line and the many-line spectrum, and of the changes in the spectrum as the conditions are varied? Is it high temperature, pressure, the electrical conditions of the circuit, or water vapor, or what? It was to obtain evidence to aid in answering these questions that the experiments described here were undertaken.

APPARATUS.

The hydrogen used in these experiments was prepared by the electrolysis of dilute sulphuric acid, and was passed through a system of tubes containing substances for drying and purifying to the Geissler tubes, and thence to another series of tubes to the mercury pumps used in exhausting, making a completely closed system about sixty feet long from generator to pump, with only one stop-cock at the generator. Calcium chloride, sulphuric acid, caustic potash, potassa-lime, and phosphorous pentoxide were used for drying, sulphur and gold foil for the removal of the mercury, and heated copper foil to remove oxygen. There was no trouble from hydrocarbon vapors, oxygen, or air after everything had been pumped out thoroughly, and most of the time there was no trouble from mercury vapor, the mercury lines not appearing in the spectrum, or else very faintly.

As a source for the electric current a storage battery of 4000 cells was constructed. For obtaining the continuous discharge the battery was connected to the vacuum tube with a water resistance of about a million ohms in series. A milliammeter was used to give the current. The oscillatory discharge was obtained by connecting the terminals of the battery to a glass plate condenser, equivalent to one or more Leyden jars, which in turn was connected through a spark-gap and coil of self-induction of 0.009 henry (which coil could be short circuited at will) to the vacuum tube.

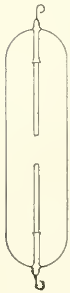


FIG. 2.

A number of tubes were used of different diameters and with different distances between the electrodes, and the spectrum was observed with a Steinheil spectroscope as the pressure of the gas, the temperature, and the electrical conditions of the circuit were varied. Some of the tubes had capillaries, but most of them had none, being of uniform diameter of from 6 to 22 mm. The electrodes in most cases were of aluminium rod 2 mm in diameter. The pressure of the gas was varied from nearly an atmosphere down to a millimeter (and sometimes a fraction of it).

CONTINUOUS DISCHARGE — EFFECT OF PRESSURE.

The continuous current of a few milliamperes from the battery always gave the many-line spectrum and the four lines all sharp and fine even at the highest pressures (500 mm, or even 700 mm). The discharge in wide tubes was characterized by a bright point on the anode (which appeared always to give the many-line spectrum), and a white velvety glow at the kathode, which appeared on the tip of the electrode at high pressures and gradually expanded until it filled that part of the tube at a pressure of a millimeter, and in which $H\beta$ could always be seen. There was no essential difference in the action of the tubes with different electrode distances, as the pressure was lower; the discharge ceased sooner in the tubes with the electrodes close together than in those with electrodes farther apart. In the capillary the spectrum was faint at high pressures, appearing first in

the green and blue, with $H\beta$ not as bright as other fine lines in that region. With decreasing pressure the spectrum first spread toward the red and violet, and then the intensity of the whole increased, and also the relative intensity of the four lines, until the point of maximum resistance was reached, beyond which the process was reversed.

VARIATION OF CURRENT DENSITY.

The same changes that occur in the glow discharge and in the spectrum during a decrease of pressure were produced by increasing the current either by an increase of voltage or a decrease of external resistance, but the changes also took place with decreasing pressure at constant current (though less rapidly than with decreasing pressure and constant external resistance).

The abruptly oscillating discharge, occurring with the condenser and spark-gap, gave the deep red spark discharge between the electrodes, and always tended to broaden the lines and to increase the relative intensity of the four lines. At high pressures, with considerable capacity, the four lines appeared very bright and broad, while the *Al* lines from the electrodes were bright and fine. With decreasing pressure the four lines gradually became narrower, $H\alpha$ first becoming fairly sharp, and then $H\beta$. All decreased in intensity, but $H\delta$ faded away relatively quite rapidly. The electrode lines disappeared in the central portion, and finally extended only a short distance from the electrodes. At a pressure of more than 100 mm the many lines appeared in the red and orange, at first broad and indistinct and massed together, but sharp and distinct at lower pressures. With decreasing pressure the many lines increased in intensity relatively to the four lines. The introduction of self-induction even at pressures as high as an atmosphere made the four lines narrow, and brought out the many lines quite distinct and fine. Excepting at very high pressures, self-induction increased the intensity of the many-line spectrum, and decreased the intensity of the four lines.

A tube 15 mm in diameter, with electrodes 6 mm apart, showed a heavy red spark discharge at high pressure, which,

when the pressure was lowered to about 100 mm, contracted to a cone with its base at the anode and extending almost to the kathode, and was surrounded by a blue sheath broadest at the kathode. The blue glow gave a faint continuous spectrum and, apparently, the many-line spectrum of hydrogen, though faint.

Fig. 3.

The red discharge gave the four lines broad and bright near the electrodes, $H\alpha$ and $H\beta$ extending all the way across and rather fine near the kathode, while $H\gamma$ and $H\delta$ were seen only near the anode. With decreasing pressure the red cone shortened until it became a point on the tip of the anode, and finally disappeared, leaving the discharge entirely blue. Another tube of a larger diameter, but in all other respects identical with this one, gave the deep red discharge throughout this entire range of pressure. It was at first difficult to see how two tubes, both connected to the apparatus and at precisely the same pressure, and differing only in diameter (and in that very slightly), would act so differently. It was easily shown, however, that the resistance of the tube was the determining factor, being high enough in the narrower tube to prevent the abruptly oscillating discharge which took place easily in the larger one. When a high external resistance was put in series with the large tube, it gave the blue discharge similarly to the narrower tube, and when the capacity was increased the red discharge occurred in the narrow tube at comparatively low pressures. In fact, in all the experiments, whenever the electrical conditions of the circuit were such as to give the abruptly oscillating discharge, the red discharge and the four-line spectrum resulted. Tubes of different diameters were employed, but they showed no differences, excepting in so far as the greater resistance of the narrower ones causes the change to the blue spectrum at higher pressures.

Fig. 4.

EFFECT OF HEATING.

If the four-line spectrum is due to water vapor, one might expect an external heating of the tube sufficient to drive off water vapor, to cause an increase in the red discharge, and a corresponding increase in the intensity and width of the four lines. Or if the four-line spectrum is due to high temperature (of the gas as a whole), we should look for the same effect when the tube is heated. However, heating the tube externally by means of a Bunsen burner, or internally by continuing the discharge for some time, instead of having the effect of increasing the amount of red in the discharge and producing a change in the direction of the four-line spectrum, had the opposite effect, decreasing the red or changing to the blue discharge.

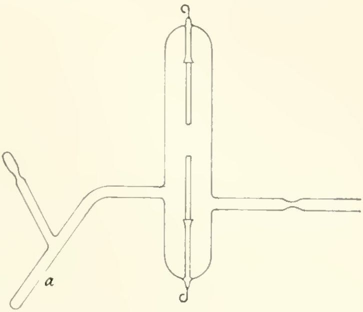


FIG. 5.

EFFECT OF WATER VAPOR.

Two tubes, as nearly identical in all respects as possible and of the form shown in the adjoining figure, were connected, one, *A*, directly to the apparatus, and the other, *B*, through drying tubes of calcium chloride and phosphorous pentoxide. In the bulb of *B* was placed about $\frac{1}{10}$ cu. cm of water, which was kept frozen while the tubes were being exhausted, washed out, and filled with hydrogen. The pressure was lowered to 44 mm, when the discharge was blue, with red extending only a short distance from the tips of the electrodes, and identical in the two tubes. The ice was melted and a short time allowed for the water to evaporate. No effect was observed then or when the water was slightly warmed. But when the water was boiled so that some passed over into the main tube, the discharge in *B* became red all the way across, and $H\alpha$, $H\beta$ and $H\gamma$ became much brighter.

The discharge remained very decidedly red after running for some minutes, when it was certain that the pressure was the same in the two tubes. Tube *A* gave the blue discharge with the red on the tips of the electrodes, just as before. On cooling the side tube in the freezing mixture, the discharge became decidedly less red. When nearly all of the water was boiled over into the main tube, the discharge became very red, and $H\alpha$ and $H\beta$ came out very bright and broad, $H\gamma$ was broad, and $H\delta$ seemed to be present, faint, and broad. On again placing the side tube in the freezing mixture and passing the current through it, the discharge became decidedly less red, and finally became somewhat blue, though not as much as the other tube.

The experiments appear to show a very marked connection between the four-line spectrum and the presence of water vapor or water. But it is to be noted that the effect was produced only when water or water vapor was present in large quantities, and did not appear in the least with only small traces of the vapor. With large quantities present some evidently was on the electrodes and in the path of discharge, and the vaporization and dissociation of the water by the current would produce a high local increase in the pressure; and the effect observed was exactly that which would have been brought about by an increase in pressure. Or, the sudden ionization of the water might well produce a violent change in the passage of the electric current—causing the abruptly oscillating discharge of which the four-line spectrum was characteristic.

When one of the tubes cracked, the presence of the air leaking in was shown by the oxygen and nitrogen bands. With the discharge without the condenser, or with the condenser and self-induction, most of the current was carried by the nitrogen, scarcely anything else being seen in the spectrum excepting the nitrogen. With capacity without self-induction and with a small spark-gap, the oxygen predominated, while with a large spark-gap giving a very definite disruptive discharge the current was carried almost entirely by the hydrogen, the four lines coming out brilliantly.

DISCHARGE IN AIR.

A system of four tubes varying in diameter from 6mm to 22mm, all with electrodes about 7mm apart, and a bulb 90mm in diameter, with electrodes 10mm apart, were first used, when filled with air, before any hydrogen had been admitted. Excepting at quite low pressures, the walls of the bulb were so far away from the discharge as not to be at all heated or apparently affected by the discharge in any way. In the narrower tubes, where the discharge reached the glass walls, $H\alpha$ and $H\beta$ were seen very bright (especially $H\alpha$) and broadened throughout a great range of pressure, while they were not seen at all under like conditions in the bulb. This pointed very strongly to the conclusion that the four-line spectrum is due to water vapor driven from the walls of the tube. But although $H\alpha$ was bright in the narrower tubes, $H\beta$ was usually comparatively faint, $H\gamma$ was never seen with certainty, and $H\delta$ not at all. When the tubes were filled with dry hydrogen, it is not likely that they contained more water vapor than when filled with air from the room, and yet, with the hydrogen, all the tubes, including the bulb, showed with the condenser discharge $H\alpha$, $H\beta$, $H\gamma$, and $H\delta$, all very bright and broad. The four-line spectrum was plainly not the same in air as in hydrogen, and certainly the evidence was not in favor of the assumption that the four-line spectrum is due only to water vapor and not at all to hydrogen as such.

It may be, as Trowbridge claims, that "a certain amount of water vapor is essential to all electrical discharge through gases," and that the four-line spectrum of hydrogen is intimately associated with the dissociation of water vapor. The idea that a perfectly dry gas of any kind not ionized is a non-conductor of electricity is certainly not a new one. In a paper published in the *Philosophical Magazine* in 1893,[1] J. J. Thomson considered the effect upon the formation of drops and said: "In a vacuum tube with electrodes at high potential, the presence of the electric charges would tend to form drops of water, if there should be any water vapor present;" and he concluded that the mole-

[1] (5) 36. 313.

cules might be dissociated into charged ions by condensation on the surface of little spheres of water or of any other liquid of high specific inductive capacity, and these charged ions in turn would facilitate the formation of other drops of water. This action would render easy the passage of a current from one electrode to the other. The presence of water vapor in this way would tend to ionize whatever gas were present in a vacuum tube, and thus render possible the passage of a current and put it into a condition to vibrate in such a way as to give its characteristic spectrum. But it cannot be said that water is absolutely necessary for this, for the same might be done by any liquid of high specific inductive capacity. In fact, anything that could ionize the gas in the tube, as X-rays, Becquerel rays, ultra-violet light, or the *Entladungstrahlen* from a spark, would render the gas conducting, and in a perfectly dry gas, not ionized with high enough potential, we may obtain a disruptive spark discharge. And a gas that from any cause whatever is carrying electricity through it, it seems fair to assume, may be giving off its characteristic spectrum, entirely apart from the presence of water vapor or any other foreign substance.

As stated before, some of the experiments described in this paper, as well as the experiments of Trowbridge, point to a connection between water vapor present in a tube and the four-line spectrum of hydrogen. It does not seem, however, that one is warranted in concluding that the four-line spectrum is the spectrum of water vapor. It may be that this spectrum is due to the particular vibrations produced at the instant of dissociation of water molecules, or of recombination of hydrogen and oxygen ions. But may it not even in that event be that the four-line spectrum is a true spectrum of hydrogen, which is due to the particular vibration of the hydrogen ion occurring at the instant of ionization, which ionization may be that of a hydrogen molecule, or of a molecule of any other compound of hydrogen as well as of water, while the natural vibrations of the free hydrogen ion give rise to the many-line spectrum? The experiments stated in this paper could be explained in this hypothesis, and could not the experiment of Trowbridge as well?

But we are not excluded from the possibility that pressure or the electrical conditions of the circuit are the determining factors. This much seems certain, whenever the discharge is comparatively steady, as with a battery, the many-line spectrum is produced, while the abrupt oscillating discharge always gives the four lines bright. The presence of water vapor in quantity (as in one of these experiments) would in the electric field result in the formation of drops of water, the vaporization and the dissociation of which would, as has been stated, cause great local increase of pressure, which would account for the facts observed, if we consider it a pure pressure effect.

The most natural conclusion from this investigation is that the four lines are increased in intensity and in width whenever anything occurs that renders the electrical discharge discontinuous and very abrupt and violent. As to what the process or mechanism is by which the characteristic mode of vibration is produced, whenever this occurs, we cannot say.

A series of photographs of the hydrogen spectrum were taken with a quartz spectrograph with capacity and self-induction, at pressures ranging from about 700 mm down to 2 mm, which gave results in harmony with the eye observations already noted. A point of interest that attracts one's attention in looking over the plates is that the lines $H\epsilon$, $H\zeta$, $H\eta$, $H\theta$, etc., assigned to the four-line spectrum, are either not present at all or else do not seem to behave in the same way as $H\alpha$, $H\beta$, $H\gamma$, and $H\delta$, which are very intense and broad at high pressures, and gradually decrease in width and intensity with decreasing pressure; while the others are faint and narrow at high pressures and change very slightly if at all with decreasing pressure.

CONCLUSIONS.

1. The "compound spectrum," "secondary spectrum," or "many-line" spectrum of hydrogen never occurs without the four lines. The four-line spectrum may occur at high pressures without the many lines.

2. The four-line spectrum is characteristic of an abruptly oscillating discharge, and the many-line spectrum of the contin-

uous discharge of the battery, the less violent oscillatory discharge which occurs with self-induction in the circuit giving a spectrum midway between the two—the many-line spectrum with the four lines very bright and fine.

3. The production of the four-line spectrum, or the broadening of the lines, may be due to high temperature occurring along the path of discharge, or locally at points where the disruptive discharge takes place, *i. e.*, it may be due to an increased translatory motion of the particles which have this mode of vibration—a point which it seems impossible to determine. But it seems certain that this is not due to high temperature, if we mean the average temperature of the gas in the tube.

4. It is likely that the presence of water vapor in a tube plays an important rôle in the conduction, because of its effect in ionizing the gas, but it seems also likely that other agencies may produce the same result. There seems to be a connection between the presence of a quantity of water vapor and the brightness of the four lines, and it may be that the process of dissociation of water gives rise to vibrations producing the four-line spectrum. But there does not seem to be any evidence for saying that the four-line spectrum is the spectrum of the water vapor. On the contrary, the "four lines" appear to be true lines of hydrogen.

In conclusion I wish to acknowledge my obligation to Professor Ames, at whose suggestion the work was undertaken, for the kindly and helpful interest he has always shown; to Professor Wood, for his suggestions; and to students who have assisted me, particularly Mr. George W. Middlekauff, who was for a considerable time associated with the work.

Johns Hopkins University,
May 1, 1903.

SOME EFFECTS OF CHANGE OF ATMOSPHERE ON ARC SPECTRA WITH REFERENCE TO SERIES RELATIONS.[1]

By A. S. King.

Since the discovery of series relations in the spectra of a number of elements, those investigators who have studied the changes produced in such spectra by variation of physical conditions have noted in many cases that lines belonging to a series are affected differently from the unrelated lines; and that when two or more series are present, they may be affected in different ways by change of condition. Such changes in physical conditions not only furnish us with a means of separating the lines of an element into groups between the members of which series relations may be looked for, but we may be enabled to draw conclusions as to the character of the vibrations which give rise to such lines by observing what conditions seem to be most favorable for their production. Any points which can be established in regard to these matters will be of great use in astronomical investigations, where the spectra of heavenly bodies show many differences in relative intensities of lines, which, if explained, would give indications as to the condition on these bodies.

The influences upon spark and arc spectra of changes in temperature, pressure, nature of discharge, and by a magnetic field have been observed, and the tendency of such changes to affect certain lines more than others has been frequently noted. Another method of separating lines into groups is the use of different gases around the electric arc or in the tube through which a spark is passed. It has long been known that such changes of atmosphere may affect certain lines in the spectrum of an element more than others, but it is only recently that any notice has been taken of the method as a means of grouping related lines.

[1] Extract from a dissertation presented in partial fulfilment of the requirements for the degree of doctor of philosophy in the University of California.

The work of Professor Crew[1] with metallic spectra influenced by an atmosphere of hydrogen has shown that the lines belonging to Kayser and Runge's series are unaffected by a change from air to hydrogen in the cases of the metals he investigated, and he points out that the use of hydrogen may offer a means of dividing spectra into series and non-series lines. R. A. Porter[2] has supplemented Professor Crew's work by the use of oxygen, nitrogen, and ammonia with several metallic arcs, but his results relate chiefly to the changes in prominent spark lines. The method thus seems to be only in the beginning of its usefulness.

The work of which an account is to be given was undertaken for the purpose of making a systematic study of the arc spectra of several metals whose lines show regularities in their arrangement, with reference to changes produced by the atmosphere surrounding the arc. The attempt has been made to connect the observed changes with the series relations among the lines of the metal, to separate the lines into groups by this means, and to deduce general conclusions as to the nature of the physical conditions which bring about the observed effects.

With this object in view, a detailed study has been made of the spectra of calcium, strontium, barium, and copper, with the arc in atmospheres of air, oxygen, carbon dioxide, nitrogen, ammonia, and the vapors of mercury and sodium. A few observations have been made of the spectra of magnesium and zinc. The use of copper terminals in some cases has given an opportunity to observe the effect of copper vapor mixed with that of the metal under observation.

Carbon terminals were used for most of the work with the metals of the calcium group. The chlorides of calcium, strontium, and barium were used, and powdered magnesium. Carbons made from calcined sugar by the method previously described[3] were used to advantage in some cases by mixing the metallic salt or powder with the powdered charcoal, then molding and baking the compound. This gives a very uniform distribution of the metal through the carbon and freedom from impurities found in the

[1] Astrophysical Journal, 12, 167, 1900. [2] *Ibid.*, 15, 274, 1902.

[3] A. S. King, Astrophysical Journal, 14, 323, 1901.

commercial carbons. Calcined rock-candy has been found to give a very pure carbon for this purpose. In using calcium chloride it is difficult to get enough of the metal in the carbon, as a large amount causes the carbon to disintegrate in the baking. Furthermore, carbons containing any hygroscopic salt must be used at once after baking, as the absorption of water vapor causes them to go rapidly to pieces. When a large amount of metal in the carbon was desired, commercial carbons were used, with their cores bored out and filled with the metallic salt or magnesium powder. These were satisfactory for the most of the work, as the spectrum showed but few impurity lines. In cases where it was desirable to eliminate the carbon spectrum, or to observe the effects of copper vapor, copper rods were used as electrodes, their ends being bored out and filled with the metallic salt.

In order to try the effects of sodium and mercury vapors, the lower carbon was passed through the bottom of a small graphite or clay crucible which was fitted around the carbon a short distance below the end, and filled with metallic sodium or mercury. The arc being started, the heat caused the sodium or mercury to give off large quantities of vapor. The use of sodium vapor with carbon terminals causes the lower carbon to disintegrate rapidly where the liquid sodium comes in contact with it, possibly on account of the presence of water in the interior of the carbon, which is attacked by the sodium. This action on the carbon limits photographic work to short exposures.

During the first part of the work the inclosure used for the arc was a glass bulb made from a round-bottomed flask by introducing a tube into a hole blown in the bottom. Stoppers were placed in this tube and in the neck of the flask, through which carbons were passed, also tubes for the passage of gases. With this chamber, either the vertical or the horizontal arc could be used. Later a heavy graphite crucible 10cm in diameter was used for the chamber. This was made almost gas-tight by inverting it upon an asbestos base through which the lower carbon and the inlet tube for gases were passed. The upper carbon and outlet tube entered through holes in the top, and the light from the arc passed through a hole in the side 1 cm in diameter, over which a glass or quartz window was placed.

The spectrum was formed by a fifteen-foot Rowland concave grating, of which the first order spectrum was usually used. Comparison photographs were made on the same plate, usually four in number, the two outer with the arc in air with different lengths of exposure, the two inner with the arc in the gas or vapor whose effect was being studied, and with exposures corresponding to those for air. An effort was made to avoid over-exposure, which would tend to equalize the intensities of lines.

CALCIUM.

The spectrum of calcium has been photographed by the writer, with the arc in atmospheres of oxygen, carbon dioxide, nitrogen, ammonia, and the vapors of sodium, mercury, and copper. The fact that the calcium salts are very volatile in the arc makes it difficult to control the conditions as to the amount of calcium vapor present in the arc. We cannot know that the same amount of calcium was fed into the arc for each of two comparison photographs, so that a general strengthening of the lines under certain conditions of surrounding atmosphere means little. It seems useless, therefore, to make any statements as to the absolute effect of a certain atmosphere on the calcium spectrum, and so I have considered only relative differences, and have looked for the causes which produce a greater intensification of some lines than of others when conditions are changed.

The most striking relative difference appears when we compare the behavior of the ultra-violet triplet of wave-lengths 3644.45, 3630.82, 3624.15 with a pair of sharp lines at $\lambda 3737.08$ and $\lambda 3706.18$. The triplet is a member of the first subseries of Kayser and Runge. The pair does not belong to a series, but has the same difference in wave-number (223) as the H and K lines, and almost the same as the pair at $\lambda 3179.45$ and $\lambda 3158.98$ (204). The triplet and pair just mentioned are close together, can be easily compared, and are observed to be affected differently by a surrounding atmosphere in nearly every photograph of the large number I have taken. The pair at $\lambda 3179.45$ and $\lambda 3158.98$ behaves very similarly to the pair just mentioned.

The action of the H and K lines is similar in a general way, but they seem to change more easily than the other pairs, perhaps on account of greater strength and frequent reversals. The changes in the triplet at λλ3644.45, 3630.82, 3624.15 are a type of the behavior of the more diffuse lines of the spectrum, including the triplets belonging to Kayser and Runge's series and most of the unrelated lines. Triplets belonging to the second subseries show a change in the same direction, but not always to the same degree as those of the first subseries. Lines belonging neither to the series nor to the pairs mentioned above show an effect intermediate between that of the sharp lines of the pairs and that of the triplets of the first subseries.

Taking the pair and the triplet as types of the two kinds of lines into which the calcium spectrum may be divided, let us notice first the effect of varying quantities of calcium in the arc, other conditions remaining the same. With a very little calcium in the arc, the pair and triplet are of about the same intensity and both sharp. Sometimes the pair is stronger than the triplet. On increasing the quantity of calcium in the arc, the pair increases but slowly in intensity and remains sharp. The triplet, on the other hand, rapidly becomes strong and diffuse, the lines broaden toward the violet, and with a large amount of calcium reverse. This behavior is typical of that of all triplets belonging to the two Kayser and Runge series, the action being most pronounced in the case of the more hazy triplets, as those whose strongest lines are at λλ3361, 3487, and 4099.

It appears, therefore, that the increase of vapor density, which should result from an increased supply of calcium, would account for the broadening observed in the case of the more diffuse lines, and that the sharp lines do not show a corresponding increase in intensity. An increase in vapor density is probably accompanied by an increase in the temperature of the arc, since when the arc is started with a large amount of the salt in the carbon, the current is above normal, and other conditions which relatively enhance one set of lines usually give a slight increase in current.

This selective effect of change in density makes it difficult to

form tables of relative intensities for spectra like that of calcium whose lines do not all behave alike. I have often noticed that the relative intensities of lines on my plates do not correspond with those given by Kayser and Runge. For example, the latter give the intensities of the lines forming the triplet λλ3644, 3631, 3624 as 1, 2, 2 respectively, and the lines λ3737 and λ3706 each as 4. The arc in air does not give this relation unless a large amount of calcium is used, as was probably done by Kayser and Runge in order to have all of the calcium lines appear distinctly.

If we now observe the effects of various atmospheres upon the calcium spectrum, we find that the theory of change of relative intensity in certain lines by increased vapor density is the only one which will harmonize the effects of change of atmosphere with the observed effects in air when the amount of metal consumed is changed. Consider first the effect of oxygen. Oxygen greatly increases the consumption of metal in the carbon arc. The carbons waste away rapidly and a large amount of calcium oxide is formed. The arc is very luminous; the current usually increases one or two amperes from its value with air around the arc. The effect on the spectrum is to enhance the diffuse triplets much more than the sharp lines which do not belong to series. This is especially noticeable in the case of the triplets with lines of greatest wave-length at λλ3361, 3487, 3644, 4099, 4586, as compared with the sharp lines λλ3159, 3179, 3706, 3737, and the group from λ4283 to λ4319, which do not broaden readily. The H and K lines are strengthened, but not so much as the *g* line, the reversal of which is widened. There is no essential difference, however, between these effects and those given by the arc in air with a large amount of calcium present. I am not now taking up the question as to whether oxygen really increases the *absolute* intensity of the calcium lines. That could be tested only by an arc formed between terminals of metallic calcium and with a constant current. The greater amount of calcium consumed by the arc in an atmosphere of oxygen would account for the general strengthening.

The effect of carbon dioxide on the calcium spectrum is very

similar to that of oxygen, but not so decided. It is probable that large quantities of carbon dioxide are generated by the use of oxygen around the carbon arc, as the oxidation rapidly reduces the diameter of the carbons.

Turning now to the effects of some atmospheres which exclude oxygen to a greater or less degree, we shall consider first the atmosphere of pure nitrogen. This was prepared from ammonium sulphate and sodium nitrite, and purified by passage through sulphuric and pyrogallic acid. Its effect seems to be to prevent oxidation without decreasing the vapor density to any great extent. The arc burns readily in nitrogen, and the same time of exposure can be used as with air or oxygen. The carbons are consumed very slowly, and no calcium oxide is formed if a strong current of nitrogen is maintained. Still, from the ready burning of the arc, it is not probable that the vapor density in the arc is greatly different from the conditions in air, and it is different from the conditions in oxygen chiefly because with the latter gas more calcium is supplied to the arc in a given time.

The effects with nitrogen are entirely in accordance with this view. The tendency of nitrogen is to equalize the intensity of the pair and triplet considered above as types of the two sets of lines. The vapor density is probably small, but not smaller than we can obtain in air with a small amount of calcium and a strong current of air pumped through the chamber. In some photographs where a large amount of calcium was used with nitrogen atmosphere, the triplets were broadened and rendered diffuse in a way often observed in air and approaching the effect of oxygen. The fact that this broadening is possible under some circumstances with a strong current of nitrogen around the arc is very much against the theory that the broadening is due to oxidation, or due in any large degree to higher temperature, as we should expect the violent combustion in the oxygen atmosphere to raise the temperature of the arc as well as that of the electrodes.

The use of sodium vapor around the arc gives another test for the effect of increased vapor density without oxidation. A

stream of sodium vapor passing up on all sides of the arc from a cup of boiling sodium around the lower carbon would allow but little oxidation, while it might be expected to crowd the calcium vapor back into the arc and increase the vapor density in the arc. The photographs taken under these conditions show a decided difference in effect upon the two sets of lines, and the difference increases as the sodium vapor becomes denser. With the sodium boiling vigorously, the pairs remain sharp and show little change in intensity, while the triplets become broad and diffuse, and the reversal of *g* is much widened. The sodium vapor increases the conductivity of the arc so that the current is greater than with air and the electrodes are consumed more rapidly, a condition which should lead to an increase of vapor pressure. An explanation may thus be given for the observed fact that the effect of sodium vapor is much like that of oxygen, though we might expect the opposite.

Turning now to the effect of ammonia, we have here a gas which in all probability reduces the vapor density in the arc. As has been observed by others, the arc is very difficult to maintain in ammonia. When it is burning, the current and fall of potential show little change from the conditions in air, but the intensity is feeble and the arc is often extinguished. The exposure must be from twelve to twenty times as long as with the arc in air. With a strong stream of ammonia sweeping away the gases formed in the arc and the feeble action which we observe, the conditions seem very favorable for a minimum vapor density in the arc. None of the other atmospheres I have used give such a relative weakening of the lines belonging to Kayser and Runge's series as compared with the pairs at $\lambda 3159$ and $\lambda 3179$, $\lambda 3706$ and $\lambda 3737$, and the H and K lines. The pairs mentioned are stronger than in air in the case of several photographs in which the supply of calcium was kept the same as nearly as possible. The triplets, on the other hand, are very decidedly weakened, and their lines rendered sharper. The reversal of *g* is much narrower in ammonia; also that of H and K, when they are reversed. The effect of mercury vapor around the arc is to diminish the triplets more than the pairs. The *g* line shows a

narrower reversal, and the effects all point toward a lower vapor density than with the arc in air. The poor conductivity of mercury vapor may account for this, giving the effects observed to a much greater degree in the case of ammonia.

The use of copper electrodes with calcium filling gives another means of producing conditions in which the vapor density is probably small. The copper arc is of feeble intensity, a much longer exposure is needed than with the carbon arc to make the lines equally strong, and then relative differences appear in the two sets of calcium lines much like those observed when mercury vapor was used around the arc. Comparing two photographs of the calcium spectrum between $\lambda 3500$ and $\lambda 4600$, one with carbon and the other with copper electrodes, with the H and K lines, as well as the pair at $\lambda 3737$ and $\lambda 3706$ of the same intensity in both, we observe a very distinct difference in the behavior of the other calcium lines. The several triplets are much weaker in the copper arc, the greatest difference being in the diffuse triplets with lines of greatest wave-length at $\lambda 3644$ and $\lambda 4512$. The *g* line is narrow and not reversed in my plates taken with the copper arc.

STRONTIUM.

The spectrum of strontium has been studied by the writer from $\lambda 3000$ to $\lambda 5000$ with atmospheres of air, oxygen, carbon dioxide, nitrogen, ammonia, and sodium vapor. The relative differences found result in the segregation of a group of strong lines which make up three pairs having the same difference in wave-number, given by Kayser[1] as follows :

λ	Vibration-Difference
4305.60 4161.95	801.6
4215.66 4077.88	801.5
3475.01 3380.89	801.1

These are homologous to the three pairs in the calcium spectrum which were observed to be affected by surrounding atmos-

[1] *Handbuch der Spectroscopie*, **2**, 541,

phere differently from the lines belonging to Kayser and Runge's series. In the strontium spectrum we again find a difference in the effect upon these two classes of lines; but in this case it is the sharp lines forming the pairs which are the more sensitive to changes of physical condition.

A comparison of a large number of photographs shows conclusively that the lines of these pairs are strengthened by atmospheres of oxygen, carbon dioxide and sodium vapor to a greater degree than the remaining lines of the spectrum, and are correspondingly weakened by atmospheres of nitrogen and ammonia. The sharp line at λ 3464.58 is acted upon in the same way as the pairs, but seems to have no companion with a similar behavior. The broad reversed line at λ 4607.52 is not strengthened by oxygen to the same degree as the pairs. In carbon dioxide it is actually weaker than in air, while the pairs are much stronger, and the remaining lines, including those belonging to Kayser and Runge's series, show little difference. In ammonia λ 4607.52 sometimes suffers a double reversal, making it broad and hazy. The pairs are distinctly weakened by ammonia, giving a general effect opposite in character to that of the first three gases. The action of nitrogen is similar to that of ammonia, but not so pronounced, as the pure nitrogen seems to make but a slight change from the conditions of the arc in air.

A NEW STRONTIUM TRIPLET.

In studying the spectrum of strontium, the writer has observed that whenever the line λ 4077.88 is of considerable intensity, three faint lines appear close to it. The wave-lengths of these three lines I have measured as follows:

4087.86
4071.22
4061.52

Not being able to identify these with any known metallic lines, I have applied several tests to determine whether they belong to strontium, to some impurity, or whether they may be "ghosts" of λ 4077.88. The last possibility was easily settled. The three lines are not at all symmetrical with respect to

λ4077.88, but a decisive test was made by comparison with the prismatic spectrum. A train of prisms was arranged which gave a dispersion almost equal to that of the first-order grating spectrum, and the three lines came out quite as strongly as with the grating. It is very improbable that they are due to metallic impurities, as they are given by "chemically pure" preparations of both the chloride and the nitrate of strontium, in a carbon arc with very pure carbons made from calcined sugar. Again, it is not likely that the lines are the heads of strontium oxide bands, as they are not reduced in intensity by atmospheres of nitrogen or ammonia.

While my measurements agree very closely with the wave-lengths of three carbon lines given by Kayser and Runge as λλ4087.88, 4071.13, 4061.53, the three lines first mentioned are certainly not carbon lines in their normal condition, as they stand out distinctly only when strontium is present.

Perhaps the strongest evidence that the three lines belong to strontium is the fact that they form a triplet having the same differences in wave-number between the component lines as the differences for four strontium triplets given by Kayser and Runge.[1] These triplets do not belong to a known series, and have their lines much closer together than the series triplets. The four triplets with their vibration-differences are as follows:

λ	Vibration-Difference
5535.01	
5504.48	100.2
5486.37	60.0
5257.12	
5229.52	100.4
5213.23	59.8
4892.20	
4868.92	97.7
4855.27	57.8
4338.00	
4319.39	99.3
4308.49	58.6

[1] *Abh. Berl. Akad.*, 1891, p. 34.

The fifth member of this group of triplets seems to be the three lines:

4087.86		
4071.22	- - - - - - -	100.0
4061.52		58.6

The members of these five triplets are the more diffuse lines in the strontium spectrum. There is a decided similarity in appearance between the last two triplets, though the one to which I have called attention is much the weaker.

MAGNESIUM.

The writer has made but few observations of the spectrum of magnesium, and these have for the most part given negative results, as far as relative differences are concerned. This may be due to the fact that nearly all of the prominent lines belong to the triplets of the first and second subseries of Kayser and Runge, and so may be expected to show the same behavior with change of physical conditions. The spark line λ 4481 does not appear in my photographs.

My plates show a very slight difference in the behavior of the triplets belonging to the first subseries in atmospheres of oxygen and nitrogen from that of the second subseries in the same atmospheres. The triplets of the first subseries are relatively stronger in nitrogen, those of the second subseries in oxygen. This is the case with the triplets with lines of greatest wave-length at $\lambda\lambda$ 3838, 3097, 2852 of the first subseries as compared with those at $\lambda\lambda$ 3337, 2942, of the second. The difference is most noticeable in the case of the very broad line λ 2852. The results so far obtained do not point to any more general conclusions than the similar behavior of lines belonging to the same series.

BARIUM.

A study of the spectrum of barium with the arc in atmospheres of air, oxygen, nitrogen, carbon dioxide, ammonia, and the vapors of copper, mercury, and sodium, shows a number of lines which are affected differently from the majority of the barium lines, and always affected in the same way by a given gas. While no

general series relation has yet been found for the barium lines, my results serve to strengthen the evidence that certain lines are related to each other.

The opposite effects of oxygen and ammonia upon a number of lines give a basis for separating a certain group from the main body of the spectrum. These lines are as follows:

5853.91	4525.19
4934.24	4166.24
4900.13	4130.88
4700.64	3906.20
4554.21	3891.97

A comparison of a large number of plates shows these lines in every case to be strengthened by oxygen and weakened by ammonia more than the remaining lines of the spectrum. Nitrogen produces scarcely any change from the appearance of the lines in air. A slight relative weakening sometimes appears, showing that the tendency of nitrogen is in the same direction as ammonia. Carbon dioxide gives a definite strengthening of these lines, comparable to that of oxygen. Both mercury vapor and sodium vapor strengthen the lines, the effect of the latter being almost as pronounced as that of oxygen. Copper terminals filled with barium salt also strengthen the lines as compared with barium in the carbon arc. With all these atmospheres, the most sensitive lines are those at $\lambda\lambda$ 4166, 4525, and 4900. With the exception of λ 3906, the group of lines given above are among the strongest in the spectrum, and are sharp with a moderate quantity of metal in the arc, though they broaden and reverse with a large quantity. They are, however, always different in appearance from the broad diffuse lines at $\lambda\lambda$ 4323, 4325, 4489, and 4494, which are not affected by change of atmosphere. This greater sensitiveness of the sharper, more prominent lines of barium is similar to the effect of corresponding atmospheres upon the strontium spectrum, and unlike that upon the calcium lines. The changes in the line λ 4525, as compared with its more diffuse companion λ 4523, are very striking.

Looking now at the relations between these lines which are affected by surrounding atmosphere, we find that almost the

only definite regularity so far discovered in the barium spectrum is the existence of four pairs of lines with constant difference in wave number, as follows:

λ	Vibration-Difference
6497.07 5853.91	1691.05
4934.24 4554.21	1691.16
4900.13 4525.19	1690.90
4166.24 3891.97	1691.47

It will be observed that all of these lines occur in my list of sensitive lines, except λ6497, which is beyond the region covered by my photographs. My results for the barium spectrum, then, serve to separate these lines, together with a few others, from the main body of the spectrum, as forming a series of related pairs, which appear to be relatively strengthened by increased vapor density in the arc.

ZINC.

My data for the zinc spectrum cover only the region from λ3000 to λ4100 with a comparison of the effects of oxygen and air. Brass terminals were used for the arc, so that the stronger copper lines appear. My results show a decided strengthening in oxygen of the group of lines

3282.42	3345.13
3702.62	3345.62
3303.03	3346.04

belonging to the first subseries of Kayser and Runge.

The lines

3572.90	3740.12
3671.71	4019.75
3683.63	4058.02

between which no relation has been found, are weaker in oxygen than in air. The difference is not as great as in the case of the series group. This strengthening of the series lines by oxygen is similar to that observed with the series lines of calcium, while

the effect on the unrelated lines is like that which we shall see in the case of the line pairs of copper.

COPPER.

The spectrum of copper shows a large number of relative differences in intensities of lines, when different atmospheres are used around the arc, the most important changes being given by an atmosphere of oxygen. The copper arc in oxygen burns with difficulty and frequently goes out, in decided contrast to the behavior of the carbon arc in oxygen. This action may be due to the formation of a non-conducting coating of copper oxide on the terminal, though it is doubtful if this forms in the arc itself. The oxidation of copper is not accompanied by the increased heat and wasting of the electrodes which accompanies the formation of carbon dioxide in the carbon arc.

In general, the more diffuse copper lines are most reduced by oxygen, the best examples of this being the pairs of wave-lengths

3247.65	3654.60
3274.06	3688.60

These pairs are almost blotted out by a strong current of oxygen. The strong pair λ4023 and λ4063 are weakened by oxygen, though not so much as the preceding; while many of the sharper lines are quite unaffected, notably those of wave-lengths:

3308.10	4378.40
3599.20	4587.19
3602.11	4651.31

The lines from λ3100 to λ5300 most reduced by oxygen are as follows:

3194.17	3925.40
3208.32	4015.80
3247.65	4022.83
3274.06	4056.80
3337.95	4062.94
3384.88	4480.59
3530.50	4531.04
3654.60	5105.75
3688.60	5153.33
3825.13	5218.45
3861 88	

It will be observed that these twenty-one lines include those of the pairs given by Kayser[1] as belonging to the first and second subseries of copper; also the pair λ 4015.80 and λ 4056.80, and the strong pair λ 3247.65 and λ 3274.06 which Kayser thinks may be a member of the principal series of copper. The lines of these several pairs have nearly the same difference in wave-number (248). Besides these pairs we have a weakening by oxygen of the seven lines:

3194.97	3530.50
3208.32	3925.40
3337.95	5105.75
3384.88	

In addition to these known relations, I have found that a number of these lines affected in the same way by oxygen form pairs with a constant vibration difference of about 3400 as follows:

λ	Vibration-Difference
3247.65 / 3654.60	3428.7
3274.06 / 3688.60	3432.5
3384.88 / 3825.13	3400.2
3925.40 / 4531.04	3405.1

This gives an interesting relation between the pair $\lambda\lambda$ 3247.65, 3274.06 and one of the pairs of Kayser's first subseries. The corresponding lines of these two pairs are seen to have nearly the same difference in wave-number. The two pairs of the second subseries have a line of each pair related to an outside line by this constant difference; but in one case it is the upper line of the pair which has an outside companion, and in the other case the lower line. The connection indicated by these constant differences does not appear to be a simple one. It is probably a result of some relation among the the vibrations giving rise to this set of lines which are affected in the same way by change of physical conditions.

[1] *Handbuch der Spectroscopie*, 2, pp. 530–531.

The use of nitrogen gives effects but little different from those with air around the arc, except in the case of the pair $\lambda 3247.65$ and $\lambda 3274.06$. These are slightly weaker with nitrogen, and the reversal is narrower, while scarcely any change can be perceived in the other lines which are weakened by oxygen.

The effect of mercury vapor on the copper lines is similar to that of oxygen, the same lines being weakened, and to almost the same degree, as by a strong current of oxygen. The action is the same with sodium vapor around the arc, though not so pronounced, only the most sensitive lines being changed.

Mixtures of other vapors with that of copper were tried by using brass electrodes, giving strong zinc lines mixed with those of copper, and by using one copper and one carbon terminal, giving the carbon and cyanogen bands. These experiments were made with the arc in the open air. No relative differences could be detected in either case by comparing the plates with those taken with copper terminals.

Looking now to see if changes in vapor density would produce the effects we observe, the first fact to be noted is that the effect of oxygen upon the burning of the arc with copper terminals is very different from that when carbon electrodes are used. With carbon terminals, the arc burns with great vigor and brightness in oxygen, the carbons being rapidly consumed, probably with the formation of large quantities of carbon dioxide. With copper terminals, on the other hand, it is difficult to maintain the arc in oxygen. Some copper oxide is formed, but no change in the flame is observed except a weakening. A longer exposure is required for the same general intensity of the spectrum, as compared with the arc in air. This agrees with the observation of Porter,[1] who used the rotating arc with electrodes of magnesium, tin, zinc, and iron in oxygen, and found the metallic lines weakened. This difference between the carbon and metallic arcs must be considered in questions of vapor density.

The conditions with oxygen about the carbon arc appeared favorable for greater vapor density, and its action in intensifying certain lines strengthened the hypothesis that the change

[1] *Loc. cit.*

was produced by greater vapor density. With the copper arc, however, the weak action in an atmosphere of oxygen appears favorable for a low vapor density in the arc, and the reduction of the broad diffuse lines by oxygen can be attributed to this cause. Both mercury and sodium vapors weaken the arc. The action of sodium vapor is not the same as with the carbon arc. The current remains the same as with the arc in air, so that the sodium does not appear to increase the conductivity, and the arc burns with difficulty, frequently going out. Nitrogen might be expected to give little change of vapor density from that of air, and it is noted that only the most sensitive lines are changed by nitrogen, whose effect is slightly to reduce the vapor density, if my hypothesis is correct.

DISCUSSION OF RESULTS.

The first general result to be noted as given by a change of atmosphere around the arc is the power of the method as a means of separating the lines of an element into groups. The several groups may then be investigated with respect to series relations between the lines. It thus offers possibilities of finding new relations between lines, especially in the case of elements having so large a number of lines that it would be difficult to obtain a starting-point without some means of separating certain groups.

The magnetic field is a well-known means of separating lines into groups, and an interesting comparison of the two methods is given by comparing my results with those given by Kayser in his discussion of the Zeeman effect.[1] From this it will be seen that a number of lines in the spectra of calcium, strontium, barium, and copper, which belong to certain types in the classification of Zeeman phenomena, are among the lines which I find affected by a surrounding atmosphere differently from the other lines in those spectra.

The fact that a certain atmosphere may not produce the same effect on lines of the same kind (sharp or diffuse) in the spectra of different metals, might present a difficulty. Especially is

[1] *Handbuch der Spectroscopie*, 2, 671.

this the case with the behavior of calcium and strontium in the carbon arc. The structure of these two spectra show many similarities, yet the diffuse calcium lines belonging to Kayser and Runge's series are the more sensitive to change of atmosphere; while in the case of strontium, the strong lines not in Kayser and Runge's series respond most readily. It is very probable, however, that the groups of strong lines actually form the principal series in these two spectra, though no numerical relation has yet been found between the lines except the constant differences. If this is the case, the state of affairs seems to be that a change of atmosphere produces the greatest difference on the principal series of strontium and on the first subseries of calcium—an effect which might easily result from the differences between the two metals, in spite of their many similarities. The more rapid volatilization of the calcium salt in the arc might bring about a change of condition sufficient to account for the different behavior of the two series.

In regard to the probable action of the surrounding atmosphere in bringing about the changes in the spectral lines, the weight of evidence seems to indicate that chemical action has little or no part in producing the effects, except as it may affect the vapor density in the arc. The chief arguments against the effect of an atmosphere of oxygen being due to chemical action are (1) that the use of pure nitrogen produces little change as compared with the arc in air, and occasionally its action is in the same direction as that of oxygen; (2) that a cloud of sodium vapor around the arc leaves most of the lines unchanged, and slightly enhances those of one group, as has been noted in several cases.

The hypothesis that seems best to accord with my results and those of others is that the effects are to be ascribed to changes in vapor density which effect the vibrations producing the lines belonging to a certain group—vibrations which are probably similar in general character, judging from the behavior of the resulting spectral lines. This theory has been kept in view during the discussion of the results, and seems to be the one which best co-ordinates the various phenomena, both as to the

changes observed with different atmospheres, and those in which the supply of metal was varied with the arc in air.

It is possible that temperature changes may play a large part in giving the effects we observe, as the most distinct changes in the spectrum are associated with conditions which probably involve change of temperature in the arc. However, the facts that the carbon arc in oxygen does not give changes proportional to its probable change of temperature, and that these changes are brought about in some degree by other conditions which should not involve much change of temperature, point to the conclusion that temperature is not the direct cause of the effects observed, though it may assist in bringing about the state which is the actual cause. Temperature changes would be closely connected with changes in vapor density, to which it seems reasonable to attribute many of the results.

It is the writer's opinion that not enough attention has heretofore been given to the effects of changes in vapor density upon the intensities of lines. Is it not possible that density is the determining element for the relative intensities, as pressure seems to be for displacements and the use of the magnetic field for polarization properties? Any notions as to the action of change of vapor density in modifying the electrical properties of the vibrating particles must be entirely hypothetical, but it is easy to conceive that such actions might be selective in their nature, resulting in a change for only a portion of the lines.

Comparing my results with some observations of the spark spectrum which have been published, we find some striking evidences that the use of atmospheres which apparently give lower vapor density tend to make the arc spectrum similar to that of the spark discharge. Sir William and Lady Huggins[1] found that when the density of calcium vapor in a vacuum tube was decreased by gradually removing the calcium from the electrodes, the H and K lines, together with the pairs at $\lambda\lambda$3737, 3706, and 3179, 3159 were weakened much less rapidly than the other lines and finally were the only lines visible. These are the lines which I find but slightly altered by changes in vapor

[1] Astrophysical Journal, 6, 77, 1897.

density in the arc. Similarly, the photograph of the spectrum of the spark discharge in calcium vapor given by Eder and Valenta[1] shows the lines λλ 3737, 3706, 3179, 3159 to have many times the intensity of λλ 3644, 3630 and the other series lines in that region. This state is approached by some of my photographs taken with the arc in ammonia, which appeared to give the lowest vapor density of the gases I have used. Again, the table given by Eder and Valenta,[2] in which their measurements of the spark lines of copper are compared with those of Kayser and Runge for the arc, shows that if the standards of intensity adopted by the two pairs of observers are at all alike, then most of the lines which I find weakened by an atmosphere of oxygen around the copper arc are much weaker in the spark spectrum than in that of the arc.

If, as is very probable, the vapor density in vacuum tubes and in the spark discharge is much less than that in the electric arc, we may arrive at a simple explanation of the action of hydrogen in intensifying certain spark lines, as observed by Crew and Porter. As the hydrogen atmosphere around the metallic arc seems to have much the same action as ammonia in causing the arc to burn with difficulty, probably reducing the vapor density, we are led to the idea that those lines which appear strongly in the spark, and faintly or not at all in the arc, may be lines whose vibrations require a low vapor density. The action of hydrogen in strengthening such lines in the arc would then be explained by the reduction of the vapor density, thereby bringing the conditions of the arc to approach those of the spark.

Further evidence in support of this view is given by the work just published by Hartmann,[3] who finds λ 4481, as given by the arc between magnesium terminals, much increased in intensity when a current as small as 0.4 ampere is used, the arc being constantly interrupted, with only a slight heating of the terminals and little vaporization — conditions which should give a low vapor density. The results show a similarity to the

[1] *Denkschriften der K. Akademie der Wissenschaften zu Wien*, 68, 534, 1900.

[2] *Ibid.*, 63, 189, 1896.

[3] ASTROPHYSICAL JOURNAL, 17, 270, 1903.

spark spectrum and to the effects given by the arc when surrounded by atmospheres which should give low vapor density.

My results, then, considered in connection with those of others, show a definite selective action in the effects of surrounding atmospheres on spectral lines—an effect evidently connected with the series relations of the element. All of the results point to a change of vapor density in the arc as the direct cause of the effects observed. The tendency of a low vapor density seems to be to make the arc spectrum more like that of the spark.

In conclusion, I wish to express my thanks to Professors Slate and Lewis for constant interest in the work and many helpful suggestions.

University of California,
Berkeley, May 1903.

ON THE SPECTRUM OF THE SPONTANEOUS LUMINOUS RADIATION OF RADIUM AT ORDINARY TEMPERATURES.[1]

By Sir William Huggins and Lady Huggins.

The discovery of an element possessing such remarkable and novel properties as radium, which in its separate and distinct form as a new chemical element we owe to the researches of Professor and M^me^ Curie, has already thrown many beams of suggestive light into the very obscure regions of the constitution of matter. In radium we have a body which appears to be spontaneously and without ceasing giving off energy in several forms. According to Professor Rutherford,[2] following upon the work of Becquerel, M. and M^me^ Curie, and others, the emanations going off from radium are at least of three kinds: first, an emanation of heavy corpuscles, larger in mass than the hydrogen atom, moving with a high velocity, and carrying a positive charge; secondly, of negatively charged electrons which form a powerful and penetrating cathode emanation;[3] and, further, of a radio-activity which diffuses from the radium as if gaseous in its nature. In addition, M. and M^me^ Curie have found that radium spontaneously maintains a temperature about 1°.50 C. above the surrounding temperature, and therefore emits heat radiations of wave-lengths falling within the infra-red part of the spectrum.

Now, in addition to these forms of radiant energy, the glowing of radium in the dark shows that it emits a luminous radiation

[1] From advance proofs of an article to appear in the *Proceedings of the Royal Society*.

[2] *Phil. Mag.*, (6) 5, 445, 561, 1903.

[3] As an illustration of the penetrative power of the radio-active effects of pure radium bromide, the following experience may be recorded here. About 1 centigram of radium bromide (Buchler & Co., Brunswick) had been placed in an upper drawer of my writing table, while in a lower cupboard of the same table was a store of photographic plates. After a week or two, all the plates, in boxes lying upon each other three or four deep, were found to be as completely fogged as if they had been exposed to light.

spontaneously at ordinary temperatures. It appeared to us probable that in this glow we had not to do with either phosphorescence or fluorescence as usually understood, but with an independent and continuous radiation set up by those more active molecules which are supposed, in consequence of a condition of internal instability, to be the source of all the phenomena of radio-activity, and which can scarcely fail themselves to be violently agitated, in connection with disruptive molecular changes — especially the flinging off of the heavy corpuscles — during which, part of the energy stored up within the molecule is liberated in the kinetic form.

Taking this view of the luminous radiations visible to the eye, it seemed highly probable that the molecular motions by which they were set up, whether we suppose all the radium molecules alike to be concerned, or those only which are in active change, would be so far analogous to the vibrations produced artificially, when radium vapor is rendered luminous in a flame, or by the blow of an electric discharge, as, in like manner, to set up radiations of certain definite wave-lengths or, in other words, to furnish a spectrum of bright lines.

A preliminary prismatic examination of the glow from pure radium bromide was attempted by eye. In consequence of the feebleness of the light under dispersion a slit spectroscope could not be used. A thin fragment of some length of radium was selected, which in the dark shone as a narrow line of light; when this was viewed through a direct-vision prism, it was seen to be dispersed into a spectrum which extended from the blue down to about D, where it became too faint to be traced farther in the direction of the red. Within this faint spectrum certain spots were distinctly brighter, due, in all probability, to the presence of bright lines at those positions in the spectrum.

The success of this preliminary observation encouraged us to hope that it might be possible by availing ourselves of the accumulative power of continuous photographic exposure, to obtain a record of the blue, violet, and ultra-violet regions of the spectrum, if the glow radiations extended so far.

We made use of a small quartz spectroscope which had been

constructed some years ago for very faint celestial objects. It consists of a compound quartz prism of 60°, consisting of two prisms of 30° of right-handed and left-handed quartz respectively. The quartz lenses are of short focus and of large angular aperture, being about $f = 1/3$. The focal length of the lenses is $5\frac{3}{4}$ inches; they are plano-convex, the marginal parts of the convex surfaces being "figured" to diminish spherical aberration.

The solid radium bromide was placed at about a millimeter distance in front of the slit, which had to be wider than if a bright object was being photographed; the width was about $\frac{1}{450}$ inch. In the case of the spark spectrum of radium and the comparison spectrum of nitrogen, a slit of less than half this width was used.

With the exposure of twenty-four hours, faint traces of two lines were seen on the plate. After several trials the negative reproduced on the accompanying plate was obtained with an exposure of seventy-two hours. The reproduction is enlarged two and a halt times. The spectrum consists of eight bright lines, and at least eight faint lines, together with a faint trace of continuous spectrum in the blue region, which does not come out in the reproduction.

In consequence of the wide slit and the small scale of the spectrum, it is not possible to measure with certainty to the fourth figure, but the probable error is, we think, not greater than two units in the fourth place.

It was seen at once that the two very strong characteristic rays of the spark spectrum of radium, in this part of the spectrum, namely, λ 3814.5 and 3649.6[1] were not present on the plate. It was clear that the spectrum was not of the radium molecule when excited by the electric discharge. It was indeed not improbable that if the radiation came alone from the most active molecules, which were suffering loss by material emanations, then, if we may accept the analogy from sound, like a filed tuning-fork they would no longer give radiations of the same wave-lengths as before.

[1] For spark spectrum of radium, see DEMARÇAY, *Comptes Rendus*, 129, 786, and 131, 258; Exner and Haschek, *Wien. Akad. Sitzber.*, 110, July 1901; RUNGE, ASTROPHYSICAL JOURNAL, 12, 1.

On comparing the new spectrum with the band spectrum of nitrogen, seven of the strongest lines were found to agree, not only in position, but also in intensity, and in character with those of the three nitrogen bands in this part of the spectrum.

The positions of the three bands are, according to Ames,[1] λ3576.85, 3371.2 and 3158.9.

Indications of other lines, besides those which can be seen in the reproduction, can be faintly glimpsed on the negative. There seems little doubt that with a longer photographic exposure a more complete spectrum will be obtained. We have now secured some radium bromide prepared by the Société Centrale de Produits Chimiques, and it is our intention to take photographs of this salt as well as photographs of the German salt with longer exposures. It may then be that indications of helium, and possibly of radium itself, may be forthcoming.

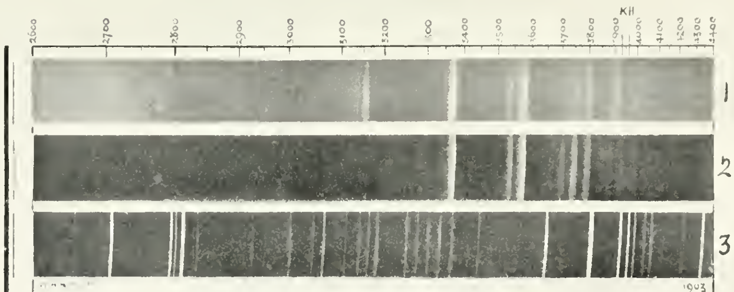
Nearly the whole of the ultra-violet radiations appear to come from nitrogen, and we think it best to refrain from any discussion at this moment. Have we to do with occluded or with atmospheric nitrogen? The remarkable fact should be pointed out that in radium we have a body which at the ordinary temperature sets up radiations which are similar to those which have hitherto only been obtained in connection with the electric discharge.

Description of the plate.—At the top is placed a scale of approximate wave-lengths. Immediately below is a reproduction, enlarged two and a half times, of the spectrum obtained from the radium bromide with an exposure of seventy-two hours. As has been already explained, this has been shifted to bring the lines into position with those of nitrogen photographed from a vacuum tube. The identity of the two spectra seems complete. The third band is faint in the nitrogen spectrum on account of the absorption of the glass of the tube.

Below is a spark spectrum of radium bromide from the Société Centrale de Produits Chimiques. The H and K lines of calcium are present, as well as faintly some of the stronger lines of barium. The characteristic lines of radium at λ3814.59 and

[1] *Phil. Mag.*, (5) **30**, 57. 1890. See also DESLANDRES, *Comptes Rendus*, **101**, 1256; and PERCIVAL LEWIS, ASTROPHYSICAL JOURNAL, **12**, 8.

Spectrum of Radium Bromide



1 Spectrum of Radium Bromide.
2 Nitrogen : Band Spectrum.
3 Spark Spectrum of Radium.

3649.7 come out strongly, as well as the strong line recorded by Demarçay at λ4340.6. A strong line about λ2710 was placed by Berndt[1] at λ2708.6. The strong line a little beyond, about λ2814 is due to radium. We do not recognize several lines recorded by Exner and Haschek[2] in this part of the spectrum.

[1] *Phys. Zeitschr.*, **2**, No. 12 (1900–1).

[2] *Sitzb. Ak. Wiss. Wien.*, **110**, July 4, 1901.

Reviews.

Problems in Astrophysics. By Miss A. M. Clerke. London: A. & C. Black, 1903.

It is not possible to separate entirely the child Astrophysics from the parent Astronomy. The two are mutually dependent in much of their instrumental equipment and in many of their methods, and their results are often intimately related. Radial-velocity determinations, for example, are based largely upon astrophysical methods, but the direct results are mostly astrometrical. However, the newer term serves an exceedingly useful purpose, and has become permanently fixed in our literature. In general, we may say that the older astronomy is concerned with the determination of the structure of the sidereal universe; with the arrangement of the sidereal units in space, and with their motions and relations in accordance with Newton's law of gravitation. Astrophysics, on the contrary, is interested in determining *what the stars really are*, and what the history of their development has been.

The beginning of astrophysical inquiry is old, but until the last half-century its results were surprisingly few. Theories of planetary evolution by Kant and Laplace, the observations and wonderfully sagacious deductions of the elder Herschel, and studies of planetary conditions and of Sun-spots, constituted the main body of the science up to 1859. But the spirit of inquiry as to the nature and history of the heavenly bodies was latent in many quarters; and Kirchhoff's superlatively important discovery that "the light which reveals to us the existence of the heavenly bodies also bears the secret of their constitution and physical condition," opened a gateway to Comte's forbidden field which many were ready to enter. The pioneer work of Secchi, Huggins, Vogel, Young, and their colleagues in the next quarter-century was very fruitful. Their results, originally scattered through the journals, were collected and systematized by Roscoe, Schellen, Kayser, and later by Scheiner and Clerke;[1] but the speed of development in the past fifteen years has been such that astrophysical literature is again widely scattered. This statement will not, I trust, be con-

[1] Miss Clerke, *The System of the Stars*, 1890.

strued into an overlooking of the colossal work on general spectroscopy now issuing from the able pen of Professor Kayser. A historical and critical survey of astrophysics is greatly needed, and the appearance of Miss Clerke's volume could scarcely have been better timed.

The author's intention, as announced in the preface, "is not so much to instruct as to suggest. It represents a sort of reconnaissance, and embodies the information collected by scouts and skirmishers regarding practicable lines of advance and accessible points of attack. . . . . Before attempting to add to our store of learning, we must realize what is already possessed." This statement of purpose has been consistently adhered to. By way of suggestions for future lines of research this book is the richest one known to me.

The physical study of the Sun forms the basis of astrophysical research more fully perhaps than astronomers themselves have realized. It is the only star near enough to be studied in some geometrical detail, all other stars giving integrated images which practically are mathematical points. It is likewise the only star bright enough to supply satisfactory metrical standards demanded in the study of other stars. Miss Clerke devotes the first third of the volume to a treatment of the Sun. After outlining "The Progress of Solar Physics," there are chapters on "The Chemistry of the Sun," "The Reversing Layer," "The Photosphere," "The Sun spots," "The Faculæ and Prominences," "The Chromosphere," "The Corona," "The Sun's Rotation," "The Solar Cycle," and "The Sun as a Whole."

The difficulties of solar study, in spite of comparative nearness and intense brightness, are very great. It is not generally appreciated that we are unable to study the body of the Sun, except by very indirect methods. Its interior is invisible; the photospheric veil hides it completely from view. The photosphere, chromosphere, prominences, corona, etc.—the only portions accessible for direct observation—are an insignificant part of its mass. They are literally the Sun's outcasts. Our knowledge of the Sun as a whole is based almost entirely upon a study of these outcasts. We might hope to reach safe conclusions as to the characteristics of a hermit nation by making a careful study of its banished subjects, provided the observed types correspond with types produced by our own civilization; but if new types, new customs, new forms, presented themselves on a stupendous scale, and were observable only at long range, our conclusions as to the country from which they were expelled should come slowly and uncertainly. It is difficult to say what the chromosphere is. To determine what the conditions

within the body of the Sun must be in order to create and maintain such an outcast shell is far more difficult.

Again, the ordinary photographic or visual image of the Sun is an integrated one. A coronal photograph secured at a total eclipse is the result of a projection upon and into one plane, at right angles to the line of sight, of all that remains of the Sun after subtracting the (approximate) cylinder of matter hidden by the Moon. The tops of some coronal streamers, the intermediate portions of others, the bases of those near the limb, the corresponding parts of prominences and chromosphere, are all projected into one point, illuminated in part by reflected and in part by inherent light. Whether every man who has gone forth to solve the riddle of the corona has fully realized the odds against success is doubtful.

Imperfections in atmospheric conditions are perhaps more liable to introduce uncertainties in solar observations than in any other class. Whether Janssen's *réseau photosphérique* phenomena are of solar or terrestrial origin is still uncertain, as Miss Clerke states on p. 17, but Hale's recent results with the spectroheliograph, in which the image is built up from one limb to the opposite limb in some two or three minutes, confirm the pretty general belief that the effect is purely terrestrial.

The impossibility at present of reproducing solar phenomena in our laboratories is recognized as our most severe limitation. There is scarcely one result to be obtained by interpolation: extrapolation is demanded everywhere, and at great distances beyond the range of observed points in the curves. Such decided extensions of our resources as the electric furnace provides seem at best but beginnings toward what is required. And, as Miss Clerke repeatedly points out, our knowledge of the effect of the presence of one vapor upon the spectrum of another vapor is almost wholly wanting. Evidence is not lacking, however, that such effects must be both common and potent.

There can be no doubt that the Sun is a very tempestuous body. It is probable that convection currents are on a scale so great and have at times a speed so high that we have difficulty in adequately realizing their immensity. I fear, therefore, that Miss Clerke's division of the solar outcasts into "several distinct envelopes" overlying the photosphere (p. 16) — reversing layer, chromosphere, corona, etc. — is somewhat too definite, even with her qualifying remark that none of them is "apparently in a condition of atmospheric equilibrium." Our direct evidence of disturbance in the photosphere is limited largely to the

visible spots, but the faculæ and prominences are almost certainly the results of disturbances generally invisible. In this connection it would be well, I think, to modify the statement on p. 74, "but faculæ exist abundantly where there are no spots," to read where there are no visible spots. Faculæ accompany very small spots as well as large ones. It is natural that there should be countless invisible spots, and that they as well as the large ones should be surrounded by faculæ. Spots one second of arc (650 kilometers) in diameter could easily escape notice. There is little doubt that the coronal streamers springing from every part of the sphere are produced in some way by powerful but invisible disturbances in all parts of the surface. There is certainly a close relation between spots and faculæ, and the latter are not confined to the spot zones; between chromosphere and prominences; and between prominences and at least the inner part of the corona. The tremendous cone of disturbance in the corona of 1901 (Sumatra) and in its apparent situation immediately above the large, and only Sun-spot visible near that date, discovered by Perrine[1] perhaps inadvertently not mentioned in *Problems in Astrophysics* — is also very significant. Not to dwell on the details longer, it seems to me that the strata above the photosphere must be highly confused, though there is no doubt that the order of elevation generally assigned to the reversing layer, chromosphere, etc., is correct. If the zeal displayed by a successful comet hunter were devoted to searching for changes in the solar spectrum, rich rewards could reasonably be expected.

These statements are not opposed to the fact that progress must be made by differentiating and isolating various kinds of phenomena; and those instruments and methods which have this power most markedly are the most promising. The spectrographic method of differentiating, including the line-displacement principle, has had no equal; and the extension of it by the two-slit spectroheliograph has been a real advance. Much can reasonably be expected from Hale's improved form of this instrument.

The value of a solar eclipse as a partial differentiator, through the elimination of photospheric light, should not be overlooked. Surprising, indeed, is the impetus given to solar study by eclipse observations. Our knowledge of the *existence* of the reversing layer, the chromosphere, the prominences, and the corona, is an eclipse product; and so are many of our present everyday methods of studying them. The

[1] *Lick Observatory Bulletin* No. 18, and CLERKE'S *History of Astronomy*, 4th ed., p. 190.

richness of eclipse results, considering the remarkably short intervals available for observation, is unique in science. Nor are eclipse problems all solved; every such event brings its returns. Whether the reversing layer and chromosphere can ever be studied equally well without an eclipse is a question. They probably cannot, by means now available, but Hale's observation of carbon bright lines near the photosphere shows clearly that the effort should be made. Limitations in quality may be more than balanced by continuity and timeliness. The reviewer agrees with Miss Clerke (p. 135) that "the prospect is dim of realizing" success in observing the corona without an eclipse, at least along the lines of attack thus far attempted. Any chance for even moderate success would seem to be limited to the gaseous inner portion, and a daily record of this would no doubt be extremely valuable, but the real problem of the corona would remain unsolved.

Miss Clerke refers to the possibility that electrical or magnetic forces are playing prominent parts in solar and stellar phenomena; and while positive evidence of their action has not yet been recognized, the idea is a valuable one, and observers would do well to apply tests for Zeeman and other effects whenever opportunity offers. In this connection, I have been more than surprised to find no mention of Arrhenius's theory of light- and heat-pressure effects in the production of the more extended solar phenomena. Nichols's proof that theory is in agreement with observation on this question requires us to consider whether such forces at the Sun's surface may not be sufficient to create observable movements of finely divided matter existing at photospheric temperatures. Is it not possible that the resultant of gravity and light- and heat-pressures is small for very minute particles, and that herein lies the explanation of the hypothetical principle of "levity" referred to by the author, on p. 51 and elsewhere?

The salient points in solar progress have received masterful treatment from Miss Clerke. Especially well is attention called to the gaps in our knowledge. Many of them could be filled, at least in part, by the energetic use of apparatus now in existing observatories, while others demand new and expensive equipments, with ample provision for their systematic application. The Sun as the great exemplar of our science is receiving but scant attention.

Part II, 375 pages on "Sidereal Physics," is a very able presentation of facts and theories for all known types of stars, and for nebulæ. It is not so systematic as the author's treatment of "Solar Physics," but the nature of the subject, and the scattered character of observed

facts, forbid. There are chapters on all the principal spectrum types, on all the types of variable stars, on all known classes of nebulæ, on the spectra of double stars, on spectroscopic binaries, new stars, dark stars, and the Milky Way, etc. ; forty-one chapters, each rich with the most salient facts of observation, with theories very wisely and conservatively discussed, and with constant suggestion as to points most in need of strengthening by observation. The impossibility of doing justice to so extensive a work in a brief review will be evident to all.

The field of stellar and nebular investigation has its own difficulties. The quantity of light received from a star or nebula is really very minute; a stellar spectrum is an integrated result, for photosphere, reversing layer, chromosphere, prominences, and corona—all fall in one small point, as viewed from the Earth; the distances, diameters, and in nearly all cases the masses, are unknown, leaving us without basis for correlating masses, quantity, and intensity of light; the amount of measurement accurate according to today's standard is small; little effort has been devoted to the ultra-violet region, so important from temperature considerations; and laboratory accompaniments are but just beginning on a requisite scale. Nevertheless, the mass of solid fact brought within the realm of knowledge in the past forty years is exceedingly encouraging.

The foundation of every philosophic treatise on the stars must rest, so far as information now avaible is concerned, upon a classification of stars according to spectral types, preferably in the order of their supposed evolution from nebulæ to dark stars. Miss Clerke treats nebulæ last of all. This is not the order of nature, but the plan has its advantages, principally because we know too little about the nebulæ and their processes of transition to stellar states. The astrophysical treatise of the future will probably begin with the nebulæ or perhaps with the origin of the nebulæ! Miss Clerke assigns all stellar spectra to eight classes: four showing absorption spectra only, and four marked by both absorption and emission spectra. The first five classes are: I, helium stars, brilliantly white, hydrogen and helium absorption predominating, the *Orion* and *Pleiades* stars as illustrations; II, hydrogen stars, with intense hydrogen absorption, feeble and thin calcium H and K, feeble iron lines, with *Vega* and *Sirius* as examples; III, solar stars, with innumerable metallic lines, the lower hydrogen lines, and broad and heavy H and K, with the Sun and *Arcturus* as types; IV, stars with fluted spectra, containing a great number of fine dark lines, with superposed heavy absorption flutings whose heads are toward

the violet, α *Orionis* and α *Herculis* as illustrations; V, carbon stars, with innumerable dark lines, very feeble in violet, strong carbon absorption flutings and many apparent bright lines, 19 *Piscium* being typical.

This scheme of classification differs very little from Secchi's four-type system. Classes I and II are subdivisions of Secchi's Type I. Class III is Secchi's II, and IV and V are Secchi's III and IV. It is in effect, also, a condensation of Miss Maury's system of more than twenty groups. It is of course understood by all readers that any such scheme of classification will divide the stellar spectra into compartments only very imperfectly: there will be many spectra on the border lines between classes, especially in Classes I, II, and III. Classes IV and V have more individuality than the others.

Secchi's classification has been a most fortunate and useful one. For general purposes it is satisfactory even today, when the observational data are almost incomparably more numerous and accurate than when Secchi formed it; and I doubt whether the time has come to replace it with another. The Harvard system is very useful, and perfectly justifiable, as an expression of the results of systematic examination and study of a great number of spectra. But its general adoption in the daily work of astrophysicists would seem to be unwise for the present, in view of our very imperfect knowledge of the temperature and other basic conditions underlying the real system of stellar development. New determinations of the spheroidal constants of the Earth, or of the aberration constant, based upon the latest data, are extremely valuable; but to adopt each improved spheroid as a basis for government surveys, or each improved aberration constant in astronomical yearbooks, would lead to hopeless confusion. For similar reasons I regret the division of Secchi's I into Miss Clerke's I and II, and the mechanical shifting of the former's II, III, and IV to the latter's III, IV, and V. The highly desirable distinctions which Miss Clerke desired to call attention to in stars of Secchi's I could have been accomplished more wisely, I think, by some subclassification analogous to Vogel's — notwithstanding the very important calcium-absorption distinctions covered by her plan. Again, her Class VI stars, having fluted spectra and showing hydrogen and other bright lines, includes only the variables of the *o Ceti* type. They are certainly very closely related to her Class IV stars. The spectra of the variables apparently differ from those of Class IV only in having bright lines, induced probably by the forces which produce variations

in brightness; and in this case a subclass of IV would certainly be satisfactory and more logical. Helium stars with bright lines and Wolf-Rayet stars are assigned to Classes VII and VIII. Miss Clerke states that "no hypothesis of growth and affinity is implied by the order of succession given to the bright line objects," though "it certainly rests upon broad and unmistakable distinctions." Class VII is certainly closely related to the Class I helium stars, and a subdivision for these under I would seem to me both advantageous and natural. Of their close relationship there is no doubt, and many facts could be cited in that connection. Space prevents reference to more than the bright lines in spectra of *Alcyone*, *Pleione*, *μ Centauri*, etc., and the absence of the bright lines from many otherwise almost identical spectra of Class I. The assignment of a special class to the Wolf-Rayet stars — Miss Clerke's VIII, Pickering's V — seems necessary at present, as no one is now able to fix their relation to other spectra. These stars, like comets, seem to stand out from any system of stellar evolution, but we should be fully justified, I think, in saying that they are comparatively new stars. It is also possible that they, as well as other stars of peculiar spectral types, are alien to the main sidereal system, and have no place in the sequence of development of ordinary stars; but this seems very improbable.

In the absence of full information as to what conditions produce bright lines in stellar spectra — whether in *γ Cassiopeiae* or in *o Ceti* — it is difficult to say why certain stars have bright lines and others have not. It is a constant surprise to me that bright-line spectra are not vastly more numerous than we have found them to be. Miss Clerke's reference to Classes I and II as *unveiled*, in contradistinction to the Sun's photospheric veil, may prove to be a very useful one. It probably has a basis in fact, though of this we have no positive proof. It would be surprising if the formation of bright lines is not favored by the absence of a photospheric veil. The study of bright line spectra is one of the most fascinating and important subjects in spectroscopy. They appear to represent critical periods in stellar development, or to accompany rapid changes, as in variable stars. Systematic study of such spectra is greatly needed. The field is undoubtedly a rich one. It is but a few years since the discovery was made of both bright and dark lines of hydrogen in the same spectrum, the bright lines growing weaker from red to violet, and the dark lines growing stronger. Enough additional evidence has been collected to indicate that a general principle is here involved; and Kayser's explanation, or some other equally

simple one, will probably establish its fundamentality. As a case in point, since Miss Clerke wrote (p. 220), "the invisibility (up to the present) of hydrogen in the carbon stars is not easily accounted for," a long-exposure spectrogram secured with the Yerkes two-foot reflector has shown strong dark $H\gamma$ and $H\delta$ lines in the red star 19 *Piscium*, whereas $H\beta$ is bright or missing in all observed stars of that type. The laboratory investigations of Hale and others on spark spectra obtained under high pressure show somewhat analogous phenomena, but of their correct interpretation I do not feel sure.

It is very unfortunate that our knowledge of temperature relations in stars of various types is so defective. The paucity of laboratory results is perhaps the major cause. The great importance and attractiveness of work along this line is evident to those who have followed investigations on the magnesium line at $\lambda 4481$ by Scheiner, Huggins, and others, or on the relation between arc and spark spectra by Hartmann. Very few such researches are sufficient to modify greatly our views on stellar temperature relations.

In this connection, attention should be called to the great need for extensive observations of stellar ultra-violet spectra, along the lines pointed out by Huggins. If the truly continuous portion of the ultra-violet in solar types is relatively stronger than in helium stars, that surprising fact should be fully established.

Miss Clerke's computations for the masses and radiative powers of various stars, leading to results very widely different from those for our Sun, are extremely interesting, and they in a sense mark the opening of a new field of great significance. Unfortunately, the available parallaxes for many of these stars are so uncertain as to introduce large uncertainty into her numerical results. The need for better and more numerous parallax data is at least as pressing in astrophysics as in the study of the distribution of the stars in space, and it is fortunate that extensive plans are forming at several observatories to supply the deficiency. However, granting the sufficiency of the author's parallax data, the tremendous radiative powers arrived at for certain stars need not wholly surprise us. There are certainly no more startling facts in all astronomy than those of *o Ceti* varying in brilliancy four-thousand-fold, with very little change in spectral type; of *Nova Cygni* varying from third to fifteenth magnitude, the spectrum at the present lower limit being continuous, without any evidence of bright lines; and of *Nova Persei* decreasing from the first in brilliancy in the northern hemisphere to below the tenth magnitude. In these cases the ratio of mass

to quantity of light radiations has had a remarkable range. It is impossible to say what the brightness of the Sun would be if its photospheric veil were suddenly removed.

Among the most interesting chapters in the book are those on variable stars. Excepting the new stars, a half dozen *Algol* and δ *Cephei* types, β *Lyrae* and o *Ceti*, our knowledge of variable star spectra is remarkably slight. No well-equipped observer has devoted himself entirely or in the main to that subject. They offer a great variety of phenomena, and afford almost exclusive opportunities to observe spectral changes in rapid progress. Miss Clerke emphasizes the need for more work. There is in my opinion no richer field awaiting systematic cultivation. The number of variables known is about 1200.

The author's urgent recommendation that the bright lines in variable star spectra be tested for Zeeman effect is timely. The reviewer has held the subject in mind for several years in connection with the triple $H\gamma$ and $H\delta$ of o *Ceti*, but when the star was in favorable position the lines have been either single or too faint for such observation. The Lick tests on the broad bands of *Nova Persei* in 1901, and of *Nova Geminorum* in 1903 — no decrease in width resulting from a rotating Nicol prism — seem to be the only applications of such tests to stars thus far made.

Many interesting features of variable- and double-star spectra, of nebulæ, star clusters, etc., merit the reviewer's attention, but lack of space prevents.

In a book written in so broad a spirit, it would be invidious for the reviewer to enumerate the fifteen or twenty minor errors noticed, but a few should be referred to. On p. 500, Keeler's result is misinterpreted — that in the hydrogen vacuum tube "the third line ($H\gamma$) always vanishes experimentally before the first ($H\alpha$), while in a nebula it ($H\gamma$) shines unfailingly, although $H\beta$[1] be imperceptible." The $H\beta$ is, I think, visually much brighter in all nebulæ than is $H\gamma$, though $H\gamma$ is brighter than $H\alpha$. Again, at the top of p. 501, the uninformed reader will undoubtedly credit the discovery of variations in the relative intensities of the *Orion* nebular lines to one of the several observers who confirmed, and not to the discoverer who bore the brunt of the unusually hostile criticisms following the announcement of his discovery.

On p. 394, l. 13, the erroneous dates "7th and 13th November"

[1] Since this review has been put in type, it seems probable that $H\beta$ in the quotation is a misprint for $H\alpha$. — W. W. C.

instead of the correct dates 9th and 13th, are extremely unfortunate, though the dates were stated correctly in the author's *History of Astronomy*, 4th ed., p. 401.

It is to be regretted that the illustrations of nebulæ, clusters, etc., are not the most modern obtainable.

Miss Clerke's book is in my opinion most ideally planned: its history is fairly, but inconspicuously, complete; only relevant facts are stated; theory has not run away with observation; the defective corners in our field of knowledge are pointed out; the style is lucid, attractive, and logical; and to me its most remarkable feature is the wonderfully correct estimate of the relative values of observations, by an author with little or no experience in making observations.

W. W. CAMPBELL.

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A REVISION OF ROWLAND'S SYSTEM OF WAVE-LENGTHS.

By J. Hartmann.

1. Rowland's system of wave-lengths has become the basis of all spectroscopic measurements made in recent years, and with the progressive increase in the precision of these measures the necessity has now arisen of testing the reliability of that important basis, and, in case it should not appear as adequate, of correcting it by new series of observations in order thus to create a foundation sufficient for all demands.

Aside from the earlier and provisional publications of Rowland, his system of wave-lengths as at present accepted appears in three different forms:

a) The standard wave-lengths in the solar spectrum, and

b) The standard wave-lengths in the arc spectra of different metals which was first published in *Astronomy and Astro-Physics* (**12**, 321, 1893) under the title "A New Table of Standard Wave-Lengths." For brevity I shall denote the wave-lengths of these two series by ☉ St. and M. St. Rowland gave in 1896 in the *Memoirs of the American Academy of Arts and Sciences* (**12**, No. 2, 101) a more extended account of the origin of these standards.

c) The measurement of the whole solar spectrum which appeared in the ASTROPHYSICAL JOURNAL, Volumes 1 to 5, in the years 1895–97, under the title "Preliminary Table of Solar Spectrum Wave-Lengths." The wave-lengths in this table I shall designate by P. T.

The question which first arises, as to how far these three series of wave-lengths are to be regarded as coincident, so that we can speak of a single Rowland system, may be answered in two different ways—first, on the basis of the history of the development of these series, and secondly, by a numerical comparison of them.

2. The history of the development of Rowland's wave-lengths is briefly as follows. By taking the mean of the absolute measures of Ångström, Müller and Kempf, Kurlbaum, Peirce, and Bell, Rowland derived for the Fraunhofer line D_1 the wave-length

$$\lambda = 5896.156.$$

To this line he connected thirteen further "primary standards" (P. St.) in the visible spectrum by visual measures of coincidences with the use of several concave gratings of $21\frac{1}{2}$ feet focus. These P. St. are single lines in only certain instances; being for the most part mean values from the wave-lengths of groups of two, three, and four lines, which are separated by distances as great as 76 tenth-meters. As the result of numerous measurements he obtained twenty-six equations of condition of the form

$$nA - mB = D,$$

where n and m represent the order of the coinciding spectra concerned, B the wave-lengths of the observed lines, and D the separation of the lines measured with the eyepiece micrometer. The measured distances of the lines observed as coincident are in part very large: only in eight cases are they smaller than ten revolutions of a screw for which one revolution = one tenth-meter in the first order of the grating concerned; in eleven cases it is more than twenty revolutions, and in two cases it even exceeds one hundred revolutions. Although the micrometer screw employed by Rowland, which he himself made in the most careful manner

and investigated, is undoubtedly of the highest excellence, there is nevertheless a certain danger in the measurement of such long distances, particularly as these distances frequently could be measured only on one side, so that it was not possible to interpolate the lines of the one order *between* those of the other order. How Rowland obtained the screw-value with sufficient accuracy for such long distances is not to be readily ascertained from his publications, which, indeed, contain so few data as to the measurements themselves that a test of them is impossible.

3. Rowland did not solve the twenty-six equations of condition for the wave-lengths of the fourteen P. St. by the method of least squares, although this would have been quite simple, as each equation contains only two unknowns with integral coefficients ranging from two to seven. He employed *per contra* the process of successive approximations, which had the advantage of greater clearness, and without doubt also led to a system of values which satisfied very well the equations of condition.

From this process of adjustment Rowland deduced for the line D_1 the wave-length 5896.160, and, in order to bring this value into better agreement with the above-mentioned mean from the absolute measurement, he reduced[1] all the wave-lengths found in the solution by $\frac{1}{200{,}000}$. In this way he now obtained for D_1, $\lambda = 5896.157$.

The P. St. determined in this way lie between the limits λ 4215.6 and λ 7040.1. Subsequently the Fraunhofer line A was also once connected in the spectrum of the first order with the nearest normal in the second order, for which purpose it was necessary to measure with the screw over a distance of 84 tenth-meters; a further connection was also obtained by the coincidence of the second and third order.

4. The 798 ⊙ St. and the 404 M. St. were now connected with these 15 P. St. as follows: The visual portion of the solar

[1] The corrections actually applied by Rowland (*loc. cit.*, p. 139) do not agree with this statement in the case of several of the standards. It cannot be ascertained whether this is due to typographical errors, of which the publication unfortunately contains a very large number, or are errors of computation, or whether certain of the standards were intentionally further altered for some reason which was not given.

spectrum between λ 4048 and λ 7714 was repeatedly measured with a micrometer screw 5 inches long, with the use of various gratings. The portions separately measured with the screw were then united by means of the overlapping portions into thirteen long strips, which rendered possible not only an interpolation of the standards, but also an adjustment of the P. St.

Eighteen photographs were made with the solar spectrum of one order situated between two strips of the solar spectrum of another order for the continuation of the work into the ultra-violet region. A large number of plates was also made which contained a solar spectrum and beside it metallic spectra of different orders. The measurement of all these plates, which was carried on by Mr. Jewell, now produced numerous combinations between the different parts of the spectrum, and thus there always occurred, along with the interpolation of the standards, also an adjustment of the P. St. Thus for each wave-length of the standards there were developed a number of individual determinations from which finally the mean was taken; these are the numbers published as ⊙ St. and M. St.

5. It follows from this origin that the whole system of ⊙ St. is entirely cóincident with that of the P. St., but that the wave-lengths of the single lines in the two systems may differ by small quantities from each other. Thus the line D_1 appears under the ⊙ St. with the wave-length

$$\lambda = 5896.154.$$

The ⊙ St. are to be regarded in every case as the better adjusted system in comparison with the P. St.

6. The conditions are less simple with the M. St. Rowland remarks[1] as follows:

In every plate having a solar and metallic spectrum upon it, there is often — indeed, always — a slight displacement. This is due either to some slight displacement of the apparatus in changing from one spectrum to the other, or to the fact that the solar and the electric light pass through the slit and fall on the grating differently. In all cases an attempt was made to eliminate it by exposing on the solar spectrum both before and after the arc, but there still remained a displacement of 0.01 to 0.02 division of Angström, which was determined and corrected for by measuring the difference between

[1] *Memoirs of the American Academy*, **12**, 116.

the metallic and coinciding solar lines, relating a great number of them, if possible.

It follows from this remark that the wave-lengths of the M. St. which were directly measured and hence accurately referred to the system of the ⊙ St., in case the apparatus was in order, were subsequently so altered by empirical corrections, now larger and again smaller, that as accurate a coincidence as possible was attained between the lines of the arc and the solar spectrum.

Jewell was the first to call attention to the erroneousness of this process, in his very valuable paper on "The Coincidence of Solar and Metallic Lines."[1] It cannot now be ascertained by what amounts the individual M. St. were falsified by the application of these corrections. More definite information could not be given to me by Mr. Jewell in response to a letter which I addressed to him several years ago. We must therefore first remember that all the M. St. are erroneous by small amounts, which cannot be recomputed, lying in the vicinity of 0.01 to 0.02 tenth-meters, and in general too large relatively to the system of the ⊙ St. In certain parts of the spectrum the ⊙ St. may be affected to a small amount by the application of these empirical corrections, since Rowland repeatedly also employed the M. St. in order to derive the ⊙ St. in a spectrum of another order by the method of coincidences.

7. While the progression of the errors which arose from the application of these corrections is very gradual, so that the error for neighboring lines is always of nearly the same amount, I must now call attention to another source of error which may have produced larger deviations of *individual* lines. As already mentioned, the ⊙ St. and the M. St. are mean values from a number of different sorts of determinations; and the number of plates employed is very different for each line. The following series of M. St. in the iron spectrum may serve as an example.

Even this short list shows that the M. St. measured on eight or nine plates either coincide very closely with the ⊙ St. or fall on the negative side of them, while all the lines measured only once or twice deviate, in some cases by very considerable

[1] ASTROPHYSICAL JOURNAL, 3, 89, 1896.

TABLE I.

M. St.	No. of Measures	☉ St.	M. St. − ☉ St.
4308.072	8	.071	+ 1
4325.932	8	.940	− 8
4352.908	1	.903	+ 5
4369.948	1	.943	+ 5
4376.108	1	.103	+ 5
4383.721	9	.721	0
4404.928	9	.927	+ 1
4415.298	8	.299	− 1
4447.912	2	.899	+13
4494.756	1	.735	+21

amounts, toward the positive side. In particular I would refer to the uncertainty of the two lines at λ 4448 and λ 4495.

The direct measures made by Rowland and Jewell on each separate plate certainly possess a very high degree of accuracy, so that the relative position of the lines was determined to a precision of a few thousandths of a tenth-meter; but since each plate has a systematic error, chiefly in consequence of the application of the empirical correction, there occur, through the above-mentioned lack of uniformity in the number of plates employed, displacements in the relative position of the individual lines which may rise as high as several hundredths of a tenth-meter.

8. The relative wave-lengths of the ☉ St. are also injuriously affected by the large variation in the number of measurements employed for the individual lines. However, the errors arising in the individual wave-lengths of the solar spectrum are for the most part rendered harmless by the extensive measurement of the whole solar spectrum which was carried out in a uniform manner, the results of which are given in the P. T. The wave-lengths of the P. T. depend upon the measurements of the photographic plates of the whole solar spectrum carried out by Jewell. In this work the newly measured lines were not merely referred to the ☉ St. by interpolation, but a solution was made for every plate by which the ☉ St. were also freed from their accidental errors, so that they constitute a homogeneous system with the lines newly connected to them. There accordingly exists between

the wave-lengths of the P. T. and the ☉ St. a relation similar to that between the ☉ St. and the P. St. (see 5). The *whole system* of the P. T. is also here to be regarded as entirely coincident with the system of the ☉ St., but the wave-lengths of the individual lines in both systems may differ from each other by small amounts. For instance, the line D_1 appears in the P. T. with the wave-length 5896.155.

9. It appears from this statement of the history of the origin of Rowland's wave-lengths that the P. T. constitutes the definitive and most reliable form of the Rowland system. If, for instance, the question should arise which is the most correct of the four values for the wave-length of the line D_1 in the solar spectrum, viz.:

$\lambda = 5896.156$ (starting value)
.157 (P. St.)
.154 (☉ St.)
.155 (P. T.)

and which most precisely represents the Rowland system, the answer would be the value 5896.155.

10. The conclusions which were reached as to coincidence of the three systems P. St., ☉ St., and P. T. on the basis of the history of their origin are confirmed by a direct comparison of the numerical values. The relation of the ☉ St. to the P. St. may be seen from Table II.

The difference ☉ St.—P. St. amounts at the maximum for a line to 0.007 tenth-meter; the sum of all these differences properly should be exactly zero; for the thirty-four lines together, however, it amounts to +0.016, whence the mean value of this difference is +0.00047 tenth-meter. Hence the two systems P. St. and ☉ St. are sufficiently in coincidence.

11. On account of its great extent I will not print here the comparison of the system ☉ St. and P. T., and I will limit myself to giving in Table III for intervals of 100 tenth-meters under the heading S the sum of the differences P. T.—☉ St. In an entirely rigorous comparison S should be, at least for long stretches of spectrum, zero. The number of ☉ St. employed is denoted by n. Several of Rowland's ☉ St. had to be omitted in this com-

TABLE II.

P. St.	⊙ St.	⊙ St.-P. St.	P. St.	⊙ St.	⊙ St.-P. St.
4215.665	.667	+2	5397.350	.346	−4
4222.381	.381	0	5405.984	.987	+3
4376.103	.103	0	5624.254	.253	−1
			5624.764	.768	+4
4494.729	.735	+6	5826.582	.580	−2
4497.048	.041	−7	5890.184	.182	−2
4501.444	.444	0	5896.157	.154	−3
4508.460	.456	−4	5914.386	.384	−2
4690.323	.324	+1	6246.530	.530	0
4691.575	.581	+6	6318.241	.242	+1
			6322.912	.912	0
4903.484	.488	+4			
4924.110	.109	−1	6563.049	.054	+5
4924.956	.955	−1	6569.461	.461	0
5060.250	.252	+2			
5064.834	.833	−1	6750.409	.412	+3
5068.946	.946	0			
			7023.747	.747	0
5269.717	.722	+5	7027.724	.726	+2
5270.497	.495	−2	7040.056	.058	+2

parison because the measurement was made upon the center of gravity of a double line, the components of which were separately measured in the P. T. M denotes the mean error of coincidence, that is, the quotient $\frac{S}{n}$, and the last column contains the largest one of the differences P. T. − ⊙ St. which occur in the particular portion of the spectrum.

Table IV contains a similar comparison for stretches of 1,000 tenth-meters.

It may be seen from the numbers of Tables III and IV that the P. T. are well related to the system of the ⊙ St., if we except the short region from λ 7000 to λ 7400, the measurement of which gave special difficulty. The difference P. T. − ⊙ St. amounts in the mean to +0.00040 tenth-meter for the whole spectrum from λ 3000 to λ 7000. The errors of individual lines of the ⊙ St. exceed 0.03 tenth-meter only in isolated cases; as a rule they amount to from 0.01 to 0.02 tenth-meter.

TABLE III.

λ	S	n	M	Maximum
3000–3100	−0.014	21	−0.0007	+0.018
3100–3200	− 13	12	− 11	− 13
3200–3300	+ 59	10	+ 59	+ 21
3300–3400	+ 15	8	+ 19	− 12
3400–3500	− 15	16	− 9	+ 20
3500–3600	+ 50	17	+ 29	+ 18
3600–3700	− 20	29	− 7	+ 30
3700–3800	− 28	32	− 9	± 16
3800–3900	+ 77	23	+ 33	+ 20
3900–4000	+ 44	25	+ 18	+ 16
4000–4100	− 18	17	− 11	− 11
4100–4200	+ 1	9	+ 1	+ 6
4200–4300	+ 28	14	+ 20	+ 12
4300–4400	+ 33	11	+ 30	+ 10
4400–4500	− 18	13	− 14	− 18
4500–4600	− 10	9	− 11	− 3
4600–4700	− 4	12	− 3	− 7
4700–4800	− 15	5	− 30	− 12
4800–4900	+ 29	6	+ 48	+ 31
4900–5000	− 28	13	− 22	− 10
5000–5100	− 13	11	− 12	− 8
5100–5200	− 32	31	− 10	− 15
5200–5300	+ 41	28	+ 15	+ 28
5300–5400	+ 18	17	+ 11	+ 30
5400–5500	− 33	15	− 22	− 27
5500–5600	− 43	16	− 27	− 31
5600–5700	− 3	16	− 2	− 14
5700–5800	+ 8	18	+ 4	± 15
5800–5900	− 7	13	− 5	− 12
5900–6000	+ 1	13	+ 1	+ 5
6000–6100	− 19	13	− 15	− 10
6100–6200	− 26	17	− 15	− 16
6200–6300	+ 70	17	+ 41	+ 26
6300–6400	− 29	13	− 22	− 24
6400–6500	+ 90	15	+ 60	+ 29
6500–6600	+ 72	12	+ 60	+ 49
6600–6700	− 39	6	− 65	− 38
6700–6800	− 37	10	− 37	− 37
6800–6900	+ 46	35	+ 13	− 32
6900–7000	+ 36	25	+ 14	+ 24
7000–7100	+ 157	13	+ 121	+ 51
7100–7200	− 21	7	− 30	− 21
7200–7300	+ 61	12	+ 51	+ 41
7300–7400	− 14	3	− 47	− 26

TABLE IV.

λ	S	n	M	Maximum
3000–4000	+0.155	193	+ 0.0008	+0.030
4000–5000	− 2	109	0	+ 31
5000–6000	− 63	178	− 4	− 31
6000–7000	+ 164	163	+ 10	+ 49
7000–7400	+ 183	35	+ 52	+ 51
3000–7000	+0.254	643	+0.00040	+0.049

12. In Table V, I give a comparison of the systems ⊙ St. and M. St. A few lines had to be excluded here also, for which it appeared, from the remarks made by Rowland, that not precisely the same lines were measured in the solar spectrum and the arc spectrum.

TABLE V.

λ	S	n	M	Maximum
3000—3500	+0.109	29	+0.0038	+0.027
3500—4000	+ 284	80	+ 35	+ 30
4000—4500	+ 90	47	+ 19	+ 21
4500—5000	− 64	15	− 43	− 25
5000—5500	− 62	21	− 30	+ 72
5500—6000	− 64	8	− 80	− 80
3000—6000	+0.293	200	+0.0015	−0.080

An almost perfect coincidence has also been effected between the systems M. St. and ⊙ St., as is seen from the mean value holding good for the whole spectrum, $M = +0.0015$ in Table V. It has already been mentioned above under 6 that this does *not* actually occur, and it is particularly striking that the difference M. St. − ⊙ St. is prevailingly positive in the stretch from λ 3000 to λ 4500; while the lines of the solar spectrum, in consequence of the greater pressure under which the absorbing strata of vapor exist in the solar atmosphere, should in general be somewhat displaced toward the red as compared with the arc lines at atmospheric pressure. Hence the difference M. St. − ⊙ St. should be prevailingly negative; as may be seen from Table V, this is true, however, only between λ 4500 and λ 6000.

The conclusions drawn above are accordingly confirmed by the direct comparison of Rowland's tables of wave-length: the P. T. constitutes the most reliable form of Rowland's wave-lengths; the ⊙ St. and the P. St. coincide on the whole with the P. T., but nevertheless individual lines have large accidental errors; the M. St. similarly contain large accidental errors, and are, moreover, systematically displaced by measurable amounts as compared with the system of the P. T.

13. At this point I will also refer to another system of wave-lengths which bears the same relation to the M. St. that the P. T. bears to the ⊙ St. This is the "Normale aus dem Bogenspectrum des Eisens," which Kayser published in the year 1900.[1] Kayser's standards, which I shall briefly designate with K., are — aside from a region near λ 3400 — precisely connected to the system of the M. St., and therefore contain the constant systematic errors of the M. St., but the careful measurement of numerous plates has so far reduced the accidental errors of the separate lines in K. that the mean error of a line amounts at the most to 0.003 tenth-meter.

14. We shall certainly not be far from the truth if we assume that the accidental error of the single lines in the P. T. is about ± 0.003.[2] It is true that a direct test of the P. T. has never been made, but everyone who has occasion to employ Rowland's wave-lengths will have become convinced of their great reliability, in so far as the relative position of lines within small regions of spectrum are concerned. The conditions are, however, changed in respect to the accuracy of the relative wave-lengths as soon as widely separated portions of the spectrum are compared with each other. Attention has already been called by various individuals to the fact that there are irregularities which slowly increase in Rowland's values, as a result of which for considerable portions of the spectrum all the wave-lengths are from 0.01 to 0.02 tenth-meter too large, and in other regions are too small. Müller[3] was the first to point out the possibility

[1] *Ann. der Phys.*, (4) 3, 195; ASTROPHYSICAL JOURNAL, 13, 329, 1901.

[2] A proof of the correctness of this estimate follows, under 16.

[3] "Beobachtungen auf dem Gipfel des Säntis;" *Publ. des Astroph. Observatoriums zu Potsdam*, 8, 49, 1893.

of such errors on the occasion of his making a comparison of the Potsdam system of wave-lengths with the ⊙ St. He found that the difference "Potsdam — ⊙ St." had a prevailingly positive sign from λ 4900 to λ 5400, while it was negative from λ 5400 to λ 6100, and then positive again up to λ 6350. Kayser[1] expressed himself more definitely when he found that the values of several M. St. at λ 3400 are too large by from 0.02 to 0.03 tenth-meter. An entirely certain explanation as to the magnitude and the course of these errors in a part of the visual spectrum was, however, first given in the research by Fabry and Perot,[2] who determined with their interference apparatus the absolute wave-lengths of 33 lines of the solar spectrum and of 14 lines of the iron spectrum. In order to compare their results with Rowland's wave-lengths they computed for the 33 lines the quotient

$$F = \frac{\text{P. T.}}{\text{F. and P.}\odot},$$

which must be constant if Rowland's figures are free from systematic errors. The values of F show, however, a clearly expressed progression which precisely coincides in respect to sense with the results of Müller. Eberhard[3] then made a more precise comparison of the Potsdam and Rowland systems, utilizing 104 lines of the P. T., and was able to furnish another complete confirmation of Fabry and Perot.

15. There can accordingly no longer be any doubt that the wave-lengths of the P. T. are up to 0.02 unit too small from λ 4900 to λ 5370; that they are up to 0.02 unit too large between λ 5370 and λ 6050, and then again too small up to λ 6500. We may assume that similar errors will be revealed in the remaining portions of the spectrum which have not yet been checked, and the question therefore arises in what manner a correction can best be applied to the Rowland system. At the same time it would be necessary to determine the systematic

[1] *Loc. cit.*, p. 198.

[2] "Mesures de longeurs d'onde en valeur absolue," *Ann. de Chim. et de Phys.*, (7) 25, 98; ASTROPHYSICAL JOURNAL, 15, 270, 1902.

[3] Systematic errors in the Wave-Lengths of the Lines of Rowland's Solar Spectrum," ASTROPHYSICAL JOURNAL, 17, 141, 1903.

difference between the systems of the P. T. and the M. St., which have been extensively discussed above.

Fabry and Perot write, after determining the values of the reduction factor F: "All recent spectroscopic measurements which have been based upon Rowland's figures contain the same errors. In order to correct them, and at the same time to reduce them to the value of the wave-length of the cadmium line found by Michelson and Benoit, one has only to divide the values by F." I would, however, by no means recommend this procedure, for, in the first place, the systematic error above mentioned between the ☉ St. and the M. St. is not contained in the F, and since most observers have used the iron spectrum for comparison and not the solar spectrum, the true value of the wave-length would not be obtained by division by F. Secondly, it certainly could not be recommended to alter so greatly all previously determined wave-lengths as would be necessary to pass from the Rowland system to that of Michelson. The reduction factor F has at this place the value 1.000034, whence it follows that all the wave-lengths in the visual spectrum would have to be diminished by about 0.2 tenth-meter if they were to be transferred to the Michelson system. This would produce great confusion.

16. I would, on the contrary, recommend that each correction of Rowland's wave-lengths should be undertaken in such a manner that his numerical values are thereby altered only by the *smallest possible* amount. This may be accomplished in the following manner. As has already been said, Fabry and Perot give the quotient $\frac{\text{P. T.}}{\text{F. and P.}} = F$ for each of the 33 solar lines they measured. The values of F lie between the limits 1.0000286 and 1.0000381. If we take the arithmetical mean of these thirty-three values,

$$F_0 = 1.0000340 \ ,$$

and multiply all the wave-lengths found by Fabry and Perot by this factor, we shall evidently obtain a system of wave-lengths which fits that of Rowland as closely as possible, but is free from its systematic errors. If we designate these errorless wave-

lengths on the Rowland system by λ, then $\lambda = (\text{F. and P.}) \cdot F_0$. I have computed the values of λ in Table VI, in which the first column contains the wave-lengths directly measured by Fabry and Perot, the second the value of λ, and the third the corresponding number in the P. T. The difference $\lambda -$ P. T. $= C'$ given in the fourth column therefore represents the correction which must be applied to Rowland's values in order to free them of their systematic errors. The sum of all the positive values of C' is $+0.178$ tenth-meters, of all the negative values -0.183; a proof that the λ system coincides precisely with the P. T.

TABLE VI.

F. and P.	λ	P. T.	C'	C	$C'-C$
4643.483	4643.641	.645	− 4	− 7	+ 3
4704.960	4705.120	.131	−11	− 6	− 5
4736.800	4736.961	.963	− 2	− 5	+ 3
4783.449	4783.612	.613	− 1	− 4	+ 3
4859.758	4859.923	.928	− 5	− 1	− 4
4923.943	4924.110	.107	+ 3	+ 1	+ 2
5001.881	5002.051	.044	+ 7	+ 5	+ 2
5090.787	5090.960	.954	+ 6	11	− 5
5123.739	5123.913	.899	+14	+14	0
5171.622	5171.798	.778	+20	+17	+ 3
5247.063	5247.241	.229	+12	+21	− 9
5247.587	5247.765	.737	+28	+21	+ 7
5339.956	5340.138	.121	+17	+13	+ 4
5345.820	5346.002	.991	+11	+11	0
5367.485	5367.667	.669	− 2	0	− 2
5409.800	5409.984	.000	−16	−11	− 5
5434.544	5434.729	.740	−11	−13	+ 2
5497.536	5497.723	.735	−12	−17	+ 5
5506.794	5506.981	.000	−19	−18	− 1
5586.778	5586.968	.991	−23	−20	− 3
5715.095	5715.289	.308	−19	−19	0
5763.004	5763.200	.218	−18	−18	0
5862.368	5862.567	.582	−15	−15	0
5934.666	5934.868	.881	−13	−11	− 2
5987.081	5987.284	.290	− 6	− 7	+ 1
6016.650	6016.855	.801	− 6	− 4	− 2
6065.506	6065.712	.709	+ 3	+ 2	+ 1
6151.639	6151.848	.834	+14	+13	+ 1
6230.746	6230.958	.943	+15	+15	0
6322.700	6322.915	.907	+ 8	+11	− 3
6335.346	6335.561	.554	+ 7	+10	− 3
6408.027	6408.245	.233	+12	+ 5	+ 7
6471.666	6471.886	.885	+ 1	+ 1	0

The accidental errors of measurement of the individual lines are still contained in the values of C'; in order to free them of

these I have graphically represented their progression by the curve reproduced in Fig. 1. From the curve we now obtain the true values C of the corrections, which are collected in Table VII. The values $C' - C$ given in the last column of Table VI correspond to the accidental errors of observation in the difference F. and P. – P. T. If we regard as equally accurate the absolute measurements of Fabry and Perot and the relative wave-lengths of the P. T., we obtain for the mean accidental error of a wave-length in the two series of measurements, ± 0.0025 tenth-

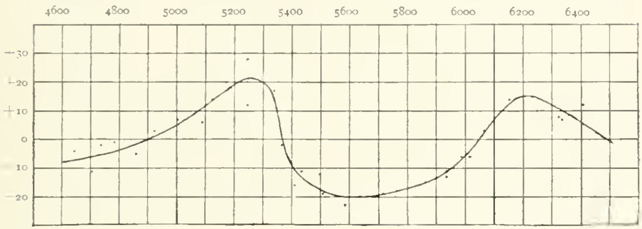


Fig. 1.

meter. The sum of all the values of $C' - C$ in Table VI is zero, whence it follows that the curve was drawn in Fig. 1 with entire accuracy.

17. It is seen that the corrections given in Table VII refer only to the stretch of the solar spectrum from λ 4600 to λ 6490 observed by Fabry and Perot, and hence there arises the necessity, on the one hand, of determining the values of C for the remaining parts of the P. T., and, on the other, of constructing a similar table of corrections for the M. St.

First, in respect to the determination of C for other portions of the solar spectrum, I made the attempt of utilizing for this purpose the plates used by Rowland himself by a new reduction. In all cases where a portion of the solar spectrum, falling within a range of Table VII, was photographed in coincidence with any other portion of the solar spectrum, the values of C for this particular new portion of the spectrum can be obtained by applying the correction from Table VII. It unfortunately

TABLE VII.

CORRECTIONS C OF THE P. T. FROM λ 4600 TO λ 6490.

(Unit is 0.001 tenth-meter.)

	0	1	2	3	4	5	6	7	8	9
46	− 8	− 8	− 8	− 7	− 7	− 7	− 7	− 7	− 7	− 6
47	− 6	− 6	− 6	− 5	− 5	− 5	− 5	− 4	− 4	− 4
48	− 4	− 3	− 3	− 2	− 2	− 2	− 1	− 1	− 1	0
49	0	+ 1	+ 1	+ 1	+ 2	+ 2	+ 3	+ 3	+ 4	+ 4
50	+ 5	+ 6	+ 6	+ 7	+ 8	+ 8	+ 9	+10	+10	+11
51	+12	+13	+13	+14	+15	+16	+16	+17	+18	+18
52	+19	+20	+20	+21	+21	+21	+21	+21	+21	+20
53	+20	+19	+18	+16	+13	+ 9	+ 4	− 1	− 5	− 7
54	− 9	−11	−12	−13	−14	−15	−15	−16	−16	−17
55	−18	−18	−19	−19	−19	−20	−20	−20	−20	−20
56	−20	−20	−20	−20	−20	−20	−20	−20	−20	−20
57	−19	−19	−19	−19	−19	−19	−18	−18	−18	−18
58	−17	−17	−17	−16	−16	−16	−15	−15	−14	−14
59	−13	−13	−12	−12	−11	−10	− 9	− 9	− 8	− 7
60	− 6	− 5	− 4	− 2	− 1	0	+ 1	+ 3	+ 4	+ 5
61	+ 7	+ 8	+ 9	+10	+11	+12	+13	+14	+14	+15
62	+15	+15	+15	+15	+15	+14	+14	+14	+13	+13
63	+12	+11	+11	+10	+10	+ 9	+ 9	+ 8	+ 7	+ 7
64	+ 6	+ 5	+ 4	+ 4	+ 3	+ 3	+ 2	+ 1	+ 1	0

appeared that the material available for this purpose was altogether insufficient, since, for the reasons above mentioned, all measurements had to be excluded in which the wave-lengths of the solar spectrum were derived from coincidences with a metallic spectrum of another order. I was able to derive the values of the correction C collected in Table VIII, which are of very slight accuracy, from the few coincidences between two parts of the solar spectrum which I could utilize.

TABLE VIII.

3500 - - - - -	−0.004 tenth-meter
3600 - - - - -	− 2
3700 - - - - -	+ 2
3800 - - - - -	+ 7
3900 - - - - -	+ 4
4000 - - - - -	− 2
4100 - - - - -	− 7

The corrections, therefore, appear to be under 0.01 tenth-meter for this reason. We may regard as comparatively safely

determined only the stretch from λ 3890 to λ 3990, as here as many as seven plates in two different orders were used for each line. In the region of λ 4300 C appears to attain more appreciable values, but for this place, as well as for the other parts of the spectrum, there are so few measurements available that it is impossible to derive even a tolerably certain value of C from Rowland's observations.

18. The necessity is thus shown of securing new observations in order to obtain the corrections to Rowland's wave-lengths, and since the apparatus necessary for the execution of such fundamental measurements is not available to me at present, I should be glad to suggest to others the prosecution of these investigations. On account of the great importance for all spectroscopic researches of the production of as accurate and errorless fundamental system as possible, it is desirable that these researches should not be carried out at one place only, but should be independently undertaken by as many observers as possible.

There are three different problems to solve which mutually complete and check each other: *first*, the corrections C for all portions of the P. T., hence for the entire solar spectrum, should be determined; *second*, the amount of the displacement of the iron lines in the solar spectrum should be accurately determined by photographs giving the solar spectrum with an adjacent arc spectrum of iron; and *third*, a table of standard lines in the arc spectrum of iron covering the whole spectrum, and rigorously reduced to the corrected Rowland system, should be constructed.

19. If absolute measurements are made for the purpose of obtaining the corrections C, either by the interference method of Fabry and Perot or in any other way, there will always be obtained in addition to C a new determination of the reduction factor F_0. However, for the determination of C it is only necessary to make relative measurements in the solar spectrum, as by observations of coincidences with a large concave grating.

20. For the determination of the displacement of the lines of

the solar spectrum in respect to the arc lines, Jewell[1] has already contributed a very valuable series of measures, which embraces the iron spectrum from λ 3424 to λ 4495, and also includes the lines of numerous other elements. Such measurement should be repeated and extended to all regions of the spectrum. The determination of these differences, which I shall designate by *D*, the sense being solar spectrum minus arc spectrum, is at the present moment a pressing necessity in astrophysics. An exhaustive study of these displacements would not only render possible important conclusions as to the distribution of the pressure and the occurrence of the metallic vapors at different altitudes in the solar atmosphere, but would be of fundamental importance in the determination of the radial velocities of the stars. It will doubtless for a long time be the case that for all stars of the second type, which on account of the sharpness of their lines are particularly adapted for accurate determinations of velocity, the most reliable wave-lengths will be taken from the solar spectrum, since we may assume that conditions similar to those in the Sun prevail in the atmospheres of those stars. If in the computation of the velocity of such a star one should employ for the comparison spectrum the wave-lengths determined in the laboratory and referred to Rowland's M. St.—as, for instance, Kayser's iron standards—the whole amount of *D*, which Rowland did away with by empirical corrections, enters into the velocity and vitiates it. If we assume that *D* has a value of +0.015 tenth-meter in the region of spectrum near λ 4400 chiefly employed for stars of the second type, then every velocity would be found about one kilometer greater than it should be. We should, however, commit precisely the same error if we were to employ the *same* wave-lengths for the stellar and comparison spectrum, as is always the case, for instance, when the method of coincidence measures is used.

21. The above-mentioned series of measures by Jewell is the only one which gives wave-lengths of the iron spectrum strictly referred to the system of the P. T., and I have therefore employed it in order to derive at least provisional values of the corrections

[1] *Loc. cit.*, p. 109.

which must be applied to Kayser's standard in order to make them also comparable with the P. T. Table IX contains the comparison of the lines common to the two observers.

TABLE IX.

Jewell	*K*	*k*	Jewell	*K*	*k*
3424.444	.430	+14	3815.988	.987	+ 1
3440.761	.762	− 1	3820.568	.573	− 5
3441.145	.138	+ 7	3824.584	.591	− 7
3445.308	.301	+ 7	3826.025	.028	− 3
3490.722	.721	+ 1	3827.969	.967	+ 2
3497.990	.989	+ 1	3856.515	.515	0
3536.704	.694	+10	3860.052	.054	− 2
3581.338	.348	−10	3886.415	.426	− 9
3617.934	.944	−10	3928.059	.073	−14
3618.912	.918	− 6	4022.022	.029	− 7
3622.155	.158	− 3	4045.964	.978	−14
3647.983	.997	−14	4062.597	.605	− 8
3687.597	.609	−12	4063.751	.755	− 4
3705.704	.714	−10	4071.898	.901	− 3
3720.075	.083	− 8	4199.263	.256	+ 7
3724.519	.527	− 8	4202.194	.195	− 1
3731.084	.102	−18	4222.396	.387	+ 9
3735.005	.016	−11	4271.917	.933	−16
3737.270	.278	− 8	4325.921	.941	−20
3745.691	.710	−19	4352.900	.910	−10
3748.406	.409	− 3	4369.937	.954	−17
3749.628	.634	− 6	4376.097	.104	− 7
3758.376	.381	− 5	4383.705	.724	−17
3763.932	.940	− 8	4404.911	.929	−18
3767.336	.339	− 3	4415.286	.301	−15
3788.023	.031	− 8	4447.886	.907	−21
3795.144	.149	− 5	4494.749	.755	− 6

The differences in the third column, $k =$ Jewell − K., exhibit some rather large leaps between the individual lines, which are chiefly due to accidental errors of measurement by Jewell. I have smoothed out these figures graphically and give in Table X the values of the correction k thus found.

22. The construction of a table of standard lines of the arc spectrum of iron rigorously referred to the corrected Rowland system, which should serve as the basis for all laboratory measurements, can be accomplished in two different ways. In the first place, we may depart from the relation given above in section 16,

$$\lambda = (\text{F. and P.}) \cdot F_c ;$$

that is, we may reduce to the Rowland system the wave-lengths

TABLE X.

For the Reduction of Kayser's Iron Standards to the System of the P. T.

(Unit is 0.001 tenth-meter.)

λ	k	λ	k
3400	+ 9	3950	−11
3450	+ 6	4000	−10
3500	+ 2	4050	− 7
3550	− 3	4100	− 4
3600	− 7	4150	− 1
3650	−10	4200	0
3700	−11	4250	− 2
3750	− 8	4300	− 7
3800	− 5	4350	−13
3850	− 6	4400	−16
3900	− 9	4450	−14
3950	−11	4500	−10

in the iron spectrum measured by Fabry and Perot by multiplying them with the factor $F_0 = 1.0000340$ found from their measures in the solar spectrum, and then connect to them lines in all parts of the iron spectrum by the coincidence method. Instead of the iron lines of Fabry and Perot we may employ the other metallic lines determined by interference methods by these observers as well as by Michelson and by Hamy, the wave-lengths of which, as reduced to Rowland's system, I have collected in Table XI.

The following remarks should be made in regard to this list of standard lines. Rowland's system gives the wave-lengths in air at 760 mm pressure, and at $+20^\circ$ C., while Michelson's cadmium standards are referred to air at 760 mm pressure and at $+15^\circ$ C. Fabry and Perot, therefore, properly ought first to reduce their wave-lengths to $+20^\circ$ before comparing with the P. T. But this correction is precisely proportional to the wave-length, for the stretch from λ 4000 to λ 7000, being here

$$\lambda_{20} = 1.0000047\,\lambda_{15} .$$

This reduction factor is therefore contained in the values of F computed by Fabry and Perot, and hence also in F_0; and accordingly the wave-lengths are at the same time reduced to $+20^\circ$ when they are multiplied by F_0. The values of λ in

TABLE XI.

λ	Element	Observer	λ	Element	Observer
4358.491	*Hg*	F. and P.	5460.9281	*Hg*	F. and P.
4662.5117	*Cd*	H.	5465.675	*Ag*	"
4680.297	*Zn*	F. and P.	5506.970	*Fe*	"
4722.325	*Zn*	"	5586.965	*Fe*	"
4736.946	*Fe*	"	5615.848	*Fe*	"
4800.0739	*Cd*	M.	5763.219	*Fe*	"
4810.699	*Zn*	F. and P.	5769.7946	*Hg*	"
4859.928	*Fe*	"	5782.287	*Cu*	"
5002.057	*Fe*	"	5782.356	*Cu*	"
5083.518	*Fe*	"	5790.8562	*Hg*	"
5085.9969	*Cd*	M.	5890.165	*Na*	"
5086.0754	*Cd*	H.	5896.132	*Na*	"
5105.717	*Cu*	F. and P.	6065.695	*Fe*	"
5153.426	*Cu*	"	6230.945	*Fe*	"
5154.8363	*Cd*	H.	6325.3853	*Cd*	H.
5209.258	*Ag*	F. and P.	6362.561	*Zn*	F. and P.
5218.379	*Cu*	"	6438.6911	*Cd*	M.
5233.132	*Fe*	"	6495.213	*Fe*	F. and P.
5302.501	*Fe*	"	6708.074	*Li*	"
5434.710	*Fe*	"			

Table XI are consequently rigorously referred to the corrected Rowland system (P. T. $+ C$). The true value of the reduction factor, depending only on the absolute measurement, or upon the comparison with the standard of length, is therefore

$$\frac{1.0000340}{1.0000047} = 1.0000293 \; .$$

The large difference between the wave-lengths of the cadmium lines at λ 5086, as determined by Michelson and by Hamy, may possibly be explained by the somewhat different character of the light sources used by the two observers; as to this point, see the articles by Fabry and Perot,[1] Hamy,[2] and Bell.[3] In view of the great importance which these figures have in the construction of a new fundamental system of wave-lengths, it is nevertheless necessary that the values of Table XI should be established with entire certainty by independent series of measurements by different observers.

23. Another way for constructing a list of iron standards accurately referred to Rowland's corrected system is as follows:

[1] *Comptes Rendus*, **130**, 653, 1900; ASTROPHYSICAL JOURNAL, **16**, 36, 1902.

[2] *Comptes Rendus*, **130**, 700, 1900.

[3] ASTROPHYSICAL JOURNAL, **15**, 157, 1902.

A perfectly correct system of relative wave-lengths of iron (λ') should be established, at first wholly without regard to the absolute wave-lengths, either by interference measurements or by means of coincidences obtained with a large concave grating. The wave-lengths of the same lines on the system of the P. T. is obtained by the above-mentioned (20) measures of displacements in the solar spectrum. F is obtained by the division of the two wave-lengths thus found for each line, and thereafter the reduction factor F_0 is obtained by taking the means, and then the wave-lengths of the iron lines measured are transferred to the Rowland system by means of F_0. Hence the reduction is made by the formula

$$F_0 = \frac{1}{n} \sum \frac{\text{P. T.} - D}{\lambda'} ,$$

where n is the number of lines measured, and then

$$\lambda = F_0 \lambda' .$$

24. For more convenient examination I will summarize here the relationships so far employed and the conditions which connect them:

The true Rowland system at present current is represented by the wave-lengths (P. T.) of the "Preliminary Table of Solar Spectrum Wave-Lengths."

The P. T. contain slowly progressing inequalities, for removing which the corrections C are to be applied. The wave-length of a line of the solar spectrum on the corrected Rowland system is therefore

$$\lambda_{\odot} = \text{P. T.} + C . \qquad (a)$$

Rowland's standards in the arc spectrum (M. St.) have been systematically vitiated by the application of empirical corrections in a manner that cannot be checked, and therefore they cannot be used as a basis for accurate measures. The system of iron standards ($K.$) of Kayser, which is accurately referred to the M. St., must therefore receive the corrections k in order to be comparable with the P. T., and the wave-length of an iron line on the corrected Rowland system is accordingly

$$\lambda_M = K. + k + C . \qquad (b)$$

There exists between $\lambda_{\odot}$ and λ_M the relation

$$\lambda_{\odot} - \lambda_M = D \ , \qquad (c)$$

wherein it must be remembered that D may change its value from line to line by leaps, and therefore cannot be interpolated, as can C and k.

It follows from equations (a) and (c) that

$$\text{P. T.} - D = \lambda_M - C \ . \qquad (d)$$

The wave-lengths of metallic lines published by Jewell give the values P. T. $- D$, and must therefore, according to (d), be corrected by the amount $+ C$, in order to furnish the correct wave-lengths:

$$\lambda_M = (\text{Jewell}) + C \ . \qquad (e)$$

For the determination of k we get from (b) and (e)

$$k = (\text{Jewell}) - K. \qquad (f)$$

If λ' are entirely correct relative wave-lengths, in order to refer them to the solar spectrum, we have either

$$F_{\circ} = \frac{1}{n} \sum \frac{\text{P. T.}}{\lambda'_{\odot}} \qquad (g)$$

$$\lambda = F_{\circ} \lambda'_{\odot} \ ,$$

whence the values of C follow from (a); or, if the λ' referred to metallic lines,

$$F_{\circ} = \frac{1}{n} \sum \frac{\text{P. T.} - D}{\lambda'_M} \qquad (h)$$

$$\lambda_M = F_{\circ} \lambda'_M \ ,$$

whence C may be obtained from (d).

If the λ' in (g) and (h) were obtained by relative measurements, the values $F_{\circ}$ have no general meaning; but if they were determined by absolute measurements—as, for instance, in the system of Michelson or Fabry and Perot—then $\frac{1}{F_{\circ}}$ is the factor for converting the corrected Rowland system into the absolute meter system; that is,

$$\lambda_{\text{abs.}} = \frac{\lambda}{F_{\circ}} \ . \qquad (i)$$

According to Michelson's determination, if the wave-lengths have been previously reduced to the same temperature of the air, the value of

$$F_{\circ} = 1.0000293 \ .$$

Use will be made of this conversion, however, only in very rare cases where the absolute values of the wave-lengths are employed. For most investigations, particularly in astrophysics, it is enough to employ the correct wave-lengths λ, which can easily be derived from all series of observations made up to the present and referred to the Rowland system, by the application of the small corrections C, D, and k.

POTSDAM, ASTROPHYSIKALISCHES OBSERVATORIUM,
June 1903.

THE PEROT-FABRY CORRECTIONS OF ROWLAND'S WAVE-LENGTHS.

By Louis Bell.

Last year in this Journal[1] I took occasion, in discussing the discrepancies between the wave-length measured by gratings and by the interferometer, to call in question the validity of the correction curve deduced by MM. Perot and Fabry for Rowland's relative wave-lengths. Their most courteous reply[2] seems to call for some comment, as it still leaves the main issue in doubt. The corrections deduced by them are actually of small magnitude, never greater than 0.03 tenth-meter and on the average less than 0.02 tenth-meter[3]—so small, in fact, that were they purely accidental, they would be negligible for most spectroscopic purposes. The serious feature of the case is that if the reality of these corrections as systematic errors is established, a grave doubt is raised as to the sufficiency of the grating for the determination of relative wave-lengths by the method of coincidences. The minute care with which Rowland's measures were made is well known, and if the method itself proves to have been faulty, spectroscopists are left entirely at sea for any practical method of establishing relative wave-lengths in ordinary spectrographic work. If an error of even 0.02 tenth-meter is due to systematic causes, errors many times greater may reasonably be expected under less favorable conditions.

For absolute measures the grating must be, as I have already shown, definitely put aside for good and sufficient cause, although I do not consider that even the beautiful work of Michelson and Benoit can yet be accepted as a final result within several times its apparent probable error. The purely metrological difficulties, well known to those who are familiar with the comparison of standards of length, are sufficient to imply some uncertainty. Michelson's value is, however, much more reliable

[1] Astrophysical Journal, 15, 157, 1902.

[2] *Ibid.*, 16, 36, 1902. [3] *Ibid.*, 15, 272, 1902.

than any other, and is very properly accepted universally as the standard.

As regards relative measurements the case is different. The closeness of agreement instanced by Perot and Fabry regarding their own measures[1] can be duplicated without especial difficulty in the higher orders of a long-focus concave grating. The relative convenience of the two procedures depends on the nature of the research undertaken. Precision of setting, in methods both essentially micrometrical, is not the issue here involved. It is of fundamental importance to determine whether the grating method is or is not beset by systematic errors which, small in the case cited, may become serious. And I must here reiterate that, while systematic errors may exist in that or any other method, the evidence adduced by Perot and Fabry is insufficient to establish them.

In the first place one must recur to that extraordinary result of Hamy[2] for the wave-length of the green cadmium line. In this instance the relative values assigned by Hamy and by Michelson differ by 0.079 tenth-meter, a quantity four times greater than the differences which Perot and Fabry have sought to establish by reference to this same green line assumed as a standard. It is quite true that the experimenters used different sorts of vacuum tubes, but the observed pressure shifts of the *Cd* lines do not account for any such difference as that here found; and if they did, it would merely show such hypersensitiveness to slight changes in excitation as would indicate that the line in question is a very shifty standard. The other alternatives are a downright error in the application of interference methods, or a shifting of the apparent place of the line under differences of illumination. Until this matter is cleared up the green *Cd* line is certainly open to suspicion, and while Perot and Fabry have not heretofore had occasion to note its possible variability, they may actually have been measuring it in the case under discussion. The point is one which can be settled only by a thorough investigation.

Another source of incertitude in measures carried out by the

[1] *Ibid.*, **15**, 263, 1902. [2] *C. R.*, **130**, 489, 1900.

method of Perot and Fabry is one common to all micrometric comparisons of dark and bright lines — viz., the difficulty of treating such lines or diffraction rings just alike in making the settings. The consistency of the results does not carry proof that there is no error introduced in locating the centers of the respective objects, and that, if such an error occurs, it will remain constant when the color of the bright line or field changes. The experiments of Reese[1] and of Hasselberg[2] show very plainly that such errors occur in micrometric measures of photographic plates and that they are by no means negligible in amount. In these cases cited they partake of the nature of a personal equation which may or may not have the same sign for different observers. Whether the difference in the readings by two observers having personal equations of the same sign would be comparable in size with their common error would be a matter of chance. The smaller the absolute error, the less likely perceptible differences in it.

The general experience of spectroscopists is that it is extremely difficult to treat a bright line and a dark line just alike with the micrometer. The same difficulty extends to comparing lines of widely different appearance in general. And even supposing the personal equations of MM. Perot and Fabry to have been zero, it still remains to be shown that the dark diffraction ring as measured, having been obtained through a rather wide slit, correctly represents the wave-length of the line as it would be set upon in the grating spectrum itself. Analogy would suggest the negative, and it would be most extraordinary if the variation remained uniform in fields of different colors and intensities.

Difficulties of this sort appear in every branch of spectroscopy when dealing with quantities of the order of magnitude attacked by Perot and Fabry, and are most aggravated in comparing bright and dark lines. Even in Hasselberg's plates, which were on a large scale and uncomplicated by the question of color differences, the errors exceeded 0.01 tenth-meter.

In my previous paper I referred to a discrepancy between

[1] ASTROPHYSICAL JOURNAL, 15, 142, 1902. [2] *Ibid.*, 15, 208, 1902.

Perot and Fabry's correction curve and their value for the *Zn* line at λ 4810 +. The difference, as they remark, is in the direction which would be produced by pressure-shift; but Rowland's measures on the arc agree with his solar value to 0.002 tenth-meter, and the pressure shift for *Zn*, as found by Humphreys and Mohler,[1] accounts for only one-fourth of the outstanding difference of 0.02 tenth-meter. The error may be Rowland's, or Perot and Fabry's, or due to both, but it is much larger than the ostensible probable errors of either.

A far more serious source of error is the apparent neglect by Perot and Fabry of the corrections for the Earth's annual and diurnal changes of relative velocity. Perot and Fabry very carefully eliminated the velocity shift due to the Sun's rotation, but I find no record of corrections for the Earth's motion, which is almost equally important. This error affects all comparisons of solar and terrestrial sources, but of course does not appear in comparisons of solar or terrestrial sources by themselves.

Frost[2] has examined these errors and finds for the correction due to the eccentricity of the Earth's orbit a value varying from 0 to ± 0.01 tenth-meter as the Earth shifts from perihelion or aphelion to quadrature, the latter being reached in April and October. The value I here note is for the green of the spectrum. Similarly the variation with hour-angle amounts in the latitude of Paris to about $0.006 \times \sin t$ tenth-meter in the green. These quantities are quite comparable with those which Perot and Fabry have sought to evaluate, and must be reckoned with in every comparison of solar and metallic spectra, although generally, if not universally, neglected.

Any one of the sources of error which I have here noted is sufficient to account for the apparent systematic variations in Rowland's relative wave-lengths found by Perot and Fabry, and until they are eliminated, one and all, it is unsafe to assume that Rowland's relative values involve errors other than accidental. The differences found are much greater than the probable errors of reading either with the grating or the interferometer, and deserve further investigation; but they certainly should not be

[1] *Ibid.*, **3**, 114, 1896. [2] *Ibid.*, **10**, 283, 1900.

charged up to systematic errors introduced in Rowland's method of coincidences until the method used to determine them is cleared of the uncertainties which have here been noted. I do not hold Rowland's work as in the least sacrosanct, and I greatly admire the elegant method adopted by MM. Perot and Fabry; but the matter at stake is the validity of relative wave-lengths measured with the grating.

The whole discussion tends to confirm the opinion, expressed in my previous paper, that at about 0.01 tenth-meter the precision of wave-length measurements rests on a somewhat shaky foundation on account of numerous and varied small sources of error, which it is to be hoped will eventually be eliminated.

The attempt of Eberhard[1] to confirm the results of Perot and Fabry from the wave-length measurements of Müller and Kempf is interesting, but somewhat unconvincing. In Fig. 1 I have plotted the variations from Rowland's tables of the individual lines studied by Eberhard, together with some others of Müller and Kempf's lines easily identified on Rowland's map. The result is ragged in the extreme, and shows the futility of using such data to define supposed errors eight or ten times smaller than the variations in the data. The similarities between Perot and Fabry's and Müller and Kempf's curves, as shown by Eberhard, seem to be mainly due to the process of averaging. The differences are conspicuous. For instance, as between Rowland and Perot and Fabry there is practically a constant difference from λ 5900 to λ 5600. But in this particular region the difference between Rowland and Müller and Kempf is varying with great rapidity. The greatest abnormality between R. and P. and F. is at λ 5200, while between R. and M. and K. there are two regions of about equally great variation, one at λ 5300, the other at λ 6300. Add to this the wideness of the apparent variations shown by M. and K. as compared with those shown by P. and F., and I do not see how one can avoid the conclusion that the former are entirely worthless as confirmatory evidence. In Fig. 1 I have shown in addition the relation of Rowland's values to those obtained in absolute measure by Thalén.[2] These, as might

[1] *Ibid.*, 17, 141, 1903. [2] *Nova Acta Upsala*, 1899.

be anticipated, are in vastly better accord with each other than the measures of Müller and Kempf, in which great skill was handicapped by poor gratings; but in relative values they do not agree with either M. and K. or P. and F. Thalén shows an abnormal region at about λ 5150, near that located by P. and F.,

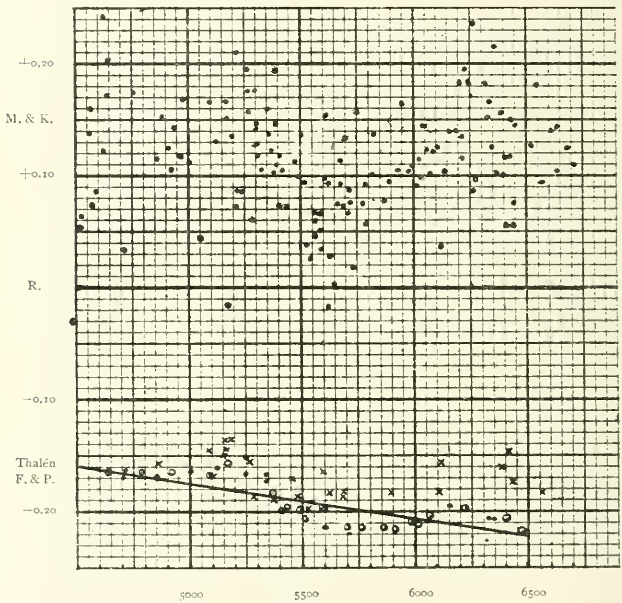


FIG. 1.

but toward the red the two sets diverge, Thalén indicating almost a constant difference from Rowland and showing no such extraordinary difference at λ 5500 as is found in M. and K. In short, Perot and Fabry, Müller and Kempf, and Thalén agree neither with Rowland nor with each other, the variations being conspicuously greater than the probable errors in each case. Rowland's values may very possibly be in error, but they obviously cannot

concurrently err by three different and considerable amounts at the same point. Considering the methods employed and the magnitudes of the supposed errors located by Perot and Fabry it is in no wise hypercritical to say that only the results of these latest experimenters can be taken seriously. I hope ere long, through the courtesy of Mr. Jewell, to have at hand a direct recomparison of certain of Rowland's lines which will confirm or reject the differences found by Perot and Fabry between the regions *circa* λ 5200 and 5700. Meanwhile these differences should be taken to heart as a lesson in the relation between probable and possible errors.

BOSTON, July 1903.

ON THE SPECTRUM AND RADIAL VELOCITY OF χ CYGNI.

By G. EBERHARD.

THE well-known variable χ *Cygni* has been observed by me at its last two appearances in 1901 and 1902 with the aid of the excellent spectrograph IV, designed by Vogel for the Potsdam photographic refractor (32.5cm). The results obtained during its appearance in 1901 have already been published in No. 3765 of the *Astronomische Nachrichten*. The observations of 1902, however, indicate departures in some respects from the star's behavior in the year preceding, and for this reason it may be

1901	G.M.T.	Plate	Observer[1]	Exposure	Slit-Width	Brightness	v from $H\gamma$	v from Fe 4308
	h m			m	mm		km	km
Aug. 2.....	9 28	IV 766	E.	150	0.025	....	— 17.5	......
9.....	9 10	772	L., E.	90	0.025	....	— 17.5	......
10.....	9 26	774	E., L.	240	0.025	$4^m.9-5^m.0$	— 19.9	......
11.....	9 20	777	L., E.	30	0.025	....	— 19.6	......
12.....	9 4	778	E.	60	0.025	....	— 18.1	......
15.....	8 3	779	E., L.	55	0.015	....	— 20.2	......
20.....	8 2	784	L., E.	23	0.025	....	— 21.5	......
21.....	8 7	786	E., L.	30	0.025	4.7	— 20.5	......
22.....	8 22	789	L., E.	30	0.020	....	— 23.8	......
23.....	8 10	793	E., L.	12	0.025	4.7	— 20.3	......
24.....	10 5	794	L., E.	60	0.025	....	— 20.6	......
31.....	7 34	803	E., L.	60	0.025	....	— 21.2	......
Sept. 1.....	8 11	805	E.	38	0.025	4.9	— 20.9	......
7.....	8 40	812	E.	60	0.025	5.0	— 21.2	— 18.6
17.....	6 59	814	E.	60	0.025	5.5 – 5.6	— 17.5	20.9
19.....	7 15	823	E.	60	0.025	5.6	— 22.5	24.2
26.....	7 36	822	E., S.	60	0.025	....	20.7	25.3:
27.....	7 49	827	E., S.	60	0.025	5.8 – 5.9	— 19.1	......
28.....	8 0	828	E.	60	0.025	....	— 17.5	......
Oct. 1.....	7 47	835	E., S.	60	0.025	6.4	— 17.2	— 16.8
3.....	8 23	838	L., S.	120	0.025	6.5 – 6.6	— 23.6	21.0
7.....	8 58	840	L., E.	60	0.025	6.7 – 6.8	— 18.3	......
15.....	7 19	844	E., L.	120	0.025	7.0 – 7.2	— 19.2	— 16.9
26.....	6 15	845	E.	60	0.025	7.8 – 7.9	— 16.4:	......
Nov. 9.....	5 40	864	L., E.	120	0.025	8.5 – 8.7	20.4	20.3
23.....	6 24	876	E., S.	180	0.025	9.0 – 9.2	— 15.7:	— 19.1:
						Mean	— 19.7	— 20.3

[1] Observer: E. = Eberhard, L. = Ludendorff, S. = Scholz.

well to insert here briefly, at the appropriate places, the principal results from the above-mentioned communication.

The star *χ Cygni* has a very weak absorption spectrum, on which an emission spectrum of very bright hydrogen lines and some essentially weaker metallic lines are superposed. While the bright $H\gamma$ line at the time of the light maximum of *χ Cygni* requires an exposure which corresponds to a star of the first type and second or third magnitude, it is necessary to expose for the continuous spectrum as long as for a star of the first type, but of about the sixth magnitude. On this account it is unfortunately impossible to obtain a good photograph of the continuous spectrum except at the time of the light maximum, and even then only on those evenings during which favorable atmospheric conditions prevail, while the bright $H\gamma$ line is easily photographable even when the brightness of the star is still less.

The measurement of the bright $H\gamma$ line and the equally bright *Fe* line at λ 4308 gave for the year 1901 the radial velocity indicated in the preceding table.

In good agreement with those figures is the radial velocity of the star obtained in 1902:

1902	G. M. T.	Plate	Observer	Exposure	Slit-Width	Brightness	*v* from $H\gamma$
	h m			m	mm	m	km
Sept. 22	7 17	IV 1158	E.	30	0.020	4.5	— 16.3
23	8 21	1161	E., S.	240	0.025	4.5	— 17.2
24	7 29	1162	E., S.	135	0.025	4.6	— 20.4
25	9 23	1163	E., S.	240	0.025	4.4	— 21.9
26	8 1	1165	E., S.	240	0.025	4.6	— 21.4
28	7 52	1167	E.	40	0.025	....	17.5
29	6 25	1169	E., S.	15	0.025	4.4 – 4.5	— 22.3[2]
30	6 34	1174	E., S.	20	0.015	....	— 20.7
Oct. 11	7 10	1176	E., S.	60	0.020	....	— 24.0
11	8 7	1177	E., S.	45	0.010	4.6 – 4.7	— 21.7
15	5 38	1178	E.	40	0.015	4.8 – 5.0	— 20.3
21	5 13	1184	E., S.	120	0.025	....	— 17.9
22	7 15	1195	E., S.	180	0.025	5.3	— 16.8
Nov. 2	6 42	1210	E.	40	0.025	5.2	— 24.3
7	6 10	1232	E., S.	30	0.020	5.3 – 5.4	— 23.6
18	5 29	1255	E.	40	0.025	6.4	— 25.9
30	5 28	1285	E.	50	0.025	7.5	— 18.9:
Dec. 12	5 9	1301	E., S.	90	0.025	7.8 – 8.0	— 27.0:
						Mean	— 21.0

[2] $H\gamma$ double?

In this table it is to be noted that the estimates of the magnitude of the variable in terms of the usual comparison stars were made only incidentally, and can therefore make no claim to precision, and the given magnitudes may perhaps be uncertain by several tenths. The $H\gamma$ line presented the same appearance during both years; it was hazy toward the red, as was also $H\delta$. This caused an uncertainty in the measurement, since it was not easy, particularly in the case of fully exposed plates, always to judge in the same manner as to the place of maximum intensity which is set upon throughout the measurement. The figures given for the single plates show this clearly. I sometimes got the impression, particularly on plate 1169, that $H\gamma$ was double; but this question could not be decided with certainty in view of the comparatively slight linear extent of the spectrograms. The much fainter *Fe* line at λ 4308 could be measured only on the plates of 1901, and it was only just at the close of the observations of the next year that it was visible at all, and then it was too faint for measurement.

Nothing in the way of a law appears in the separate figures of the above tables, so that we are well justified in assuming that the radial velocity of the portions of χ *Cygni* showing the emission lines is constant and about -20 km. The comparatively large departures of the individual values from the mean are probably less to be attributed to alterations in the form of the $H\gamma$ line than to the uncertainty of the observer as to the proper place at which to make the setting.

The continuous spectrum of χ *Cygni* varies much from that of the Sun between λ 4200 and λ 4435, perhaps most closely resembling that of α *Herculis*. The intensities of many lines — as, for instance, those of *Ca* and *Va* — are wholly different from those in a star of the second type. Many of the characteristic iron lines are partially concealed by the *Fe* emission lines. In all about seventy lines could be recognized between λ 4200 and λ 4435 on the fully exposed plates, and I have made the attempt to identify these with known lines of the elements.

The presence of *Ca*, *Ti*, *Cr*, *Va*, *Fe*, and *Mg* could be established, and, in fact, practically all the principal lines of these

metals were present. Several of the absorption lines of iron were in 1901, as already stated, partially concealed by the iron emission lines; for instance, those at λ4415.29, 4383.71, 4325.94, 4308.06. But, on the contrary, no such superposition could be determined, and, indeed, the *Fe* emission lines did not appear at all until the brightness of the star was again rapidly decreasing.

There were a large number of lines, particularly of the broader ones (presumably groups), which I could not assign to any element with any degree of probability. I do not give these in detail here, since a minute knowledge of the spectrum of *χ Cygni* will not be attained until a bright star of the same, or at least of a similar, character (such as *o Ceti* at a principal maximum) has been investigated with an instrument of the greatest possible dispersion. On account of our still too limited knowledge of the continuous spectrum, there is only a small number of tolerably unobjectionable lines at our disposal for a determination of velocity. Based upon twelve and eighteen such selected lines the radial velocity of the portion of *χ Cygni* producing the continuous spectrum resulted as follows:

1901	+2.5 km	Plate 772	1902	−1.3 km	Plate 1162
	+2.3	774		−3.3	1165
Mean	+2.4 km			−2.3 km	

I was at first of the opinion that this not particularly large difference of the values of the radial velocity was perhaps to be attributed to flexures in the apparatus during the comparatively long exposure of from two to four hours. This is, however, not very probable, since with these long exposures the *Fe* comparison spectrum was for precaution taken both at the beginning and end as well as at the middle of the exposure; moreover, the plates were made at neither very large nor very different hour angles, the middle of the exposures falling in every case near the meridian. Furthermore, if such flexures actually occur, the radial velocities of these plates, as deduced from the bright $H\gamma$ line, should exhibit especially large deviations from the mean values, which is not the case. Hitherto an appreciable effect of

flexure has not appeared on the plates taken with spectrograph IV, and temperature changes are excluded by the continuous careful supervision of the electric heating, so that we may assume for the present, with a high degree of probability, that the portion of χ *Cygni* producing the continuous spectrum has a variable radial velocity. Further observations will be required, however, for the certain decision of this matter, and I therefore plan to continue the spectrographic observations of χ *Cygni* during this year, particularly in respect to the above question, the solution of which is of the greatest importance for the explanation of the phenomena presented by a variable of this sort.

It is in any case quite remarkable that χ *Cygni* exhibits the same behavior as *o Ceti* in the fact that for both stars the emission spectrum is displaced toward the violet of the absorption spectrum, contrary to the manner in the case of the temporary stars. On June 1 (Plate D 115; exposure 90 m) and June 9, 1899 (D 116; exposure 150 m), two plates of χ *Cygni* were made by Dr. Ludendorff and myself with spectrograph III, which showed $H\gamma$, $H\delta$, $H\zeta$, $H\theta$, and $H\iota$ to be bright, $H\epsilon$ being lacking on account of the strong calcium absorption. $H\delta$ is by far the strongest line. A bright line having the wave-length 3905.8 also appears, and therefore coincides with the principal line at 3905.7 (Rowland) in the arc spectrum of silicon. I followed up this interesting fact and found that the line also occurred in a spectrum of *Mira Ceti* obtained by Dr. Ludendorff and myself in 1899, and that it was also doubtless measured by Vogel[1] in earlier plates of this star (λ 3906). The fact that the bright $H\beta$ line was not present, although observed by Miss Maury,[2] is to be attributed to the fact that light of this wave-length is united too far from the focus of the region ($H\gamma$ to $H\zeta$) ordinarily used, both for the photographic refractor and for spectrograph D.

On the plates of 1899, 1901, 1902 there is further present between λ 4000 and λ 4200 a series of strong brightenings, giving the impression of broad bright lines. There is a particu-

[1] "Ueber das Spectrum von *Mira Ceti*," *Sitzungsberichte der K. Akad. der Wiss. zu Berlin*, **17**, 395, 1896.

[2] *Annals of Harvard College Observatory*, **28**, I, 98.

larly strong band of this sort at λ 4138, and one lying at the side toward red of $H\delta$. Inasmuch as spectrograph IV does not permit any accurate measurements of wave-lengths in this region, I have not been able to attempt the identification of these bright bands.

In concluding I would also mention that the bright lines $H\gamma$, $H\delta$, and *Fe* λ 4308 altered their intensities in different ways in 1901. From August 2 until September 19 $H\delta$ was considerably brighter than $H\gamma$; from October 3 to 15 $H\gamma$ and $H\delta$ differed little from each other; on October 26 they were equal; and on November 9 and 23 $H\gamma$ was brighter than $H\delta$. The *Fe* line λ 4308, on the contrary, increased in brightness the fainter the star became. While it was not present on August 24 and 31 with an exposure of 60 minutes, it appeared on September 7 in an equal exposure time and remained of this intensity until about the last plate, on which it was equal to $H\gamma$ in brightness. The other bright *Fe* lines — for instance, λ 4402 — were only just barely visible with the exposures chosen and were in any case not measurable.

POTSDAM, June 23, 1903.

POSTSCRIPT.

(July 14, 1903.)

In *Bulletin* No. 41 of the Lick Observatory Mr. Stebbins has just published his pretty investigations of the spectrum of *o Ceti*. It is evident that my similar paper on χ *Cygni* in *A. N.* No. 3765 was unknown to him, for otherwise the highly interesting fact would not have escaped him that both stars exhibit a precisely identical behavior, both as to the spectrum and as to the variations of the spectrum; whence it is highly probable that this sort of spectrum is typical for the long-period variables of that class.

ON DOUBLE REVERSAL.

By W. J. HUMPHREYS.

APPARENTLY not much attention has been given to double reversal; that is, the occurrence of a narrow bright line on the dark space of a broadly reversed one. Liveing and Dewar[1] speak of having often momentarily seen, though rarely photographed, this condition on the introduction of fresh material into the arc. Kayser[2] says that he has photographed it very rarely ("nur ungemein selten"), and that he doubts the genuineness of the two cases selected from his plates to illustrate this phenomenon; a doubt well founded, as will appear presently. Jewell,[3] on the other hand, states that a certain result exhibited by double and multiple reversal, displacement of the line due to quantity of material, "is shown in a large number of cases." Still in these different papers, and elsewhere, so far as I am aware, the phenomenon is soon dismissed, and no experiments are described that help to determine just what is essential to its appearance.

Double reversal has long been known to be both persistently and well marked in the Sun's faculæ, so that there can be no doubt about its actual existence, under proper conditions; and thereby its claim to specific investigations is established.

I do not know how many photographs I have taken of arc spectra, but certainly several thousand, and of practically all known elements that can be so used. The great majority of these were secured at atmospheric pressure, but a few hundred were obtained at higher pressures, up to nearly fifteen atmospheres. The volume of the current has been multiplied more than a hundredfold, and the quantity of material in the case of several elements variously altered from mere traces to solid rods of the metal in a reasonably pure state. Still with all these changes, until a method presently to be explained was adopted, I never observed visually a single case of double reversal, and

[1] *Proceedings of the Cambridge Philosophical Society*, 4, 264.

[2] *Handbuch der Spectroscopie*, 2, 363. [3] ASTROPHYSICAL JOURNAL, 3, 95, 1896.

only a few of my plates give any evidence of it. Besides in each of these few the phenomenon is, I am convinced, entirely spurious.

An interesting case of apparent double reversal is shown in Fig. 1 of the accompanying plate. This line in the reversal of D^1 is supposed by Kayser[1] to be due to sodium, and he regards it as possibly a true double reversal, and also says[2] that it shows the displacement of the center of the reversal when a large amount of material is used from the position of the corresponding line given by a small amount of the substance.

During the course of this investigation I got what I shall call the true double reversal of the sodium D's, as shown in Fig. 4. Here both the D's are doubly reversed, and the fine lines are centrally placed. After some further experiments it was found that by mixing iron and sodium together and burning them in a common arc Fig. 1 was always obtained, but never so when the iron was left out. The conclusion therefore is, and this is supported by careful measurements, that the fine line in question is the ultra violet iron line λ 2948.00, second order, superimposed on D_1, λ 5896.16, first order.

This must not be taken as a wilful criticism of Kayser's work. All of us who have the pleasure of making investigations in this fascinating field acknowledge ourselves immeasurably indebted to him, and therefore feel it a duty as well as a privilege to make, whenever we can, even the slightest addition to or correction of his masterly treatise.

Fig. 1 seems to indicate that, while the outer layers of relatively cool sodium vapor can strongly absorb light of the wavelength given by D_1, it cannot so cut out light of double that frequency. Indeed, no theory, so far as I know, would lead us to suppose that free particles, such as fill the electric arc, could absorb vibrations other than those which they can also emit. However, that idea in this particular case was experimentally tested, as follows:

Two arcs were connected up in series, and placed, about 5 cm apart, in line with the slit of the spectroscope. The arc next the

[1] *Handbuch der Spectroscopie*, **2**, 363. [2] *Ibid.*, 365.

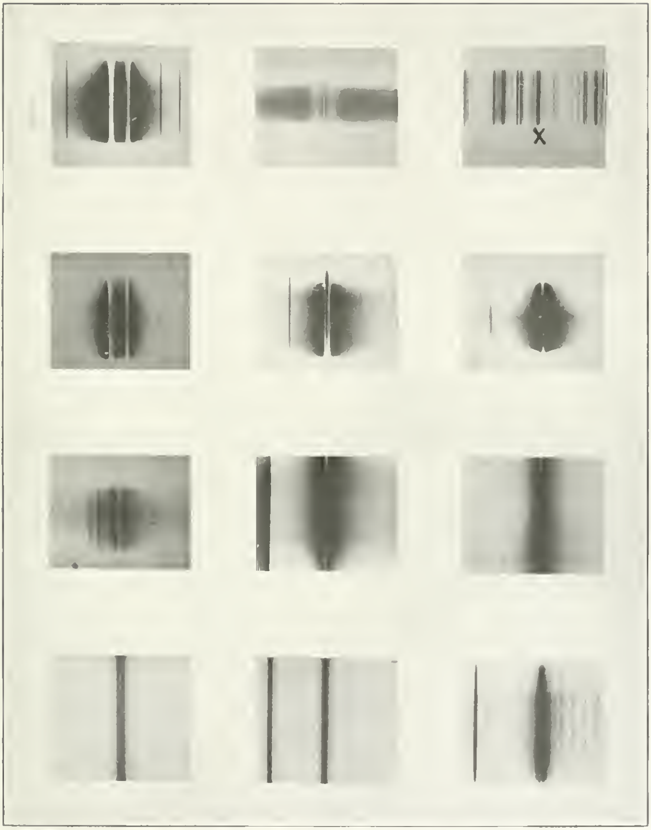
slit contained a large amount of sodium, while the one farther away had a small amount of iron. On developing the spectrogram secured with this arrangement, the fine iron line came out nicely on the reversal of D_1, and showed that its light had not been very greatly absorbed. When, instead of iron, a small amount of sodium was used in the more distant arc, no trace of the D's thus obtained could be seen on the broad reversals due to the sodium in the nearer arc. It would seem then, to borrow a musical term, that in this case the particlis that cut out one note cannot absorb its octave. Neither will it absorb the sub-octave, as is shown by mixing a trace of barium with thallium, by which the barium line λ 5535.69, first order, is superimposed on the reversal of the thallium line λ 2767.97, second order.

A somewhat different case of apparent double reversal is given by adding some iron to aluminium. Here the reversal of the aluminium line λ 3092.84 contains the iron line λ 3092.87. In this case the relatively cool vapor, in the outer parts of the arc, that cut out an aluminium radiation of a certain wave-length, either will not absorb an iron radiation of the same wave-length or else, as seems to me more likely, its absorptive power at this place is not equal to the combined corresponding emission of the two elements.

With materials free from iron, one could easily determine experimentally which of these suppositions is correct, but, as my carbons and aluminium both contain enough iron to render the results doubtful, I have not as yet undertaken it.

Occasionally two lines of the same element produce results markedly like double reversal. Thus the iron line λ 2966.99, third order, contains in its reversal the iron line λ 4450.44, second order. Also the two iron lines λ 3020.70 and λ 3021.15 both reverse in such way as to appear like a single broadly reversed line with a fine line a little to the violet of its center. But the best example I have seen of this is given, when a large amount of iron is used, by the lines λ 2973.17 and λ 2973.41. This combination, as it appears on one of my plates, is shown in Fig. 3. The fine line, nearly centrally placed in what seems to be the reversal of a single line, is, as shown by a careful determination

PLATE V.



Double Reversal.

of its wave-length, λ 2973.28, only the unabsorbed remnants of the red and violet sides respectively of the two reversed lines. Clearly then one side of each line is largely absorbed by the vibrating particles that produce the central parts of the other, a result in accord with the idea that in any particular case the absorption of radiant energy depends upon its wave-length and not upon its origin.

Fig. 2 illustrates probably the most common case of apparent double reversal. This line λ 3383.00 was obtained by putting a button of metallic silver in the arc. At first the arc burned violently, and most of the metal was quickly driven out. This period was followed by one of comparatively quiet action with a smaller amount of silver. The result was, therefore, a negative, produced by successive exposures, indistinguishable in appearance alone from true double reversal. In the same way I have obtained one plate that shows an apparent triple reversal of the silver lines.

As stated above, true double reversal is found in the faculæ, and it was during an attempt to explain to Professor Ormond Stone its probable origin in this case that it occurred to me to test the effect of two arcs connected in series and so placed that the light of one could reach the slit only by passing through the other. By putting a small amount of the substance to be studied in the arc nearest the slit and a large amount in the other, which was about 5 cm farther removed, the phenomenon of double reversal was always produced. On interchanging the arcs no such result appeared—only the dark reversals due to the large amount of the substance then in the nearer arc.

The following table gives a list of the lines that were thus obtained doubly reversed. All those in the visible region were repeatedly observed, and, with the exception of the lithium line, also photographed. This list, which a little care would extend, is, I trust, sufficient to show how double reversal may always be obtained; that is, by the use of two independent sources of the radiation in question, the one giving the broadly reversed line shining through the nearer source which contains a relatively small amount of the material.

The doubly reversed lines in the spectra of faculæ may therefore be interpreted as due to two practically distinct sources of light: a source deep in the Sun's atmosphere, where the material is abundant and the consequent reversals broad; and above this a more or less detached, self-luminous cloud containing a relatively small amount of the substance.

It might at first seem, if this is the explanation of double reversal, that ordinary arc spectra should often show the phenomenon, due presumably to the arc burning in two parts, one behind the other, and containing a different amount of the material. This, however, would really be two parallel arcs, and, as is well known, they will not burn that way. A split arc is therefore not a thing of any permanence, if indeed it has even a momentary existence; and thus true double reversals, as above explained, seldom, if ever, appear in arc spectra.

DOUBLY REVERSED LINES.

Element	Wave-length	Remarks	Element	Wave-length	Remarks
Ba	4554.21		*Pb*	4057.97	
"	4934.24		*Li*	6708.20	Small line complex
"	5535.69	Small line confused	*Mg*	2852.22	Fig. 7 of plate, small line also reversed
Cd	3261.17	Fig. 10 of plate			
Ca	3933.83		*Ag*	3280.80	Fig. 5 of plate, small line also reversed
"	3968.63				
"	4226.91	Fig. 8 of plate	"	3383.00	
Cu	3247.65		*Na*	5890.19	Fig. 4 of plate
"	3274.06		"	5896.16	Fig. 4 of plate
Pb	2614.26		*Sr*	4077.88	
"	2802.09		"	4215.66	
"	2823.28		"	4607.52	Fig. 9 of plate
"	2833.17		*Tl*	5350.65	Fig. 6 of plate
"	3639.71		*Sn*	3175.12	Fig. 11 of plate
"	3683.60	Fig. 12 of plate	"	3262.44	

The doubly reversed lines offer a favorable opportunity for studying the effect of quantity of material on wave-length. The lithium line λ6708.2 shows a double reversal very excellently, and even the smaller line is also often reversed. In this case the larger part of the small line is to the red of its own reversal, while when not reversed the fine line is to the blue of the center of the larger reversal.

This line was found by Michelson[1] to be a confused double, the difference in wave-length between the components being about 0.14 tenth-meter. The components are unequal in intensity, but there is no record as to which has the longer wave-length. My own observations would be fully accounted for by assuming the line to be double, as Michelson found it, and with the feebler component to the red of the heavier one.

The barium line λ 5535.69 also seemed either confused, or closely accompanied by a line due to some other element, probably the iron line λ 5535.52.

It is well known that many lines are more or less complex, some of them exceedingly so, and it may be that the brightest component, when a small amount of material is burned, does not remain so when the substance is used in quantity. At any rate, the complexity and unsymmetrical spreading of many lines make it doubtful whether the wave-length of any given component is ever changed by merely varying the amount of material producing it.

However this may be, careful and repeated measurements in both directions over my plates fail in most cases to show with certainty any displacement of the fine line from the center of the broad reversal. Even the sodium D's, *g* of calcium, the strontium line λ4607.52, and some others, that are greatly shifted by mechanical pressure are within errors of my measurements centrally placed.

UNIVERSITY OF VIRGINIA,
August 5, 1903.

[1] *Phil. Mag.*, September 1892.

PHOTOGRAPHIC OBSERVATIONS OF BORRELLY'S COMET AND EXPLANATION OF THE PHENOMENON OF THE TAIL ON JULY 24, 1903.

By E. E. Barnard.

The quick-acting lantern lens (which is really a doublet or portrait combination, is specially suited for photographing large diffused masses of faint light in the sky. For small objects or for minute details it is, of course, unsuited because of its small scale. This kind of lens, therefore, has its limitations as well as its advantages. As an auxiliary to a larger photographic telescope for the delineation of such phenomena as the extent of comets' tails, etc., it is indispensable. The rapidity of such a lens for cometary work is well shown in the photographs made here of Borrelly's comet of this year.

The only suitable lens available at the time for photographing the comet was a small lantern lens of 1.6 in. aperture and 6.3 in. focus (see Astrophysical Journal for January 1903). With this lens attached to the 12-inch equatorial a series of photographs was obtained of the comet. These were made with the intention of accurately locating the position of the comet's tail.

Mr. R. J. Wallace, of this Observatory, becoming interested in the subject, began an independent series of photographs with the same lens at the close of my exposures, and thus covered several nights on which other work prevented my photographing the comet; for it was intended to make the set as complete as possible. By this means there were several dates on which the comet was being photographed here from dark until dawn.

In general the two pictures for any one night showed no decided difference in appearance. On the night of July 24, however, a remarkable change was shown to be taking place in the comet.

During the period of visibility of this object with the naked eye, it appeared as a bright hazy star between the second and

PLATE VI.

July 23, $15^h\ 27^m$ – $18^h\ 2^m$ G.M.T. July 29, $16^h\ 5^m$ – $18^h\ 5^m$ G.M.T.

W. E.

July 24, $17^h\ 59^m$ – $20^h\ 29^m$ July 24, $14^h\ 57^m$ – $17^h\ 34^m$

COMET c 1903 (BORRELLY).

third magnitudes. Feeble traces of the tail could be seen with the naked eye for some 4°. In a small telescope this could be traced for even a less distance. With every advantage in its favor—a close approach to both the Earth and Sun; above the horizon all night at a high altitude, with no moonlight to interfere; and with a head as bright as some of the "great" comets—it was decidedly disappointing so far as the naked eye was concerned. The nucleus seemed to be very inactive in the production of the ordinary phenomena of a comet's tail. The lantern-lens photographs, however, showed that the comet really had a considerable extent of tail—as much as 17°.

As stated, with the exception of the length and the accurate location of the tail, the small lens did not show much of interest until July 24, when a very remarkable and important phenomenon took place, which was almost unique in its character. Some two or three degrees back of the head the tail was apparently broken off and the outer portion shifted bodily toward the direction from which the comet was receding and in a line parallel with the remaining portion of the tail—as if some force had suddenly broken off a large section of the tail and pushed it to one side of its former position. Mr. Wallace's photograph, which was begun at the close of mine, shows that the apparent separation was taking place rapidly, as if the fragment were being left behind by the comet in its flight through space.

This singular phenomenon is not entirely unique. One of the photographs of Gale's comet of 1894 with the Willard lens showed a similar, though not so pronounced, appearance.

A photograph made on July 25 showed no traces of this peculiarity, the tail having apparently assumed its normal condition while another photograph on the 23d had shown nothing abnormal about the tail. The phenomenon was, therefore, confined to an interval of a few hours only.

The following measures were made of the photographs of July 24. "The tail" refers to the short tail attached to the head, while "the section" refers to the separate portion some 9° long, which was nearly parallel with the direction of the small tail:

PLATE I (E. E. B.).

Length of tail = 2° 51′.
Distance from head to nearest end of section = 2° 14′.
Distance between the axes at the point of separation = 0° 12′.4.
Width of tail just back of head = 0° 4′.

PLATE II (R. J. W.).

Length of tail = 4° 7′. It diffuses out beyond this.
Distance from head to nearest end of section = 2° 48′.
Distance between the axes at the point of separation = 0° 18′.0.
Width of tail just back of head, not so narrow — more diffused.

Following are the exposure times of the plates:

	Plate I		Plate II
Begin	8^h 57^m		11^h 59^m
End	11 34		14 20
Middle	10 15		13 14
Interval		2^h 59^m	

As will be seen, the duration of exposure was essentially the same in each case.

In the *Bulletin de la Société Astronomique de France* for August is reproduced a photograph of the comet, made by F. Quénisset, of the Observatory of Nanterre, on July 24–25 from 23^h 9^m to 0^h 9^m. The scale of this photograph appears to be 5.05 times that of the lantern lens — from measures of stars on the two photographs. Assuming the above to be Paris times the following are the Greenwich times:

Begin July 24	11^h 00^m	G. M. T.
End	12 00	
Middle	11 30	

As my exposure was made at 16^h 15^m G. M. T. and Mr. Wallace's at 19^h 14^m, it will be seen that Mr. Wallace's photograph was 3^h 0^m ± later than mine, and that of M. Quénisset 4^h 35^m earlier. I have measured the distance between the head of the comet and the nearest end of the section on the Nanterre photograph, and get 1° 36′. By a combination of the times of these photographs, we get the following values of the motion of separation of the section and the head:

Nanterre and first Yerkes Observatory, -	10$'$.1 hourly
First and second Yerkes Observatory, - -	10.7 hourly,

or, taking the mean of these — assuming the difference of the two results as accidental, which may not be the case — we have 10$'$.4 for the mean hourly motion of separation of the nearest end of the section and the head With this motion it can be shown that the separation of this section of the tail from the nucleus or the head of the comet occurred at 2^h 30^m G. M. T., July 24.

With the aid of the elements of the comet's orbit, and assuming that the direction of the tail was coincident with the prolongation of the radius vector—from which it will deviate but little at most—the hourly rate of separation was 104,000 miles, or 29 miles a second. A large portion of this was due to the comet's approach to the Sun, which amounted to 22 miles a second. The remaining 7 miles a second must, hence, be due to the motion of the tail away from the Sun, which therefore is the actual velocity of the particles of which the tail is composed.

This leads one directly to an explanation of the phenomenon observed on the photographs. The appearance in question would seem to be due to a sudden change in the direction of emission of the particles. This must have occurred between 2 and 3 o'clock, G. M. T., on July 24. If the emission had suddenly ceased altogether, then the tail would recede from the comet bodily until it dissipated in space or ceased to be luminous. Suppose, however, that, instead of stopping, the direction of emission should suddenly be shifted slightly to one side. So far as the tail at that moment was concerned, it would be the same as if the emission had ceased entirely and it would recede bodily into space; but at the same moment a new stream of particles would begin to form a new tail in the new direction, and we should have exactly the phenomenon presented by the comet on July 24. By the 25th the disconnected tail or stream of particles would have vanished by dissipation, or by ceasing to be luminous from some other cause, and the new tail would have assumed the normal appearance.

This theory may be illustrated by a jet of steam issuing from

a nozzle. If the nozzle is suddenly slightly changed in direction, the old stream will recede for a few moments before it dissipates, and a new one will follow it closely in a slightly different direction.

That the particles of the tail take a definite time to traverse its length shows that the tail may actually move as a body—a stream of small particles. Should this moving stream encounter a resisting medium of any kind, the tail would become distorted and broken, just as was shown to have occurred in the case of Brooks' comet in 1893. It is well known that swarms and streams of meteors exist within the solar system. Whether they are dense enough to produce the disturbances shown in the tails of some comets or not, it is not the purpose of this paper to say, but the explanation seems a probable one. Or there may be other currents or sources of resistance in space of which we know nothing.

There is one feature that must be taken into account in considering the motion of these particles. Previous to its free existence, the particle was moving toward the Sun with a velocity of twenty-two miles a second—as a part of the comet. When it became independent and subject to the repulsive action of the Sun, it would still approach until the repulsive force checked its speed, when for a moment it would become stationary, and then begin to recede, at first slowly, and then with rapidly increasing velocity. The hourly motion from the Yerkes Observatory plates is greater than that from the Nanterre and Yerkes Observatory plates. This would be in accordance with the above idea, except that one would expect a greater difference. The velocity derived from these photographs seems small. In many comets' tails the particles must move with a speed many times this amount.

An inspection of Mr. Wallace's photograph shows that the new tail, at its end, is growing more rapidly than the motion of the section. This would imply that some of the particles move with a much greater speed than others. By using the near end of the section, therefore, which is the only available method, one would get the velocity of only the more sluggish particles.

Following are the measures of the angles made by the tail with

the direction of the motion of the comet; that is, they are the angles the tail made with the star trails near the comet's head. It is not possible to determine this angle with great exactness, because the tail was seldom perfectly straight when the photographs were examined with a glass. The measures, therefore, represent the general direction of the tail with reference to a tangent to the apparent direction of motion of the comet.

PLATES BY B.

Date	Exposure, C. S. T.	Angle of Tail	Length	Sky
July 18.......	$10^h 24^m$ to $12^h\ 2^m$	42°	13°	Fair
19.......	9 22, about 20^m	39	13	Clouds
22.......	9 0 11 30	38	14	Clouds
23.......	9 27 12 2	38	13	Fair
24......	8 57 11 34	40	15	Clouds
25.......	8 50 11 ±	36	12	Clouds
26.......	9 9	37	13	Clouds
29.......	10 5 12 5	34	8	Clouds
30.......	10 35 11	..	4	Clouds
August 16....	8 34 9 30	..	10	Thick sky

Mr. Wallace's plates have been measured in the same manner.

PLATES BY W.

Date	Exposure	Angle of Tail	Length	Sky
July 20.......	$9^h 40^m$ to $12^h\ 5^m$	40°	11°	
22.......	12 1 14 31	41	15	Good
23.......	12 40 14 43	38	14	
24......	11 59 14 29	38	15	Very poor
29.......	12 30 13 40	41	7	Clouds

The long and unjustly neglected stereoscope when applied to certain astronomical subjects has become of great importance in recent years. The combination of two views of the Moon, similar with respect to phase, made on different dates, reveals its spherical form in a most beautiful manner. Dr. Max Wolf has used this instrument recently for detecting variable stars or moving asteroids on celestial photographs with great success. Two photographs of the same region of the sky with the same instrument being properly adjusted for the instrument, the asteroid or variable star will stand out conspicuously from the other stars. In 1902

he presented to the Royal Astronomical Society some stereoscopic views of Perrine's Comet of that year, in which the comet stood out in beautiful relief, apparently suspended in space before the observer. These were exhibited at the November Meeting of the Royal Astronomical Society in 1902.

In the August 1903 number of the *Bulletin* of the French Astronomical Society is published a stereoscopic picture of the present Borrelly's Comet made by F. Quénisset at Nanterre, France, which when placed in the stereoscope shows the comet in beautiful relief.

Mr. Wallace has combined one of his photographs and one of mine of this comet made on the night of July 22 for the stereoscope. The result is surprisingly beautiful. The comet appears to be suspended in space between the observer and the stars in a most realistic manner. The result is what really happens in space, but which cannot be seen in any manner except with the aid of the stereoscope. A reproduction of Mr. Wallace's beautiful combination is given here, Plate VII, with the other pictures of the comet, and when placed in a stereoscope will exhibit the comet in splendid relief.

Following are descriptions of all of the photographs of the comet obtained here:

1903, July 18 (B.).—Very slight curvature of the tail, convex side preceding. It is narrow back of the head and gradually widens a little. The axis is not symmetrical with the center of the head. There are indications of a thin tail or fragment of a tail symmetrical with the head and preceding the main tail.

July 19 (B.).—Tail faint and feeble and apparently straight.

July 20 (W.).—Tail narrow back of head, and gradually widens out considerably, symmetrical with axis.

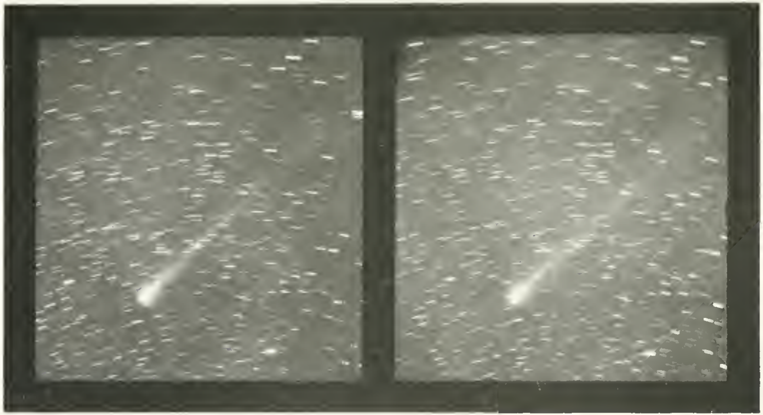
July 22 (B.).—Several strands in tail back of head, one of which is sharply defined and thread-like and is in the following side of the tail. About 6° from the head there becomes visible a faint strip 9 long, which runs parallel with the following side of the tail and distant from it 0°2. This seems to have no connection with the head. Back of the head the tail is not so narrow as before.

July 22 (W.).—Three thread-like strands back of head. These fade out into the tail farther back. The faint strip shown on B.'s plate not visible.

July 23 (B.).—Indications of several strands from the head for some 3 to 4 . The tail bulges out slightly, preceding.

PLATE VII.

July 22, $15^h\ 0^m$ $17^h\ 30^m$ G.M.T. July 22, $18^h\ 1^m$ $20^h\ 31^m$ G.M.T.



COMET c 1903 (BORRELLY). FOR THE STEREOSCOPE.

July 23 (W.).—Shortly after leaving the head the tail is faintly double, as if a fainter portion ran along beside it following.

July 24 (B.).—The tail is very narrow back of the head, and is sharper on the following side. The section is equally sharp on both sides; the nearer end is narrow and more or less pointed.

July 24 (W.).—The main tail is narrow. Near the break it brushes out somewhat preceding. The section of tail is diffused on its preceding side, and apparently slightly double in places; the following edge of it is sharper.

July 25 (B.).—Faint and straight. No signs of the fragment of last night. The exposure was so interrupted by clouds that the utter dissipation of this fragment is not certain. If present, it must have been extremely faint.

July 26 (B.).—The tail is faint and straight, but is somewhat sinuous 3 back of head, where it bulges out slighly, preceding.

July 29 (B.).—Slightly sinuous about 2°2 back of head, and bulging out slightly, preceding. The tail gradually widens out. Its end is cut off by defect in negative.

July 29 (W.).—The tail is faint and apparently straight. If continued, it would cut through ϵ *Ursae Majoris.*

July 30 (B.).—Faint traces of tail for 3°5. It seems to be straight, but is cut off by defect in negative.

August 16 (B.).—Tail extremely faint clear up to the head. It appears to be straight.

It is evident that the faint strip of tail shown on B.'s plate of July 22 is a stream of particles whose emission had ceased or changed its position some hours before.

In *Knowledge* for August is a reproduction of a photograph of the comet on July 24, made by Dr. Roberts. This shows the phenomenon exhibited on our plates for that date. Dr. Roberts has failed to give the time of this photograph, or of the others reproduced with it, and hence unfortunately the picture at present cannot be used for determining the motion of the fragment.

YERKES OBSERVATORY,
August 1903.

Minor Contributions and Notes.

AN APPLICATION OF THE CROSSLEY REFLECTOR OF THE LICK OBSERVATORY TO THE STUDY OF VERY FAINT SPECTRA.[1]

INTRODUCTION.

WHILE engaged in photographing the brighter nebulæ with the Crossley Reflector, Professor Keeler, director of the Lick Observatory, was surprised at the relative photographic brightness of the central star in the Ring Nebula in *Lyra*, though the fact had been previously remarked by many observers. This star, which is visible only in the largest telescopes, was found to be brighter photographically than the nebula itself, which is an easy object in even a comparatively small telescope. It is true that a large faint area, such as a nebula, is more readily seen than a faint stellar point, but considering that the available telescopes magnify the Ring Nebula several hundredfold, and in effect leave the star unmagnified; that the image of the nebula photographed with the Crossley Reflector is virtually magnified but twentyfold; and that the star photographs in about half the time required by the nebula, under good atmospheric conditions — making allowance for the physiological effect referred to, and for the proportions of the instrument, the photographic brightness of the star suggested that its spectrum contains an unduly large proportion of actinic rays. Professor Keeler also noticed that the central star in the Ring Nebula in *Cygnus* (H IV 13) behaved in the same manner. He tried to examine their spectra, both with a direct-vision spectroscope and with a prism held in the path of the rays, but in neither case was the spectrum bright enough to be seen. He therefore decided to construct a spectrograph for the study of these objects.

Such an instrument, to preserve and utilize the enormous advantages of the silver-on-glass reflecting telescope for work in the violet and ultra-violet regions, called for a design radically different from those of conventional spectrographs. It would evidently be difficult for a star near the limits of vision to be centered and kept upon a narrow slit; the method of guiding by means of a reflecting slit would not

[1] *Lick Observatory Bulletin* No. 35.

answer; and it would be important to avoid the usual losses due to a slit. The instrument as originally designed consisted, in outline, of a 50° quartz prism with circular aperture of 27 mm, placed directly in the converging beam of light from the great mirror, at a distance of 15 cm inside the focus; of a plate-holder suitably placed; and of a guiding eyepiece working on the same principle as that employed in ordinary nebular photography.

Director Keeler had thought that by placing the prism approximately at minimum deviation for the ray coinciding with the collimation axis, the confusion in the image due to the very large angle of the cone of incidental light, would be so slight as not to interfere with the qualitative purposes of the instrument; especially since the dispersion would be small—only 3 mm from λ 350 to λ 500. It should be said, however, that he had doubts as to the success of this plan, but felt that it would be well worth a trial.

The spectrograph was constructed by the Observatory instrument-maker. It was completed on the day Professor Keeler left Mount Hamilton for the last time, about a fortnight before his death.

A few weeks thereafter Dr. Campbell and I tried the spectrograph, and found that the beam of light could not be brought to a focus. On account of the approaching opposition of *Eros*, and the plans for completing, if possible, Professor Keeler's program of photographing the brighter nebulæ during the following spring, the instrument was set aside until April 1901.

Director Campbell then asked me to design such changes in the instrument as would permit the insertion of a double concave quartz lens immediately in front of the prism, to render parallel the rays incident upon the prism, and of a double convex similar lens immediately behind the prism to receive the refracted rays and form the image on the sensitive plate. The lenses were constructed by the John A. Brashear Co., from constants supplied by Professor Wadsworth. These additions to the optical train necessitated several changes in the support of the plate, in the guiding fixtures, etc. The alterations were completed by the instrument-maker on June 3, and to me was assigned the duty of testing its capabilities.

DESCRIPTION OF THE APPARATUS.

The Crossley Reflector has been described in detail by Professor Keeler.[1] Nothing need be said about it here, except that, for the sake

[1] ASTROPHYSICAL JOURNAL, 11, 325, 1900.

of completeness, its focal length is 534 cm and its clear aperture 92 cm, affording a ratio of aperture to length of 1 : 5.8.

The general features of the spectrograph are shown in Fig. 1. The tube *TT* fits into a similar tube on the side of the telescope, which holds it in position just as the plate-holder and guiding apparatus for ordinary photography are held in place. The latter tube can be moved in or out by rack and pinion to focus the plate. Since it would be

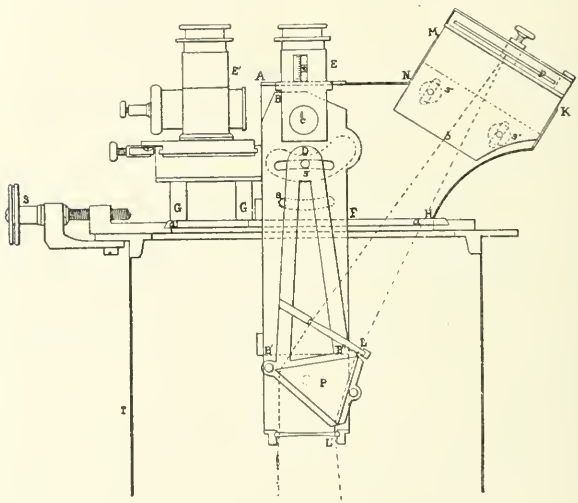


FIG. 1.

difficult to move the whole telescope to correct for any errors in following, the spectrograph is attached to the tube *TT* by two slides at right angles to each other, and the motions of these slides are controlled by two screws. In the drawing only the declination screw *S* is shown. The right-ascension screw would occupy a position in front of the middle of the apparatus, in the center of the drawing, and it is therefore omitted. The right-ascension screw has a coarse double thread to enable the observer quickly to correct the abormally large irregularities in the diurnal motion of the telescope, to which Professor Keeler's article refers at length. (The sources of much of this trouble

have been discovered and removed by Assistant Astronomer Perrine in the past year and a half, and the following in right-ascension is now much easier and better.) As the motion in declination is always small, the declination screw has a fine thread.

The base-plate *aa*, movable east and west between guides by means of the right-ascension screw, carries both the guiding eyepiece E' GG and the supporting framework AB' B'' $F H K M N$ of the spectrograph proper. This framework is built up of sheet brass, and is about 42 mm thick. It directly supports the plate-holder attachment *bp*, the finding and focusing eyepiece E, and the prism-box BL'; and the prism box in turn supports the minimum deviation apparatus DP.

The converging beam of light from the telescope first passes through the concave lens L', which converts it into a parallel beam; then through the quartz prism P; and finally through the convex quartz lens L, which brings it to a focus on the plate p. The prism and convex lens are attached directly to the minimum deviation apparatus. This consists of the framework extending from D down to and around the prism, turning about an axis P near the center of the prism. It is held in any desired position by the set screw S' which works in a slot in the prism-box. A much larger slot in the spectrograph case allows the screw to be manipulated when the apparatus is in place on the telescope. The convex lens should not, theoretically, be attached to the minimum deviation apparatus, but it was the only support available for it unless extensive changes were made in the design. However, it was set with the prism in an approximately correct position, so that its axis of collimation is nearly perfect.

The concave lens L' is attached directly to the prism-box. The plate-holder attachment *bp* can be moved toward or away from the convex lens in order to bring the plate into focus. It is held in any position by four set screws s'', two on either side. These screws work in slots in the spectrograph case. The prism-box moves about the tube C as an axis, so that the lenses and prism can be moved far enough to the right to allow the light from a star to pass without obstruction into the focusing eyepiece E. It is held in position by the set screw S, working in a slot in the spectrograph case.

The eyepiece E serves to identify the object under investigation, to set the telescope so that the spectrum will fall on the center of the plate, and finally to focus the spectrograph. The eyepiece can be moved in or out, and its position be read off on the accompanying

small scale. It is held at any desired reading by a set screw (not shown in the drawing). It is so adjusted that when the spectrum has by trial been focused on the plate, the *object itself* will be in focus in the eyepiece if the prism-box be moved aside. The cross-wires in the eyepiece are illuminated by a small electric lamp placed outside the spectrograph case. It shines through the tube *C*, about which the prism-box turns.

The guiding eyepiece E' is connected with the staging through two slides working at right angles to each other, similar to the two main slides carrying the entire spectrograph. When the star under observation has been brought to the intersection of the cross-wires in the focusing eyepiece E, the eyepiece E' can be moved over an area $15' \times 45'$ in search of a suitable guiding star. When such star is found, the eyepiece is clamped to its staging, with the star on the intersection of the cross-wires. Since this staging is rigidly attached to the plate *aa* carrying the spectrograph, the spectrum will remain in correct position so long as the guiding star is kept on the cross-wires. The lamp for the guiding eyepiece is shown just below E'. The electric light used for illuminating the wires in both eyepieces is first passed through red glass, to guard against fogging the sensitive plate.

ADJUSTMENTS AND METHOD OF OBSERVATION.

The first adjustment consists in bringing the plate to the focus of the convex lens, and this is done before prism and concave lens are mounted. A positive eyepiece is fitted in the place for the plate-holder, and is so adjusted that its focus will be in the plane of the plate. The instrument is then turned to a distant object and focused by shifting the plate-box into position, whereupon the box is clamped. In this adjustment it is necessary to use blue glass between the eye and eyepiece, as the convex lens by itself is not achromatic. When the adjustment is made in this manner, the blue light is the only light which will pass through the prism in a parallel beam, but the angles of the cones of the other colors will be too small to do harm.

After the prism and concave lens have been inserted, the focusing eyepiece and minimum deviation apparatus are adjusted. For this purpose the telescope is turned to a bright star; and using the eyepiece employed in focusing the convex lens, the prism is set at minimum deviation by shifting the deviation apparatus in the usual manner, and clamping it in the correct position.

The eyepiece E is focused best by a modification of Hartmann's

method. In this case, a rectangular diaphragm is placed over the center of the concave lens with its longer dimension at right angles to the edges of the prism. When the instrument is out of focus we shall see two parallel lines, each the continuous spectrum of the star. The instrument is now focused approximately by means of the eyepiece, after which the prism is turned to the side and the eyepiece E is focused. The reading of the scale on this eyepiece gives approximately the correct position of the eyepiece. To find the accurate position of the focus, we use Hartmann's method of taking two photographs of the spectrum, one considerably inside focus, and the other considerably outside, measuring the distance between each pair of images, and from these data interpolating the true position. In this case we set the focusing eyepiece to read, first, one or two millimeters less than the approximately correct reading, and then one or two millimeters more than its correct reading. After each setting the whole spectrograph is moved until the star is in focus in this eyepiece, then the prism is turned into position and a short exposure is made. During the exposure it is necessary to guide very carefully, as any error in guiding tends to widen the two spectra. After determining the correct focus from the two plates thus obtained, the eyepiece is set in the computed position and clamped. The spectrum then and thereafter is focused merely by moving the spectrograph in or out until the object is in the focus of the eyepiece.

When the object to be photographed has been brought to the center of the field of view, the instrument is focused and a guiding star found. This should be of about the eighth magnitude, though a fainter one can be used if necessary. It is unavoidably situated so far from the axis of collimation that its image is always very much distorted. The shape of the image can be altered by moving the guiding eyepiece in or out. For some reason, the focus of the telescope changes during a long exposure, and by noting any alteration in the form of the star image a change in the focus can be detected. By using a form of image which is sensitive to slight changes, we have a very accurate method of adjusting the focus in the progress of the exposure. After adjusting the guiding eyepiece, the optical train is turned into position and the exposure begun.

In observing some of the stellar nebulæ whose positions are not accurately assigned, difficulty is often encountered in identifying the objects. If they are brighter than the eleventh magnitude, they can usually be identified by their green color; but when they are fainter

than this, recourse must be had to their spectra. Nearly all of their visual rays are concentrated in the two green nebular lines, which are so close together in this instrument that they cannot be resolved. The result is that their spectra appear simply as dots, while a star with its continuous spectra appears as a line.

The length of exposure depends entirely on the brightness of the lines. Usually the extreme ultra-violet lines require much longer exposure than the others, though there are a few cases in which they record themselves as quickly as some of the lines in the lower part of the hydrogen series of the same spectra. The faintest continuous spectrum photographed was that of a fifteenth-magnitude star, with an exposure of four hours. The spectrum was not intense, but it was abundantly strong for purposes of interpretation.

As there is no slit, sky light and moon light exert their full effects on the plate, and it is therefore not possible to photograph a faint object on a moonlight night. Several photographs of fairly bright objects have, however, been obtained within three or four days of full moon, without serious injury to the spectrum.

The plates were measured on a Zeiss comparator. With this it is possible to measure to 0.1μ; but as only rough determinations of wave-lengths could be obtained with such a small dispersion, they were measured to 1μ only. Even this is accurate enough to determine the wave-lengths by computation to one Angström unit, but the probable errors were so great that the results were taken merely to the nearest $\mu\mu$. To determine the wave-lengths of unknown lines, a dispersion curve was first constructed. This curve was based upon the measure of known lines on several plates, after combining them by the method of least squares, using the Cornu-Hartmann formula. It was found that the least square determination was unnecessary, as the corrections which it gave were practically zero. This curve was plotted on such a scale that the measures could be laid off to 0.01 mm, and the wave-lengths read at a glance to $1\mu\mu$.

In working with a spectrograph of this kind, where no comparison spectrum is possible, all the measures must refer to some known line or lines. The interpretation of unique and unknown types of spectra would have to depend upon a comparison of the positions of the object and surrounding stars with the relative positions of the unknown spectral features and surrounding spectra on the same plate. Fortunately, in the spectra photographed thus far with this instrument, many features could always be identified; and I have in every case used the bright $H\gamma$ line as the point of reference and departure.

OBSERVATIONS.

As soon as the spectrograph was completed and adjusted, photographs were taken of the spectra of various objects to find out for what work an instrument of this kind is best adapted; and to determine its limitations. It is evident that, because of the small dispersion, it cannot as a rule be used to advantage on dark-line spectra, but some stars have been photographed with dark bands broad enough to show. This limits it very largely to bright-line spectra, though it has great value for determining the nature of very faint spectra of all kinds. Moreover, since there is no slit, the object must not be so large that the various monochromatic images will overlap too much, and this condition confines the field to objects no larger than planetary nebulæ.

Large numbers of objects having various spectral types promised to yield valuable results; but, on account of the limited time at my command, my program was confined, with the director's approval, to comparatively few objects. These included:

a) Such planetary nebulæ as were in observing position, both for purposes of comparison with results reached by observers using other instruments, and in the hope of discovering new features in the ultra-violet region. It was not expected that results already obtained in the $H\epsilon$–$H\beta$ region of bright objects, by observers using large telescopes and powerful spectroscopes, would be surpassed or even equaled, on account of the feeble dispersion of this instrument.

b) Such new stars as were in observing position.

c) The central star in the Ring Nebula in *Lyra*, and other similar stars and stellar nuclei.

d) A few Wolf-Rayet stars.

After the most of my observations were made, Professor Pickering suggested that the long-period variables be observed, to determine just when the bright hydrogen lines disappear; but lack of time prevented me from doing so.

An interesting series of observations somewhat similar to mine was made by von Gothard in 1892 (*Astronomy and Astro-Physics*, 1893, p. 51). His program included seven of the brightest planetary nebulæ, the Ring Nebula in *Lyra*, and *Nova Aurigae* after its reappearance. His telescope was a small reflector, the scale of his photographs was about the same as mine, and his results for the brighter nebulæ in the region $H\eta$–$\lambda 5007$ agree well with mine. Some of his nebulæ, however, were too large for the Crossley instrument.

Altogether, twenty-one nebular spectra were photographed by me between June and September 1901. After my departure, Mr. Joel

Stebbins, Fellow in the Lick Observatory, kindly secured for me with the same instrument some very good spectrograms of *Nova Aurigae*, *Nova Persei*, and a Wolf-Rayet star.

The general character of the different spectra is shown in Fig. 2. The scale of the original negatives was too small to permit photographic enlargements to a convenient size to be made, and they have been enlarged by drawing them to a scale of one inch to the millimeter. In the drawing, the edges of the disks appear much sharper than on the original negative.

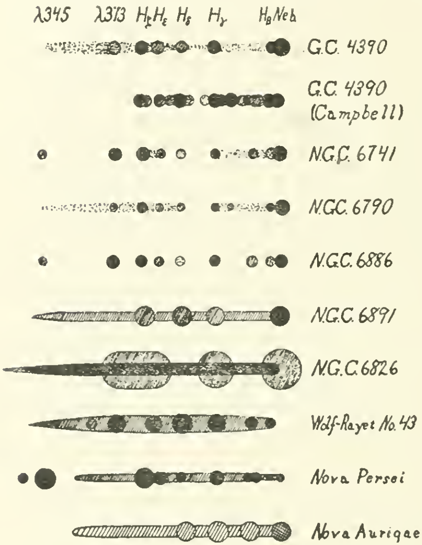


FIG. 2.

It will be noticed that in general the intensity of the continuous spectrum of the nebulæ has a minimum between $H\delta$ and $H\gamma$. This fact is confirmed by nearly all the negatives, and is too general a characteristic to be passed without notice. It seemed hardly possible that the spectrum of the nucleus, to which it was supposed that this belonged, should have an absorption band at this place, and another explanation was sought. This was found by plotting Dr. Campbell's observations of *G.C.* 4390 on the same scale, and comparing the result with the spectra obtained with the slitless spectrograph. His observations were made with a large spectroscope attached to the 36-inch refractor. A glance at the spectrum marked *G.C.* 4390 (Campbell) will show that he found the bright lines much fewer in the region where the continuous spectrum is weak in the spectrograms taken with the small instrument. His observations of *N.G.C.* 7027 give the same results. From this it seems safe to infer

that the apparently greater relative strength of my spectrograms in the $H\gamma$–$H\beta$ region is largely due to the presence of many faint overlapping bright lines, whereas, in the region $H\delta$–$H\gamma$ fewer bright lines exist. A further support of this theory is afforded by nebulæ *N.G.C.* 6826, and *N.G.C.* 6891, each of which has a stellar nucleus. In each case the spectrum of the nucleus has an almost uniform intensity throughout its entire extent.

A simple inspection of Fig. 2 will show one of the marked advantages of the reflector and quartz prism and lenses over the large refractor and glass optical train, in spectrum observations. Even in the favorable nebula *G.C.* 4390, Campbell's results stop at $H\zeta$, whereas the slitless spectrograph record extends to about λ 337.

NEBULA *N.G.C.* 6210.

This is described as very bright in Dreyer's *New General Catalogue*. Three photographs were taken with exposures of 1 minute, 5 minutes, and 30 minutes, respectively. The last is the best, the others being underexposed. The images are rather large, as the nebula is not stellar, but most of them are pretty distinct. The continuous spectrum seems to be of the kind made up largely of overlapping bright lines, with its maximum intensity in the neighborhood of $H\delta$. The following bright lines were found: principal nebular ($= \lambda\, 501 + \lambda\, 496$), $H\beta$, $H\gamma$, $H\delta$, $H\epsilon$, λ 386 and λ 373.

N.G.C. 6439.

This is a stellar nebula rated at thirteenth magnitude in the *N.G.C.* Satisfactory plates could not be obtained because there is a pretty bright star about 2′ due north of it, and the nebular spectrum appears to overlap the stellar spectrum. Three plates were made, the first with 1 hour and the other two with 2 hours' exposure. The principal nebular lines, $H\beta$, $H\gamma$, and $H\zeta$, are shown, though the last two are faint.

N.G.C. 6537.

This is a stellar nebula, and the images are small. Two plates were taken, with exposures of 1 and 1¾ hours, respectively, but both are underexposed. The latter shows the following lines: nebular, $H\beta$, λ 466, $H\gamma$, $H\delta$, $H\zeta$. There is no continuous spectrum recorded.

N.G.C. 6543.

This is a planetary nebula, marked "v B, p S" in the *N.G.C.* It has a diameter of about half a minute of arc, which is as large as can be photographed to advantage with this instrument. Two photo-

graphs of it were taken, with exposures of 15 minutes and 40 minutes, respectively. The former is the better of the two. The lines are very little brighter than the continuous spectrum; and being large they could not be measured accurately. Those identified were: nebular, $+H\beta$, $H\gamma$, and $\lambda 373$. Between $H\gamma$ and $H\beta$ the continuous spectrum is narrow, whereas beyond $H\gamma$ it is almost as wide as the nebula, as though above $H\gamma$ it were composed of overlapping bright lines. On a nebula as large as this one, it would take very few lines to cause such an appearance, the hydrogen series alone being practically sufficient. This continuous spectrum also extends above $\lambda 373$ to $\lambda 336$ as bright as it is below $\lambda 373$. It would seem that there must be other lines present in that region.

N.G.C. 6567.

This is a stellar nebula of the eleventh magnitude. But one photograph was obtained, with an exposure of 1 hour 30 minutes. The images are quite small, though not small enough to show the $H\beta$ line separated from the nebular lines. It contains the following lines: nebula, $+H\beta$, $H\gamma$, $H\delta$, $H\epsilon$, $H\zeta$, and $\lambda 373$.

The continuous spectrum recorded is of the type described above as probably due largely to overlapping bright lines. Between $H\gamma$ and $H\delta$ it is faint, and between $H\zeta$ and $\lambda 373$ it is almost invisible, but everywhere else it is bright.

G.C. 4390.

The spectrum of this nebula is shown in Fig. 2, as well as the spectrum obtained by Dr. Campbell, reduced to the scale of these spectrograms. The nebula is nearly stellar, described as "v B." Two photographs were obtained, with exposures of 10 and 15 minutes, respectively. The former is the better, although both are good.

N.G.C. 6741.

This spectrum is also shown in Fig. 2. It is a stellar nebula, but an exposure of three hours was able to bring out a continuous spectrum only from $H\beta$ to $H\gamma$ and from $H\epsilon$ to $H\zeta$. Another exposure of one hour failed to show any continuous spectrum. A new nebular line is strongly suspected at $\lambda 345$, but it is not absolutely certain.

N.G.C. 6790.

This is a stellar nebula of the 9.5 magnitude; and as it has been observed by Campbell with a larger spectroscope, little need be said about it here. Three photographs of 20 minutes', 1 hour's, and 3 hours'

exposure, respectively, were obtained. The first two show no continuous spectrum, but the third records one with a gap between $H\gamma$ and and $H\delta$. This spectrum is illustrated in Fig. 2. The continuous spectrum above λ 373 seems to be fluted, with condensation at $\lambda\lambda$ 363, 356, and 352, but they are very faint and their existence is doubtful.

N.G.C. 6803.

This is a stellar nebula showing only a very faint continuous spectrum. Two photographs were obtained, one with an exposure of 1 hour, and the other with 2 hours 30 minutes. The latter shows just the faintest trace of a continuous spectrum from $H\beta$ to $H\gamma$, and from $H\delta$ to a little way beyond λ 373. The bright lines which were identified are: nebular, $H\beta$, λ 468, λ 446, $H\gamma$, $H\delta$, $H\epsilon$, $H\zeta$, λ 373, and perhaps one at λ 365, though the existence of the last is very doubtful.

N.G.C. 6807.

This is a stellar nebula without the slightest trace of a continuous spectrum. Three photographs were obtained, with exposures of 45 minutes, 57 minutes, and 3 hours, respectively. They contain: nebular, $H\beta$, $H\gamma$ $H\delta$, $H\epsilon$, and $H\zeta$ lines.

N.G.C. 6818.

This is a nebula described in the *N.G.C.* as "B, v S," but is nevertheless almost too large for this kind of work. Two photographs were obtained, with exposures of 10 minutes and 32 minutes, respectively. The former is underexposed. The spectrum consists of a pretty strong continuous spectrum extending out to λ 372, with the following bright lines: nebular, $H\beta$, $H\gamma$, $H\delta$, λ 375 and λ 372.

N.G.C. 6826.

This is described as "B, p L" in the *N.G.C.* The nebula is rather large, and contains a bright stellar nucleus which gives a continuous spectrum. A drawing of the spectrum is given in Fig. 2. Two photographs were obtained, with exposures of 15 minutes and 1 hour, respectively. The former is the better, though in both the spectrum of the nucleus is overexposed. The lines are probably: nebular, $H\beta$, $H\gamma$, $H\zeta$, and λ 373. The continuous spectrum extends to λ 333.

N.G.C. 6833.

This is one of the finest stellar nebulæ. It is the only case in which $H\beta$ was suspected visually, the image seen in this case being somewhat elongated. On the plate it is distinctly separated from the nebular

lines. But one plate was taken, with an exposure of 1 hour 55 minutes. It contains the following lines: nebular, $H\beta$, $\lambda 446$, $H\gamma$, $H\delta$, $H\epsilon$, $H\zeta$, and $\lambda 373$. The continuous spectrum extends to $\lambda 348$, with a slight minimum of intensity between $H\gamma$ and $H\epsilon$. The variation in the brightness of the continuous spectrum is less noticeable in this case than in most of the others.

N.G.C. 6881.

This is a stellar nebula, with spectrum quite different from most of the others. Two photographs were taken, with exposures of 2 hours and 3 hours, respectively, and they show only: nebular, $+ H\beta$, $\lambda 468$, $H\gamma$, $\lambda\lambda 422$ and 388. There is a continuous spectrum from about $\lambda 450$ to $\lambda 360$, which seems to be fluted between the $\lambda 422$ and $\lambda 388$ lines. Still, the spectrum is too faint to allow us to decide. An exposure of at least four hours would be necessary to make sure of this point, and opportunity to secure it did not offer.

N.G.C. 6884.

This is a typical stellar nebula. The one photograph secured with exposure of 1 hour shows the following lines: nebular, $H\beta$, $\lambda 467$, $H\gamma$, $H\delta$, $H\epsilon$, $H\zeta$, and $\lambda 376$. There is a continuous spectrum from $H\beta$ to $H\gamma$, and from $H\epsilon$ to $\lambda 348$.

N.G.C. 6886.

This is a fine stellar nebula. But one photograph was obtained, with an exposure of 1 hour. It is an excellent one, and shows the images round and small. A drawing of it is given in Fig. 2. This was the first nebula in which the new line $\lambda 345$ was noticed. There is no trace of a continuous spectrum.

N.G.C. 6891.

This is a planetary nebula with a stellar nucleus. One photograph was taken with an exposure of 30 minutes. A drawing of it is given in Fig. 2.

N.G.C. 6905.

This is described as "!!, B, p S" in the *N.G.C.* It is simply a patch of nebulous matter, shading off gradually toward the edges. One photograph was taken with an exposure of 1 hour, which gives only one image of the nebula, due, probably, to the principal nebular lines.

N.G.C. 7009.

This is described as "!!!, v B" in the *N.G.C.* For so small a nebula it shows a great deal of structure. Three photographs were taken, with exposures of 5 minutes, 10 minutes, and 20 minutes, respectively. They record the nebular, $H\beta$, $H\gamma$, $H\delta$, and $H\zeta$ lines, as well as a continuous spectrum belonging to the center of the nebula. This continuous spectrum extends out to λ 345. At the $H\delta$ line it widens out to the width of the nebula, as though from there out to the ultra-violet the whole nebula gives out faint continuous light, or that there are several overlapping faint lines.

N.G.C. 7027.

This is the brightest of the stellar nebulæ photographed, being given as 8.5 magnitude in the *N.G.C.* Two photographs were taken, with exposures of 15 minutes and 30 minutes, respectively. The latter is the better of the two, although the nebular lines are so overexposed as completely to cover $H\beta$. In this nebula the λ 345 line is as bright as $H\delta$. There seems to be a very faint line at λ 337. It is very faint, but its existence is scarcely doubtful. The continuous spectrum has a minimum between $H\gamma$ and $H\epsilon$, but it does not extend quite so far as the λ 345 line.

N.G.C. 7354.

This is described as "B S" in the *N.G.C.* One photograph was taken with an exposure of 2 hours, which shows simply a single fairly bright image of the nebula, due to the nebular lines, and an exceedingly faint continuous spectrum.

NOVA PERSEI.

Several plates of *Nova Persei* were taken from September 1901, to March 1902, but the best plate was secured for me by Mr. Stebbins on October 14, with an exposure of 30 seconds. A drawing of this is given in Fig. 2. A note on this spectrum has been published by Mr. Stebbins in *Lick Observatory Bulletin* No. 8. In general the spectrum is like that of the stellar nebulæ, but resembling that of *N.G.C.* 7027 most strongly. The main difference between them is that in *Nova Persei* the brightest lines are in the ultra-violet, the green nebular lines being faint. The line suspected in *N.G.C.* 7027 at λ 337 is present here, to add to the similarity; and the line at λ 345 discovered by me in *N.G.C.* 6741 and *N.G.C.* 6886 is an exceedingly strong line in *Nova Persei*. The continuous spectrum also ends at about the same place in *N.G.C.* 7027 and the *Nova*.

NOVA AURIGAE.

Three photographs of this star were taken for my use, at the director's request, by Mr. Stebbins on September 12 and 13, 1901, with exposures of 30 minutes, 1 hour 30 minutes, and 3 hours 20 minutes, respectively. The last is the best, and is shown in Fig. 2. The visual magnitude of the star was about 13. Von Gothard[1] observed this with his objective prism on September 15, 1892, when its magnitude was 10.5. His observations are the best to compare with mine, because his dispersion was almost the same as mine. He found seven lines in this part of the spectrum, and a rather faint continuous spectrum, with a maximum brightness in the neighborhood of $H\epsilon$. Our plate made nine years later shows four lines on a nearly uniform background of continuous spectrum almost as bright as the lines themselves. Plainly the bright lines are growing relatively weaker.

NOVA CYGNI 1876.

One photograph was obtained on August 12, 1901, with an exposure of four hours. This showed that the spectrum has become continuous. At that time the *Nova* was estimated at about the fifteenth magnitude, yet the spectrum is very distinct. The plate records spectra of several stars as faint as fifteenth visual magnitude. The last published observation of this star was by Copeland and Lohse (*Copernicus*, II, 105) in 1881. At that time they surmised that the spectrum had become continuous, because they could see nothing of it, whereas, had it been a bright-line spectrum, they thought it should have been visible.

WOLF-RAYET, NO. 43.

$\alpha = 19^h\ 30.9^m,\ \delta = +30^\circ\ 19'$.

Mr. Stebbins took two photographs of this star, with exposures of 15 minutes and 60 minutes, respectively. The spectrum is shown in Fig. 2. The most noticeable difference between this and the nebulæ is the absence of the nebular lines. The continuous spectrum is nearly uniform.

RING NEBULA IN *LYRA*.

As the central star in this nebula was the principal object in mind when Professor Keeler designed the spectrograph, it was one of the first objects observed after the instrument was completed and adjusted. Three plates were taken of it, two with exposures of 30 minutes, and one with 2 hours; but the results obtained for the star were mostly

[1] *Publications A. S. P.*, 1893, 152.

negative. The rings are so large that they overlap, and only the two brightest images can be distinguished — those due to the nebular lines $+H\beta$, and to λ 373. The hydrogen lines are probably present, as the whole region between these images is darkened, probably by the overlapping images. The peculiar thing about it is that the λ 373 line is so much brighter than all of the others. Von Gothard also noticed this fact. His instrument was better adapted to this object than the Crossley Reflector is, because, having a shorter focal length, the rings were smaller; but having the same dispersion, their centers were just as far apart and consequently did not overlap so much. The only trace of the central star on these photographs is an extremely faint line, present on all three, extending from the violet edge of the green nebular ring to the red edge of the λ 373 ring. This line is too faint to be seen, except with the unaided eye. The two-hour exposure shows a dot near the middle of the λ 373 ring, but if this is due to the central star the image is a little more refrangible than λ 373. This dot is also present in the 30-minute exposure, though it is so faint that its presence would not be suspected had it not been first noticed in the long exposure.

From the fact that the spectrum of the central star is not conspicuous on these plates, it seems probable that it is almost wholly continuous, with perhaps a bright line near λ 373. The photographs taken by Professor Keeler showed that photographically the star is brighter than the nebula; and the theory that its light is spread out into a band, while the light of the nebula is concentrated in a few bright lines, seems to be the only one which explains the observed phenomena.

CONCLUSIONS.

One of the principal purposes of this investigation was to determine the efficiency of the slitless quartz spectrograph attached to the Crossley Reflector, for the photography of very faint spectra. Its power in this respect is surprisingly great. It is well illustrated in the case of *Nova Cygni*. The visual magnitude of this star was estimated by Professor Barnard in 1901 to be 15.6. My four-hour exposure, with fair following and focus, recorded its continuous spectrum in good strength, well up into the ultra-violet. The character of the spectrum could have been determined had the image been considerably less intense. This spectrum may be relatively stronger in the photographic regions than ordinary stellar spectra, though the spectra of many adjacent fifteenth-magnitude stars are recorded satisfactorily

on the same plate. It would be perfectly practicable to carry the exposures up to ten hours or more; the guiding could be more accurate, now that the principal sources of the irregularity in diurnal motion have been removed; and perhaps the more stable new telescope mounting will eliminate the present unfortunate changes in the focus during exposures. Considering these facts, I am reasonably confident that this instrument can record the continuous spectrum of the faintest star visible in the 36-inch telescope under the corresponding atmospheric conditions. Further, in the case of stellar nebulæ, etc., whose spectra contain well defined bright lines, I do not doubt that the instrument will record the principal bright lines of objects too faint to be seen in our most powerful telescope.

It is only for work on very faint spectra that this spectrograph is efficient. Its dispersive power is very low, and it should seldom be used on spectra, or on those portions of spectra, bright enough to be recorded by relatively longer exposures with instruments giving greater dispersion.

For qualitative studies of general spectral features this spectrograph is efficient on spectra of small objects, whether continuous or bright line; but for quantitative results its usefulness is limited largely to small and faint bright-line spectra.

For most of the objects observed a 60° prism, and for some of the objects two 60° prisms, would have been an improvement over the present 50° prism; requiring, of course, the reconstruction of the instrument.

For some of the stellar bright-line objects a convex lens of twice the present lens's focal length would have had an advantage in separating close lines on the photographic plate, permitting at the same time more accurate determinations of wave-lengths. This would probably alter the character of the field of the combination somewhat.

It is possible that a similar instrument on twice the linear scale, *i. e.*, with lenses and prism having 50 mm apertures — with perhaps a 60° prism — would be efficient and practicable for the objects of medium brightness on my list.

New nebular lines at λ 337 and λ 345 were discovered. These lines were later found to exist in the spectrum of *Nova Persei*, from the plate taken October 14, 1901. The nebular character of the *Nova* spectrum is thus established for the extreme ultra-violet region, as had previously been done for the other regions.

The spectrum of *Nova Cygni*, which in 1876 was of the usual "new star" type, and later changed into the nebular type, has now become continuous, with no signs of bright lines.

The bright lines in the nebular spectrum of *Nova Aurigae* are now relatively faint, and the spectrum appears to be approaching the continuous type.

These complete and astonishingly rapid changes of spectral types observed in the cases of *Nova Cygni* and *Nova Aurigae*—and likewise those observed in *Nova Normae, Nova Sagittarii, Nova Persei*, etc.—leave little doubt that the masses of these objects are small.

All the new stars of recent years should be re-observed with this spectrograph every year, to keep account of the progressive changes in their spectra. The importance of carrying out this plan can scarcely be overestimated.

The relative intensities of nebular and other bright lines, as well as of continuous spectra, are shown more accurately on the plates secured with this spectrograph than on those made with instruments absorbing the blue and violet very strongly.

The spectrum of the central star in the Ring Nebula in *Lyra* seems to be not of an unusual type, unless perhaps it contains a bright line near $\lambda 373$. It is quite possible that the star is somewhat brighter photographically than visually; at least, photographic evidence is not opposed to this theory.

The great intensity of the $\lambda 373$ ring in the Ring Nebula perhaps explains one point in nebular photography. Professor Keeler found that for most nebulæ an exposure of three hours gave the best results, whereas the best photograph of the Ring Nebula was obtained in ten minutes. While the Ring Nebula is brighter than the average bright nebula, this brief exposure of ten minutes is out of proportion to its visual brightness; but its radiations being mostly in the ultra-violet, it should have a somewhat greater photographic than visual brightness.

I wish to thank Mr. Joel Stebbins, Fellow in the Lick Observatory, for assistance in taking some of the photographs, and for making several exposures during my absence from the Observatory; and Dr. Campbell for his many suggestions during the course of the work.

HAROLD KING PALMER.

JUNE 1, 1902.

ERRATUM.

Plate III, in the September number of this JOURNAL was wrongly printed. It should be removed, and the one sent herewith inserted in its place.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

Authors are particularly requested to employ uniformly the metric units of length and mass; the English equivalents may be added if desired.

If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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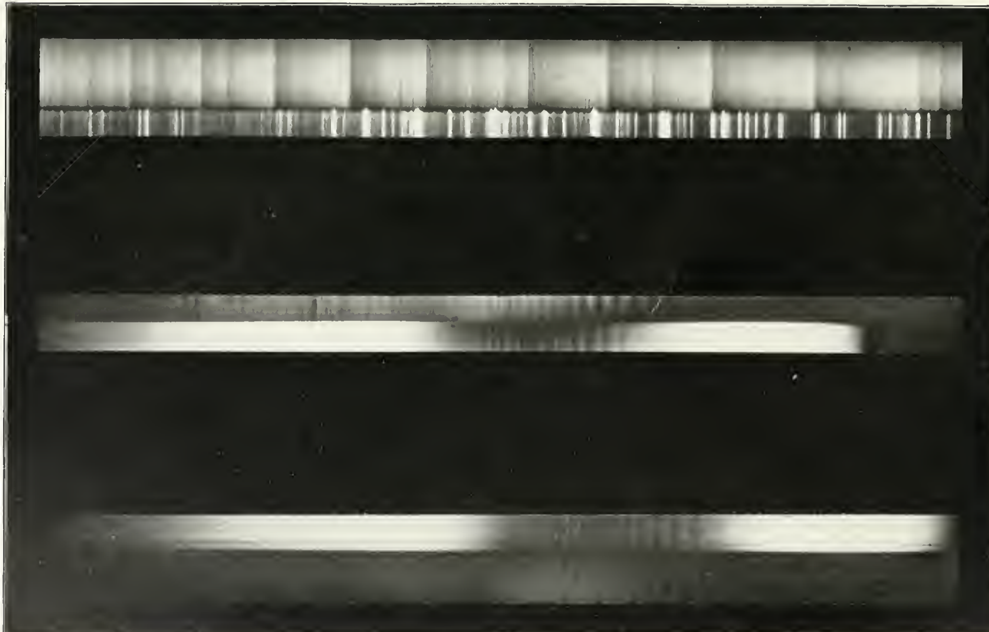

I (*a*). Absorption spectrum of sodium vapor. (Region from λ 4750 to λ 5100.)
(*b*). Corresponding comparison spectrum of iron.
II (*a*). Fluorescent spectrum. (The solar spectrum showing at the left half of the plate is due to diffuse light.)
(*b*). Absorption spectrum, taken with same grating.
III (*a*). Absorption spectrum, (*b*) fluorescent spectrum, showing their complementary character.

THE

ASTROPHYSICAL JOURNAL

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SPECTROGRAPHIC OBSERVATIONS OF STANDARD VELOCITY STARS (1902–1903).

By EDWIN B. FROST and WALTER S. ADAMS.

DURING the past year one part of the program of work with the Bruce spectrograph of the Yerkes Observatory has been the observation of the list of stars adopted for co-operative observation by those engaged in line-of-sight work with the largest spectrographs now in use. The scheme[1] of co-operation that seemed to represent the views of those interested called for three spectrograms per year of each of the ten stars (all having spectra of the solar type), taken if possible at about the date of the star's opposition and thirty days before and after opposition. It was recommended that additional observations should be made of *a Boötis* and *a Persei* throughout the year. The choice of comparison spectrum and mode of reduction was to be left wholly to the different observers, variety being considered desirable. A style of publication was suggested, uniform as regards the minimum amount of data to be communicated.

We have endeavored to carry out the plan of co-operation as

[1] "Co-operation in Observing Radial Velocities of Selected Stars," ASTROPHYSICAL JOURNAL, 16, 169–177, October 1902.

closely as possible since July 1902, but the weather conditions on the two nights weekly allotted to the spectrograph have not permitted us to obtain the plates as nearly on the schedule dates as we desired. In fact, we were able to obtain but a single plate of β *Leporis* during the past winter. The list of stars, with the approximate dates at which they were to be observed, if possible, is as follows:

α *Arietis*,	Oct. 1, Nov. 1, Dec. 1.
α *Persei*,	Oct. 23, Nov. 23, Dec. 23.
β *Leporis*,	Nov. 10, Dec. 10, Jan. 10.
β *Geminorum*,	Dec. 12, Jan. 12, Feb. 12.
α *Crateris*,	Feb. 14, March 14, April 14.
α *Boötis*,	March 13, April 13, May 13.
β *Ophiuchi*,	May 15, June 15, July 15.
γ *Aquilae*,	June 22, July 22, Aug. 22.
ϵ *Pegasi*,	July 24, Aug. 24, Sept. 24.
γ *Piscium*,	Aug. 13, Sept. 13, Oct. 13.

With the exception above noted, we have obtained three measurable plates of each of the above stars, and several extra plates of α *Boötis*, but unfortunately no extra plates of α *Persei*. We have also secured a few plates of the supplementary stars proposed by M. Bélopolsky as substitutes for the southern stars of the above list, which would be beyond his reach. Control plates of the Moon and planets have also been taken at intervals to test the performance of the spectrograph. The results for these plates are given below for the period covered by the plates of standard stars.

A detailed description of the Bruce spectrograph, with illustrations, has been given in this JOURNAL,[1] and need not be repeated here. The principal changes which have been made since that article was written have been: (1) the use of the lens instead of the mirror for projecting on the slit the image of the spark which furnishes the comparison spectrum; (2) the use of a small disk of ground glass at a distance of 20 mm in front of the slit, for diffusing the light of the spark, and insuring the complete and uniform illumination of the collimator by the rays from the spark; (3) a Hastings isokumatic collimator lens has

[1] 15, 1–27, January 1902.

been substituted for the triple of the same size (51 mm) and focal length (958 mm) at first used, resulting in some slight improvement in the previously excellent performance of the collimator.

The camera lenses which have been employed in the work included in this paper are designated as A, B, and B′. A is a Zeiss anastigmat of the "Protar" type, of 71 mm aperture and 449 mm focus, purchased of Bausch and Lomb; B is a triple lens, designed by Professor Hastings and constructed by Brashear, of 76 mm aperture and 607 mm focus; and B′, a triple from the same source as B, is of 57 mm aperture and of the same focus as B.

As the scale of the plates taken with camera B is one-third larger than for A plates, and as their definition is somewhat better at the center, we have naturally preferred to use camera B, which practically requires the same exposure time as A. B, however, has at times shown a disposition to become astigmatic, probably due to pressure on its cemented surfaces within its cell, and A has always been used when the trial plates suggested any lack of sharpness in the lines given by B. Hence A has been employed for about one-third of the plates concerned in this paper. Despite its smaller scale, camera A performs quite as well, if not better than B, as inferred from the accordance of the velocities deduced from different lines on the same plate. (See the values of ϵ in the details of measurements.) Camera B′ is also a cemented system, but has 25 per cent. less aperture than B, and is also mounted in a spring cell, so that no untoward effects are to be feared from pressure.

The performance of the great refractor has been most satisfactory in work with the spectrograph. The addition of a long-focus finder, without tube, having a six-inch object-glass attached beside the cell of the forty-inch lens with an eyepiece near the spectrograph, enables the observer to bring a star very quickly upon the slit, regardless of the flexure of the great tube.

The temperature-case of the spectrograph has also operated in a very satisfactory manner, as will appear from an examination of the records of the temperature in the outer and inner cases given in the journal of observations.

Additional details regarding the instrument, together with a new photograph of it and its accessories, may be found in our paper on the "Radial Velocities of Twenty Stars having Spectra of the *Orion* Type" in the forthcoming Vol. II of the *Publications of the Yerkes Observatory.*

The dispersion and scale of the plates for cameras A and B are as follows:

Wave-Length	Angular Dispersion for One Tenth-Meter	Tenth-Meters per mm	
		Camera A	Camera B
4300	45″.7	10.0	7.4
4500	33.8	13.5	10.1
4700	26.0	17.6	13.1

On well-exposed spectra of the solar type a sufficient number of good stellar lines is usually found within a range of about 130 tenth-meters (or 13 mm for camera B), with the center at about λ 4480, which is the ray passing the prisms at the angle of minimum deviation.

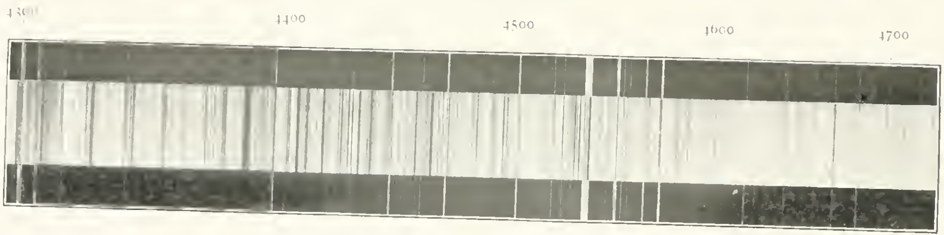
The titanium spark has been chiefly employed for the comparison spectrum on these plates, but electrodes of iron and chromium, and a helium tube can be readily rotated into the proper position for use when desired. Plate VIII, taken from the already cited article in the *Publications* of this Observatory, illustrates the distribution of the comparison lines of the *Ti* spectrum, and shows the quality of stellar spectrum obtainable. The use of a small self-induction coil obviates the presence of any air lines in the comparison spectrum, which, without the coil, are less pronounced with *Ti* than with most metallic electrodes.

MEASUREMENT OF THE PLATES.[1]

The plates have been measured with two screw-comparators constructed by Wm. Gaertner & Co., of Chicago. The screws have a pitch of 0.5 mm, and the head is large, so that the single

[1] An extended discussion of this topic, and of the various sources of errors in our determinations of radial velocities, is given in the volume of the *Publications* already alluded to, which has at this writing been in type for about eight months. For details the reader is referred to that paper.

PLATE VIII



α Bootis, with Titanium Comparison Spectrum.

Enlarged vertically fortyfold, horizontally fourfold.

divisions (0.001 mm) can be easily read through a small magnifier and estimated to tenths. A distance of less than 20 mm is ordinarily measured on the spectra here in question—in fact, for A plates usually only about 10 mm, and for B plates about 13 mm. Four settings are made on each of the comparison and stellar lines which the observer desires to employ, and then the plate is reversed under the microscope (so that the violet side shall be toward the right instead of the left), and the process is repeated.

The measurements in the two positions are combined to means, and then reduced with the aid of the simple Hartmann formula, the constants of which are determined from three standard comparison lines selected near the middle and ends of the portion of spectrum measured. The computed wave-lengths of the various other comparison lines distributed along the plate, always selected so as to be as near as possible to the best star lines, serve to give successive checks upon the accuracy of the "fit" of the formula. The corrections to formula thus obtained are applied to the corresponding or intervening star lines, with such (linear) interpolation as may be necessary. We have feared the dangers of abitrariness in any process of smoothing out the accidental errors of settings by a curve, so that the accordance of the velocities deduced from different lines on the same plate is less than it might be otherwise; but the mean of the values probably may be regarded as quite as trustworthy, and is certainly free from errors due to prepossession in curve-drawing. From the values of the apparent wave-lengths of the stellar lines thus derived, the displacements, and hence the corresponding velocities, are immediately inferred. It is hardly necessary to remind the reader that the full amount of any error in the relative wave-length[1] of any line enters into our result, a relative error of 0.01 tenth-meter producing an error of 0.7 km per second in the velocity deduced for that line.[2] It will, however, be easy to correct our results later, as subsequent laboratory

[1] Taken from ROWLAND'S "Preliminary Table of Solar Spectrum Wave-Lengths."

[2] Unless the star line is identical with the comparison line, so that a direct displacement is measured.

observations shall gradually remove the relative errors in Rowland's tables. In our computations we carry the wave-lengths to the third decimal place, as given by Rowland, but of course we do not attach significance to measures on any given plate beyond the hundredth of a tenth-meter. Similarly, though we carry out our reductions to the hundredth of a kilometer, it is clear that the uncertainties of the wave-lengths, quite aside from the accidental errors of measurement, seriously affect the tenths of a kilometer.

Tests made by ourselves and by others indicate that the measurement of the plate in the two positions under the microscope largely eliminates the physiological error due to the difference in the nature of the process of setting a dark thread upon a white star line and the same dark thread upon a dark comparison line (on the negative). Our systematic errors of this sort would be quite large, about 4 km for Frost and 2 km for Adams, but with opposite signs.

Preliminary investigations of the periodic errors of the screws of the two comparators were made during the progress of this work, and errors were found, reaching a maximum of 4μ for one machine and of 3μ for the other. It was found that they changed when the nut was more tightly clamped to the screw, which was occasionally necessary, as the screws tapered slightly from both ends toward the center. Corrections have therefore not been applied for the errors of the screws in this year's work, and new screws are being supplied to replace the present ones. However, for reasons fully set forth in our article on the *Orion* stars, it does not appear to us probable that the radial velocity of any solar star deduced from measures of one plate will be affected by over 0.1 km on account of screw-errors. The general accordance in the measures of the same plate by the two observers with the two different machines is confirmatory of this opinion.

We desire to emphasize the independent character of the measurements of a plate by the two observers. The selection of the lines of both comparison and stellar spectrum was made by the measurer without reference to his own or the other observer's

measures of other plates of the same star, and no comparisons were made until all the reductions were completed. There has been no attempt to secure conformity in the habit of the two observers in making settings. Thus, for instance, A. is accustomed to make his settings on the inner tips of the comparison lines, while F. has followed the practice of setting upon a certain point estimated as one-third of the length of the comparison line from the inner tip. Of course, the corrections for curvature[1] have been different for these two distances from the center of the star spectrum, which distance is the argument in our table of curvature corrections. But if any serious error had entered in the habit of either observer in regard to the point at which the setting is made, it would appear as a persistent systematic difference in the results, which we cannot discover.

It is not to be doubted that the observers' mode of setting may change with time and experience, but we have not found any such changes, of magnitude sufficient to be regarded as real, during our work since the Bruce spectrograph has been in operation. The differences that occur in the results of the independent measurements of the same plate by F. and by A., tabulated below, are perhaps disappointingly large; and the deviations of the separate plates from the mean may be larger than would be anticipated by those who have not directly engaged in line-of-sight work; but they seem in any event to be of an accidental character. It is perhaps safe to say that at present, with a stable spectrograph, errors due to personality are quite as much to be feared as errors due to instrumental sources.

We have elsewhere[2] discussed the effect of our assumption of solar wave-lengths for spark lines. If arc and spark wave-length are absolutely identical, we might expect that a systematic correction to our results would be required to compensate for the displacement of lines due to pressure in the stellar atmosphere. There are not at present sufficient observational data available for making such corrections (to the solar wave-lengths),

[1] These are applied to the mean of the settings on each comparison line, and do not further appear in the reductions.

[2] *Loc. cit.*, p. 155.

and furthermore our control plates on the Moon and the planets do not reveal such a systematic difference from the computed velocities.

The excellent character of many lines in the stellar spectra, which are made up of two or more lines too close to one another to be separated with the dispersion employed, has necessitated the use of a number of blends. In all such cases we have followed the usual practice of assigning weights to the component lines according to the intensities given them in Rowland's table, and taking the weighted mean of their wave-lengths as the wave-length of the compound line. The maximum distance between two lines for which blending is admissible depends upon the practical separating power of the instrument. On lantern-slide plates of the solar spectrum the least distance at which two lines can be seen as distinct is about 0.15 tenth-meter. The less fine-grained plates used for stellar spectra do not admit of the resolution of such close lines, and with them the limit is usually from 0.25 to 0.30 tenth-meter, according to the quality of the plate and the character of the lines involved. In addition to the matter of separation, it is, of course, necessary to determine the intensity which a line must have in order to exert an appreciable influence upon a closely adjacent line. The answer to such a question is naturally somewhat difficult; an examination, however, of solar and planetary plates has led us to the conclusion that, upon a plate of quality suitable for good measurement, a line of intensity 0 on Rowland's scale is appreciable, while fainter lines may, in general, be neglected. A few exceptions are found to this rule, but these occur mainly in cases of uncommonly strong lines, when the influence of the faint component appears to be negligible.

The arbitrary character of the assumption of equal intensities for the same lines in the Sun, and in stars which are classed broadly under the solar type, finds its justification in the agreement of the values deduced from lines whose wave-lengths have been obtained under this assumption with those derived from uncompounded lines whose wave-lengths are subject to no uncertainty due to this cause; and also in the accord of the com-

pounded lines with the same lines in planetary and lunar spectra where the above method of blending is evidently rigorously correct. In the case of the stars discussed in this paper the wave-lengths of the blended lines, found in the way described above, have proved quite satisfactory for all except *a Persei.* The spectrum of this star, however, is of a less developed type than that of the other stars, its most striking characteristic being the prominence of the "enhanced" metallic lines. When such lines, accordingly, have formed the most important part in the composition of the blends used for the other stars, the wave-length of the enhanced line alone has in each case been used, and the results obtained have shown this to be unquestionably the correct procedure.

The table which follows contains the wave-lengths of all of the blended lines which we have employed two or more times in the course of our reductions, together with the elements to which they are due. Those which have been employed but once are indicated, where they occur in the tables of detailed reductions, by the letter B, following the wave-length of the line.

Elements	Wave Length	Elements	Wave Length	Elements	Wave-Length
Ni; Ti, Cr	4399.903	*Ni; Fe*	4466.701	*—; Cr; Fe*	4526.644
V; Fe; V	4408.549	*Fe; Ag*	76.214	*Ti;—*	27.518
Ti; Fe	27.420	*—, Fe; Fe*	82.376	*Ti-Co;—*	34.168
Fe; Ti	34.021	*Ni; Fe,—*	90.900	*Ni; Fe; Ni*	47.196
Ca; Fe	35.184	*Cr; Zr*	97.046	*Ni, Ti; Fe*	60.225
Mn; Ca	56.030	*Cr, Mn; Ti*	4501.422	*Cr; Co-Fe*	65.750
Ti; V, Zr; Mn	57.656	*—; Fe*	15.475	*Ca; Co, Fe*	81.634
Ni; Fe; Fe, Cr	59.304	*Fe?; — Ti*	22.853	*Cr; Fe*	4611.455
V; *Mn;—*	60.460				

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The table which follows contains the observational data for all of the plates discussed in this article. As usual the series letters A, B, and B′ refer to the three cameras already described earlier in this paper. The middle of the star exposure is given in Greenwich Mean Time, and the hour angle at the end of the exposure is added to indicate the position of the telescope at the time.

The exposure of the plate to the comparison spectrum has commonly been made at the beginning and end of the star

Object	Series and Number	Date		Middle of Exposure		Duration	Hour Angle at End			Slit-Width
		1902		h	m	m		h	m	mm
Moon	A 350	July	16	14	28	40	E	0	20	0.030
ε *Pegasi*	A 364	July	31	19	15	62	W	0	45	0.032
β *Ophiuchi*	B 378	Aug.	7	15	06	77	W	1	15	0.038
ε *Pegasi*	B 379	Aug.	7	16	37	70	E	1	20	0.038
γ *Piscium*	B 381	Aug.	7	19	23	120	W	0	20	0.038
α *Persei*	B 382	Aug.	7	20	53	30	E	3	00	0.020
γ *Aquilae*	B 398	Aug.	27	17	02	100	W	2	40	0.038
Moon	B 401	Aug.	27	15	47	37	E	3	25	0.038
α *Boötis*	A 373	Sept.	6	7	43	15±	W	4	50	0.025
Moon	B 408	Sept.	13	10	13	35	W	1	32	0.038
γ *Piscium*	B 415	Oct.	8	14	38	153	E	0	05	0.038
γ *Aquilae*	B 417	Oct.	9	14	12	125	W	2	55	0.038
ε *Pegasi*	B 418	Oct.	9	16	14	93	W	2	40	0.038
α *Arietis*	B 420	Oct.	9	19	08	40	W	0	45	0.038
Moon	B 423	Oct.	15	15	44	30	E	0	46	0.028
γ *Cephei*	B 428	Oct.	16	17	44	70	W	2	25	0.038
α *Arietis*	B 430	Oct.	29	13	23	50	E	3	35	0.038
α *Persei*	B 431	Oct.	29	14	21	40	E	4	00	0.038
γ *Piscium*	B 436	Oct.	30	14	39	120	W	1	05	0.038
Moon	B 444	Nov.	6	13	38	40	W	3	15	0.032
ι *Aurigae*	B 446	Nov.	6	15	31	75	E	3	35	0.032
β *Leporis*	B 449	Nov.	6	19	55	90	W	0	25	0.038
α *Persei*	B 458	Nov.	19	13	27	33	E	3	35	0.038
γ *Cephei*	B 464	Nov.	27	10	35	75	W	4	45	0.038
α *Arietis*	B 465	Nov.	27	18	35	52	W	3	30	0.038
β *Geminorum*	B 477	Dec.	31	20	13	20	W	1	30	0.038
Mars	B 478	Dec.	31	20	56	32	E	2	25	0.038
		1903								
ε *Leonis*	B 483	Jan.	8	21	56	88	W	2	20	0.038
Moon	A 387	Jan.	16	14	28	25	E	0	30	0.038
α *Crateris*	B 491	Feb.	4	19	55	148	W	1	23	0.045
α *Bootis*	B 492	Feb.	4	21	39	20	E	1	25	0.045
β *Geminorum*	A 398	Feb.	5	19	31	18	W	3	05	0.038
Mars	A 400	Feb.	5	22	15	50	W	0	50	0.038
α *Bootis*	B 497	March	24	22	48	13	W	2	55	0.038
γ *Cephei*	A 424	April	3	15	41	68	W	11	28	0.038
β *Geminorum*	A 426	April	8	15	14	27	W	3	00	0.038
ε *Leonis*	A 427	April	8	16	11	70	W	2	20	0.038
Moon	A 429	April	8	18	35	25	W	3	05	0.038
α *Bootis*	A 433	April	8	21	46	16	W	2	50	0.030
α *Crateris*	A 438	April	16	17	24	132	W	3	18	0.038
ε *Leonis*	A 443	April	22	14	48	52	W	1	40	0.030
α *Crateris*	A 444	April	22	16	56	157	W	3	25	0.042
Mars	A 445	April	22	18	46	35	W	3	10	0.030
β *Ophiuchi*	A 451	April	30	19	55	95	E	0	20	0.038
α *Bootis*	B′ 499	May	6	14	44	20	E	2	15	0.038
γ *Cephei*	B′ 501	May	6	17	42	100	E	8	00	0.038
Mars	B′ 502	May	6	19	20	50	W	5	00	0.038
Venus	B′ 505	June	16	14	11	9	W	5	10	0.020
β *Ophiuchi*	B′ 508	June	26	15	32	83	E	1	00	0.040
γ *Aquilae*	B′ 509	June	26	17	25	110	E	0	55	0.040

Series and Number	Comparison Beginning	Comparison End	Temperature Beginning i	Temperature Beginning o	Temperature End i	Temperature End o	Seeing	Observer	Remarks
A 350	*Ti* 5	*Ti* 5	23.4	23.9	23.6	24.2	2; 1	A	Light cirrus
A 364	*Ti* 45	*Ti* 45	25.5	25.4	25.5	25.4	3; 3	A	clouds.
B 378	*Ti* 30	*Ti* 30	21.5	21.5	21.5	21.5	3-4; 3-4	F	
B 379	*Ti* 20	*Ti* 33	21.5	21.4	21.4	21.1	4; 3-4	F	
B 381	*Ti* 24	*Ti* 28	21.5	21.5	21.5	21.5	4; 4-3	F	
B 382	*Ti* 80	*Ti* 19			21.4	21.1	4; 4	F	
B 398	*Ti* 50, *Cr* 90	*Ti* 40, *Cr* 90	22.2	22.2	22.2	22.2	3; 3-4	A	Image very
B 401	*Ti* 40	*Ti* 45	22.2	22.4	....	....	3; 3	A	unsteady.
A 373		*Ti* 115	20.3	20.2	....	....	3-0; 2	A	Frequent
B 408	*Ti* 40	*Ti* 40	12.7	12.7	12.7	12.4	3; 1	A	clouds.
B 415	*Ti* 45	*Ti* 35	20.1	20.0	20.1	20.3	3; 2	A	
B 417	*Ti* 20 at 13:35	*Ti* 40	12.4	12.2	12.4	12.3	2; 2	F	
B 418	*Ti* 30	*Ti* 15	12.4	11.8	12.4	12.0	2; 2-3	F	
B 420	*Ti* 20	*Ti* 40	12.4	12.0	12.3	11.8	2; 2	F	
B 423	*Ti* 30	*Ti* 40	15.4	15.4	15.4	15.3	3; ...	A	
B 428	*Ti* 20	*Ti* 20	10.3	10.0	10.2	9.9	2-3; 2-3	F	
B 430	*Ti* 40	*Ti* 35	7.4	7.6	7.5	7.6	2; 2-1	A	
B 431	*Ti* 40	*Ti* 25	7.5	7.6	....	....	2; 2	A	
B 436	*Ti* 30	*Ti* 30	12.8	12.4	12.8	12.5	4; 4	F	
B 444	*Ti* 60		7.1	7.1	7.1	6.9	3; ...	F	
B 446	*Ti* 30	*Ti* 30	7.2	7.1	7.3	7.1	4; 4	F	
B 449	*Ti* 25	*Ti* 25	7.2	7.3	7.2	7.3	4; 4	A	
B 458	*Ti* 35	*Ti* 25	9.8	9.9	9.8	9.8	3; 3-4	A	Heavy dew on
B 464	*Ti* 25	*Ti* 30	−0.2	0.0	−0.1	+0.4	4; 3	F A	object-glass.
B 465	*Ti* 25		−0.1	−0.6	−0.1	−0.4	4; 4-3	F	Star's light
B 477	*Ti* 20	*Ti* 18	−1.2	−1.2	−1.2	−1.3	4; 2	A	cut down by
B 478	*Ti* 24	*Ti* 16	−1.3	−1.4	−1.3	−1.2	4; 1	A	snow clouds after 15 : 10.
B 483	*Ti* 25	*Ti* 25	−10.2	−10.4	−10.2	−10.3	3; 3	A	
A 387	*Ti* 20	*Ti* 20	−0.3	−0.1	−0.4	−0.8	3; ...	A	
B 491	*Ti* 15	*Ti* 15	−4.8	−4.8	−4.8	....	3; 2	F	
B 492	*Ti* 15	*Ti* 15	−5.0	....	−4.9	−4.6	3: 2	F	
A 398	*Ti* 25	*Ti* 25	−6.1	−6.0	−6.1	−6.3	3; 4	A	
A 400	*Ti* 23	*Ti* 20	−6.1	−6.4	−6.1	−6.6	3; 3	A	
B 497	*Ti* 40		0.0	0.0	....	....	2; 2	A	
A 424	*Ti* 30	*Ti* 30	1.3	1.2	1.4	1.4	3: 3	F	
A 426	*Ti* 30	*Ti* 25	14.7	14.8	14.8	14.8	3; 1	A	
A 427	*Ti* 25	*Ti* 25	14.8	14.8	14.8	14.8	3; 1-2	A	
A 429	*Ti* 25	*Ti* 25	14.8	14.9	14.8	14.5	3; ...	A	
A 433	*Ti* 32	*Ti* 30	14.8	14.9	14.8	14.8	3; 2	A	
A 438	*Ti* 20	*Ti* 25	8.3	8.0	8.2	7.8	4; 3	A	
A 443	*Ti* 30	*Ti* 35	9.7	10.5	10.2	9.7	4; 4+	F	
A 444	*Ti* 25	*Ti* 30	9.9	10.4	9.9	10.1	3; 4	F	
A 445	*Ti* 25	*Ti* 35	9.9	10.0	9.9	9.3	3; 4	F	
A 451	*Ti* 25	*Ti* 25	4.5	4.4	4.5	4.0	3-0; 3	A	Cloudy from
B′ 499		*Ti* 120	13.1	12.6	13.1	12.9	3; 3	F	13:00–
B′ 501	*Cr* 45, *Ti* 40	*Ti* 55, *Cr* 40	13.1	13.0	13.1	12.3	2-3; 2-3	F	13:35.
B′ 502	*Cr* 60, *Ti* 80		....	....	13.2	12.4	2-1; 2	F	
B′ 505	*Ti* 180, *Cr* 150		19.5	19.6	....	....	3; 2	A	
B′ 508	*Ti* 50	*Ti* 85	23.3	22.9	23.1	22.5	2; 3	F	
B′ 509	*Ti* 45	*Ti* 65	23.1	24.5	23.1	23.3	3; 3-4	F	

exposure, and the length in seconds is given by the number following the symbol of the element employed. The temperature conditions during the star exposure are indicated in the column headed "Temperature." The letter *i* refers to the thermometer within the prism box, *o* to the thermometer in the large outside temperature case. The readings are given only for the beginning and end of the exposure, as these indicate in the case of all the plates considered here the maximum range of temperature through which the air within the prism-box has passed. The last column of the table gives the estimated transparency of the sky and the steadiness and quality of the star's image, in the order named, and on a scale of 0 to 5, 5 denoting perfect conditions in each case.

The kind of photographic plate employed is not stated in the journal of observations. Previous to December 1902, Seed's "Gilt Edge 27," were used almost exclusively. Since that time the majority of photographs have been made with Seed's double-coated non-halation plates, with the outer emulsion also of the "Gilt Edge 27" quality. The principal developers used have been rodinol and edinol.

We are indebted to Mr. F. R. Sullivan, engineer in charge of the telescope, for much efficient assistance in the labor of guiding during the exposures entered in the list.

DETAILED REDUCTIONS.

The tables of detailed reductions which follow are divided into two parts, the first containing the results given by the plates of the Moon and the planets which have been taken as a check upon the behavior of the spectrograph, and the second the results for the stars whose discussion forms the subject-matter of this article. We have endeavored, so far as possible, to make the form of tabulation the same for both.

The time and hour angle given for each plate refer to the middle of the exposure. The former is expressed in Greenwich Mean Time. The first column of the table gives the wave-length employed for the stellar line, and following it in parallel columns is the velocity found for that line by each observer. The

mean for each observer is given to hundredths of a kilometer in order to avoid error in the formation of the final mean, but the latter is given only to tenths of a kilometer. On account of the great amount of repetition involved, it has not seemed desirable to give the wave-lengths of all the comparison lines used in the reduction of the plates, but the number of lines so used, as well as the wave-lengths of the standard lines employed in the determination of the constants of the reduction formula, is found at the foot of the table for each plate.

All of the velocities given, as well as the mean errors, are, of course, expressed in kilometers per second.

CONTROL PLATES OF THE MOON AND THE PLANETS.

The series of Moon and planet plates given below covers practically the whole interval over which the stellar results given in this paper extend. The first five plates of the Moon are summarized in a short table, the details of their reduction being given in Vol. II of the *Publications of the Yerkes Observatory*.

The theoretical velocities both of the Moon and of the planets have been computed with the aid of the formulæ given in so convenient a form by Professor Campbell.

One plate of *Mars*, B′502, has been excluded from the list given, as it was found on development that the glass was broken diagonally across, the photographic film alone holding the fragments together.

MOON.

Series and Number	Date	Hour Angle	Measured by	Number of Lines	Mean	Computed Velocity	O.–C.	Quality of Plate
A 350	1902, July 16, $14^h\ 28^m$	E $0^h\ 42^m$	A.	13	+0.4	+0.4	0.0	Fair
B 401	Aug. 27, 21 47	E 3 44	A.	14	−1.1	−1.5	+0.4	Fair
B 408	Sept. 13, 16 13	W 1 14	A.	12	+0.1	+0.2	−0.1	Good
B 423	Oct. 15, 15 44	E 1 01	A.	13	−0.6	−0.4	−0.2	Good
B 444	Nov. 6, 13 38	W 2 54	A.	14	+0.7	+0.6	+0.1	Good

MARS—B 478.

1902, December 31, $20^h 56^m$. Hour angle, E $2^h 41^m$. Measured by A.
Planet spectrum a trifle weak; comparison spectrum weak.

Line, Wave-Length in Sun	Velocity	Line, Wave-Length in Sun	Velocity
4468.663	−17.11	4565.750	−17.07
82.376	14.94	81.634	17.27
4501.422	17.92	84.018	17.66
28.798	16.49	4611.455	19.44
48.938	18.06		
54.211	15.74		
63.939	18.33		(11)
Number of comparison lines	9		
Ti standards used	4468.663 4534.139 4617.452	Mean Computed velocity	−17.3 −17.1

MOON—A 387.

1903, January 16, $20^h 40^m$. Hour angle, E $0^h 42^m$.
Moon spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4427.420	+0.14	−1.56	4494.738	−3.60	−1.27
42.510	−1.15	−1.82	97.046	−3.60	−1.53
43.976		−1.62	4501.422	−2.13	−1.33
47.892	+0.94	0.00	22.853		+0.27
56.030	−1.95		28.798	+0.33	−0.73
57.656	−2.49	+0.87	34.168		−1.19
68.663	−0.40		54.211	−1.51	
76.214	−0.07				
82.376		−0.80		(12)	(12)
No. of compar. lines	11	11	Means	−1.29	−0.89
Ti standards used	4427.266 4481.438 4555.662	4427.266 4481.438 4527.490	Mean Computed velocity	−1.1 −0.6	

MARS—A 400.

1903, February 15, $22^h\ 15^m$. Hour angle, W $0^h\ 25^m$.

Planet spectrum fair; comparison spectrum fair.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4427.420	−15.85	−16.46	4494.738	−16.21	−15.61
42.510	14.31	14.65	4501.422	14.32	16.12
43.976	12.69	13.97	28.798	14.37	15.03
47.892	13.48	14.56	34.168		15.21
56.030	14.47		54.211	19.62	
57.656	15.20	15.40			
68.663	13.76	12.55			
76.214	10.38				
82.376		13.98			
90.900		16.09		(12)	(12)
No. of compar. lines	13	9	Means	−14.56	−14.97
Ti standards used	4427.266 4481.438 4555.662	4427.266 4481.438 4527.490	Mean −14.8 Computed velocity −15.5		

MOON—A 429.

1903, April 8, $18^h\ 35^m$. Hour angle W $2^h\ 53^m$. Measured by A.

Moon spectrum fair; comparison spectrum good.

Line, Wave-Length in Sun	Velocity	Line, Wave-Length in Sun	Velocity
4427.420	+2.91	4482.376	+2.74
42.510	2.16	97.046	2.00
47.892	2.97	4501.422	0.60
56.030	1.55	17.702	3.19
57.656	2.35	28.798	0.20
68.663	0.00		
76.214	0.54		(12)
Number of comparison lines...	10		
Ti standards used.........	4427.266 4481.438 4527.490	Mean +1.8 Computed velocity +1.3	

MARS—A 445.

1903, April 22, 18ʰ 46ᵐ. Hour angle W 2ʰ 52ᵐ.

Planet spectrum fair; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+4.13	+3.39	4476.214	+7.70	+6.10
35.184	5.81		82.376	7.29	6.49
41.881		6.21	94.738	3.94	
42.510	5.87	5.87	97.046	1.53	3.20
43.976		7.02	4501.422		3.73
47.892	5.19	4.18	15.475	3.92	6.57
56.030	5.18	4.78	26.644		6.82
57.656		5.45	28.798	6.42	4.37
59.304	5.31		54.211	7.04	
68.063	2.64	3.22		(14)	(15)
No. of compar. lines	14	11	Means	+5.14	+5.16
Ti standards used	4427.266 4496.318 4555.662	4427.266 4481.438 4527.490	Mean +5.2 Computed velocity +5.2		

VENUS—B 505.

1903, June 16, 14ʰ 11ᵐ. Hour angle W 5ʰ 6ᵐ. Measured by A.

Planet spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity	Line, Wave-Length in Sun	Velocity
4427.420	−13.39	4482.376	−11.44
42.510	11.53	94.738	14.01
43.976	11.32	97.046	13.07
47.892	11.58	4501.422	11.99
56.030	12.57	26.644	13.99
57.656	13.31	28.798	14.05
68.663	14.55		
76.214	11.72		(14)
Number of comparison lines...	10		
Ti standards used.........	4427.266 4481.438 4527.490	Mean −12.8 Computed velocity −13.2	

The mean value of O.−C. for the above eleven plates taken with regard to sign is +0.10 km. A quantity of this size might easily be due to accidental causes, so that the conclusion may

be drawn that the control plates show no evidence of the existence of any systematic error in the spectrograph.

STANDARD VELOCITY STARS.

The following tables give the results of our measures upon the plates of standard velocity stars. The ten stars of the principal list are arranged in order of right ascension, followed by three stars which we have observed in the supplementary list.

In addition to the data given for the control plates, we append, for the star plates, the mean error ϵ of the determination of the velocity from a single line for each observer, and the mean error ϵ_0 of the final velocity of the star deduced by each observer from one plate, viz.:

$$\epsilon = \pm\sqrt{\frac{\Sigma v^2}{n-1}}\,, \quad \text{and} \quad \epsilon_0 = \pm\sqrt{\frac{\Sigma v^2}{n(n-1)}}\,.$$

The reduction of the star's velocity to the Sun is given in its two parts, V_a denoting the correction due to the Earth's orbital motion, and V_d the correction due to the Earth's diurnal rotation. Schlesinger's tables of star constants for the determination of V_a have been employed throughout.

Reference should be made to the fact that all of the measurable plates obtained are included in the reductions given below, and that none have been omitted because of inferior quality or lack of agreement with other plates.

α *ARIETIS* — B 420.

1902, October 9, $19^h\ 08^m$. Hour angle W 0^h 25

Star spectrum very good; comparison spectrum very good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−23.03	−23.37	4482.376	−21.34	−22.82
28.711		21.53	94.738	23.68	24.02
41.881		23.70	97.046	22.40	23.74
42.510	23.42	22.89	4501.422	24.25	23.58
43.976		23.62	12.906	21.33	
47.892	22.38		26.644		23.12
56.030	24.09	22.01	28.798	22.12	23.64
60.460		23.20	48.938	23.40	
68.663	21.20		54.211	24.29	
76.214	23.65	21.78		(14)	(15)
No. of compar. lines	12	11	Means	−22.90	−23.07
ϵ	±1.10	±0.78	Mean	−22.98	
ϵ_0	±0.29	±0.21	V_a	+9.70	
Ti standards used	4427.266	4427.266	V_d	−0.03	
	4481.438	4481.438	Reduction to Sun	+ 9.67	
	4527.490	4527.490	Radial velocity	−13.3	

α *ARIETIS* — B 430.

1902, October 29, $13^h\ 23^m$. Hour angle E $4^h\ 0^m$.

Star spectrum very good; comparison spectrum very good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−13.82	−13.75	4499.900	−16.49	
28.711		15.23	94.738	15.41	−16.28
42.510	12.02	13.64	97.046	11.14	11.80
43.976		13.77	4501.422		15.59
47.892	12.81	13.14	12.906	16.15	16.01
57.656	12.65	11.70	15.475	14.67	
60.460	14.05	16.13	17.702		15.46
68.663	10.06	12.41	26.644	13.38	
76.214	14.40	16.48	28.798	11.98	12.12
82.376	13.65	12.38			
				(15)	(16)
No. of compar. lines	12	11	Means	−13.51	−14.12
ϵ	±1.80	±1.74	Mean	−13.82	
ϵ_0	±0.46	±0.44	V_a	−0.14	
Ti standards used	4427.266	4427.266	V_d	+0.27	
	4481.438	4481.438	Reduction to Sun	+ 0.13	
	4527.490	4527.490	Radial velocity	−13.7	

α *ARIETIS* — B 465.

1902, November 27, $18^h\ 35^m$. Hour angle W $3^h\ 4^m$.

Star spectrum very good; comparison spectrum very good.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4427.420	+0.07	+1.08	4482.376	+2.27	+2.21
28.711		−0.20	94.738	−2.13	−0.87
41.881		−0.47	97.046	+0.73	+1.00
42.510	+1.69	+2.23	4501.422	−0.80	+0.60
43.976		−0.74	12.906	+2.53	+1.26
47.892	−0.13	0.00	28.798	+2.05	+0.86
56.030	+1.35	+0.54	54.211	+0.07	
57.656	+0.81	+2.35			
60.460		−0.87			
68.663	+1.68	+2.42			
76.214	+0.94	+0.40		(14)	(17)
No. of compar. lines	13	11	Means	+0.80	+0.69
ϵ	±1.29	±1.14	Mean	+ 0.74	
ϵ_0	±0.34	±0.28	V_a	−14.58	
Ti standards used	4427.266	4427.266	V_d	− 0.23	
	4481.438	4481.438	Reduction to Sun	−14.81	
	4555.662	4527.490	Radial velocity	−14.1	

α *PERSEI* — B 382.

1902, August 7, $20^h\ 53^m$. Hour angle E $3^h\ 15^m$.

Star spectrum fair but unsymmetrical; comparison spectrum fair.

4443.976		−24.36	4515.508	−25.16	−25.76
47.892		27.71	20.397	26.54	
59.301	−26.02		22.802	26.19	24.40
66.727	25.04	26.52	28.798	27.41	28.01
68.663	25.50	24.56	54.211	26.53	25.61
76.214	27.47	28.07	58.827		23.74
89.351	28.99		63.939		23.65
91.570	24.97		72.156		25.18
94.738	31.02	28.49			
4501.445	26.24	27.18			
08.455	27.93	28.20		(14)	(15)
No. of compar. lines	12	15	Means	−26.79	−26.10
ϵ	±1.68	±1.74	Mean	−26.44	
ϵ_0	±0.44	±0.45	V_a	+24.48	
Ti standards used	4455.485	4443.976	V_d	+ 0.17	
	4512.906	4501.445	Reduction to Sun	+24.65	
	4555.662	4563.939	Radial velocity	− 1.8	

α *PERSEI* — B 431.

1902, October 29, $14^h\ 21^m$. Hour angle E $4^h\ 20^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4443.976		−11.27	4508.455	−12.17	−11.24
44.728		12.68	15.508	11.69	10.82
47.892		10.85	20.397	12.88	13.80
59.301	−14.32		22.802	11.74	
66.727	14.23	15.30	28.798	13.18	13.91
68.663	11.94	11.07	34.139		10.71
76.214	10.45	12.06	39.759		13.74
94.738	12.61	13.61	54.211	11.39	13.10
4501.445	12.59	13.19		(12)	(15)
No. of compar. lines	12	12	Means	−12.43	−12.49
ϵ	±1.13	±1.44	Mean	−12.46	
ϵ_0	±0.32	±0.37	V_a	+10.60	
Ti standards used	4457.600	4449.313	V_d	+ 0.20	
	4512.906	4501.445	Reduction to Sun	+10.80	
	4555.662	4555.662	Radial velocity	− 1.7	

α *PERSEI* — B 458.

1902, November 19, $13^h\ 27^m$. Hour angle E $3^h\ 51^m$.

Star spectrum good; comparison spectrum slightly strong.

4427.420		−2.51	4515.508	−4.25	−2.92
43.976		2.36	20.397	6.63	6.24
44.728		6.61	22.802	3.45	
47.892		7.15	28.798	7.02	6.69
59.301	−2.02		34.139		3.37
66.727		4.23	41.690		2.51
68.663	4.50	4.23	54.211	6.52	2.70
76.214	7.17		58.827	4.41	5.99
89.351	3.81		63.939	4.73	3.68
91.570	2.80		72.156	4.98	
94.738		4.20			
4501.445	5.46	3.40			
08.455	4.99	6.98		(15)	(17)
No. of compar. lines	11	13	Means	−4.85	−4.46
ϵ	±1.47	±1.75	Mean	−4.66	
ϵ_0	±0.37	±0.42	V_a	+1.48	
Ti standards used	4457.600	4427.266	V_d	+0.19	
	4512.906	4481.438	Reduction to Sun	+1.67	
	4563.939	4555.662	Radial velocity	−3.0	

β LEPORIS—B 449.

1902, November 6, $19^h 55^m$. Hour angle E $0^h 20^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−24.04	−26.75	4501.422	−24.58	−22.98
42.510		23.56	08.455	25.41	25.21
43.976		23.55	15.475	25.30	26.83
47.892	22.92	22.65	25.285 B	26.50	
56.030		26.18	26.644		26.23
57.656	25.70	24.55	28.798	22.84	23.64
68.663	23.89	23.82	46.129	23.74	
76.214	24.19	23.45	54.211	24.56	
82.376	23.48	23.75			
94.738		26.49			
97.046	21.40	22.87		(14)	(16)
No. of compar. lines	11	11	Means	−24.18	−24.53
ϵ	±1.32	±1.50	Mean	−24.36	
ϵ_0	±0.35	±0.37	V_a	+11.95	
Ti standards used	4427.266	4427.266	V_d	+ 0.03	
	4481.438	4481.438	Reduction to Sun	+11.98	
	4555.662	4527.490	Radial velocity	−12.4	

β GEMINORUM—B 477.

1902, December 31, $20^h 13^m$. Hour angle W $1^h 20^m$.

Star spectrum good; comparison spectrum weak.

4427.420	−3.12		4522.853	−1.86	
42.510	3.44	−2.02	28.798	2.45	−2.32
43.976		1.15	34.168	2.18	1.92
47.892		1.75	47.196	2.51	
56.030		2.49	48.938	5.08	
57.656	2.09	1.68	54.211	3.23	2.30
60.460		1.01	63.939	3.28	3.55
68.663	2.89	2.48	71.275	5.90	2.36
76.214		2.55			
82.376		3.14			
97.046	3.87	4.33			
4501.422	5.33	3.86		(14)	(16)
No. of compar. lines	12	10	Means	−3.37	−2.43
ϵ	±1.28	±0.92	Mean	−2.90	
ϵ_0	±0.34	±0.23	V_a	+6.30	
Ti standards used	4427.266	4443.976	V_d	−0.10	
	4501.445	4501.445	Reduction to Sun	+6.20	
	4563.939	4563.939	Radial velocity	+3.3	

β *GEMINORUM* — A 398.

1903, February 5, $19^h\ 31^m$. Hour angle W $2^h\ 56^m$

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4399.903		+13.83	4460.460		+15.60
4406.810		16.47	68.663	+14.29	16.64
07.851 B		17.14	76.214	14.27	
08.549		16.39	82.376	15.45	16.66
11.240		14.55	94.738		15.68
27.420	+15.58	17.95	97.046	15.60	14.94
42.510	16.60	16.33	4501.422	14.45	15.39
43.976		14.85	12.906	15.08	17.61
47.892	16.45	14.83	22.853	17.30	
56.030		17.83	28.798	17.28	16.03
59.304	15.80			(12)	(18)
No. of compar. lines	10	13	Means	+15.68	+16.04
ϵ	±1.07	±1.18	Mean	+15.86	
ϵ_0	±0.31	±0.28	V_a	−12.16	
Ti standards used	4427.266	4399.935	V_d	−0.21	
	4481.438	4457.600	Reduction to Sun	−12.37	
	4527.490	4512.906	Radial velocity	+3.5	

β *GEMINORUM* — A 426.

1903, April 8, $15^h\ 14^m$. Hour angle W $2^h\ 46^m$.

Star spectrum very good; comparison spectrum very good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+31.77	+32.23	4468.663	+31.74	+32.74
28.711		33.53	76.214	33.90	32.09
35.184	33.47		82.376	35.19	34.39
41.881		31.80	97.046	33.41	34.07
42.510	32.94	33.95	4501.422	31.11	31.84
43.976	33.40	33.47	22.853	34.74	
47.892	32.09	33.23	28.798	32.24	34.04
56.030	32.64	31.96	47.196	32.97	
57.656		35.32	54.211	34.24	
				(15)	(14)
No. of compar. lines	15	10	Means	+33.06	+33.19
ϵ	±1.16	±1.10	Mean	+33.12	
ϵ_0	±0.30	±0.30	V_a	−29.42	
Ti standards used	4427.266	4427.266	V_d	−0.20	
	4481.438	4481.438	Reduction to Sun	−29.62	
	4552.632	4527.490	Radial velocity	+3.5	

α CRATERIS—B 491.

1903, February 4, 19 h 55 m. Hour angle W 0 h 9 m.

Star spectrum too weak; comparison spectrum strong.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4457.656		+ 30.49	4565.750		+ 31.59
68.663		32.49	71.275		31.10
76.214		31.30	81.634		30.18
97.046		29.15	84.018		29.83
4500.451		29.99	90.126	+ 29.33	30.58
01.422		32.52	91.614 B		29.52
15.475		31.62	4611.455	27.50	30.04
28.798	+ 31.45	33.85	25.227	33.78	
46.129	26.64		26.358	27.67	
54.211	29.96		54.743 B	31.44	
60.225		31.70	73.370 B	27.08	
63.939	29.56			(11)	(16)

	F.	A.			
No. of compar. lines	10	11	Means	+ 29.44	+ 31.00
ϵ	± 2.18	± 1.26	Mean	+ 30.22	
ϵ_0	± 0.66	± 0.32	V_a	+ 16.66	
Ti standards used	4527.490	4457.600	V_d	− 0.01	
	4617.452	4527.490	Reduction to Sun	+ 16.65	
	4682.088	4617.452	Radial velocity	+ 46.9	

α CRATERIS—A 438.

1903, April 16, 17 h 24 m. Hour angle W 2 h 12 m.

Star spectrum weak; comparison spectrum strong.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+ 61.84		4497.046	+ 60.48	+ 60.21
41.881		+ 61.23	4501.422		60.62
42.510		62.91	15.475	59.23	61.69
47.892		62.08	22.853	66.83	
57.656	60.61	61.42	28.798	63.36	62.78
60.460		63.40	47.196	62.44	
68.663		63.01	48.938		62.89
76.214		63.58	54.211	65.71	61.44
82.376	61.75		63.939		60.58
85.846		61.83	65.750		62.12
94.738	57.50				
				(10)	(16)

	F.	A.			
No. of compar. lines	9	13	Means	+ 61.98	+ 61.99
ϵ	± 2.82	± 1.04	Mean	+ 61.98	
ϵ_0	± 0.89	± 0.26	V_a	− 14.52	
Ti standards used	4427.266	4449.313	V_d	− 0.18	
	4496.318	4496.318	Reduction to Sun	− 14.70	
	4555.662	4555.662	Radial velocity	+ 47.3	

α *CRATERIS* — A 444.

1903, April 22, $16^{h}\ 56^{m}$. Hour angle W $2^{h}\ 6^{m}$.

Star spectrum weak; comparison spectrum strong.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4427.420	+69.22		4512.906	+63.53	+65.32
35.184	65.65		28.798	65.68	62.18
45.641		+64.14	44.845 B		62.09
47.892	69.16	65.86	47.196	61.72	
60.460		64.07	63.939		64.91
68.663	67.77	65.42	71.275		62.91
76.214	65.26	63.32	81.634		64.08
82.376	64.29	64.96	86.191 B	67.86	
90.900		63.36	4606.404		64.48
97.046	66.01		11.455		64.37
4501.422	63.94	62.81		(12)	(16)
No. of compar. lines	12	14	Means	+65.84	+64.02
ϵ	± 2.32	± 1.15	Mean	+64.93	
ϵ_0	± 0.67	± 0.30	V_a	−16.73	
Ti standards used	4427.266	4427.266	V_d	− 0.17	
	4512.906	4512.906	Reduction to Sun		−16.90
	4590.126	4617.452	Radial velocity		+48.0

α *BOÖTIS*—A 373

1902, September 6, $13^{h}\ 43^{m}$. Hour angle W $4^{h}\ 38^{m}$.

Star spectrum excellent; comparison spectrum excellent.

4427.420	+10.63	+10.84	4482.376	+11.84	+10.90
28.711		11.92	94.738	10.67	11.81
34.021	13.59		97.046	11.47	10.40
35.851		11.15	4501.422		10.39
42.510	12.28	11.05	12.906	12.03	10.76
43.976	10.32	11.88	15.475		11.69
47.892	12.20	11.93	28.798	11.06	11.26
56.030	11.17	11.24	48.938	13.45	
57.056	11.30		54.211	12.77	
66.701		11.61			
68.663	13.62				
75.026		10.40			
76.214	12.06	11.59		(16)	(17)
No. of compar. lines	17	11	Means	+11.90	+11.28
ϵ	±1.05	±0.57	Mean	+11.59	
ϵ_0	±0.26	±0.14	V_a	−15.79	
Ti standards used	4427.266	4427.266	V_d	− 0.30	
	4496.318	4465.975	Reduction to Sun		−16.09
	4552.632	4527.490	Radial velocity		− 4.5

a *BOÖTIS*—B 492

1903, February 4, $21^h\ 39^m$. Hour angle E $1^h\ 35^m$.

Star spectrum good; comparison spectrum weak.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4427.420	−31.02	−29.60	4501.422	−27.31	−26.64
41.881		30.58	12.906	25.98	26.31
42.510	27.61	27.47	26.644		29.41
43.976		28.95	28.798	29.93	30.26
47.892	27.10		46.129	30.60	30.08
56.030	29.06	27.72	48.024		29.66
57.656	29.33	28.72	54.211	29.89	
68.663	26.84	26.64	63.939	29.63	
76.214	28.01	28.61	65.750		28.76
82.376		27.30		(13)	(16)
No. of compar. lines	13	12	Means	−28.64	−28.54
ϵ	±1.59	±1.38	Mean	−28.59	
ϵ_0	±0.44	±0.34	V_a	+24.03	
Ti standards used	4427.266	4427.266	V_d	0.13	
	4501.445	4501.445	Reduction to Sun	+24.16	
	4563.939	4563.939	Radial velocity	− 4.4	

a *BOOTIS*—B 497.

1903, March 24, $22^h\ 48^m$. Hour angle W $2^h\ 49^m$

Star spectrum very good; comparison spectrum very good.

Line, Wave-Length in Sun	F.	A.	Line, Wave-Length in Sun	F.	A.
4427.420	−13.82	−13.81	4512.906	−12.82	−14.09
28.711	13.00		18.198		14.93
41.881		14.78	28.798	13.18	13.97
42.510	13.16	12.29	34.953	14.74	
47.892	14.56	13.55	46.129	16.49	
56.030	15.75	14.53	48.024	13.19	
57.656	14.80	12.44	48.938	11.40	
68.663	13.02	12.41	54.211	14.09	
76.214	14.40	13.53	71.275	15.54	
82.376	13.38	13.05			
94.738	15.14	14.68			
97.046	13.54	12.27			
4501.422	16.79	14.45		(20)	(15)
No. of compar. lines	17	12	Means	−14.14	−13.65
ϵ	±1.35	±0.96	Mean	−13.90	
ϵ_0	±0.30	±0.25	V_a	+8.85	
Ti standards used	4427.266	4427.266	V_d	−0.22	
	4481.438	4481.438	Reduction to Sun	+ 8.63	
	4555.662	4527.490	Radial velocity	− 5.3	

α BOÖTIS—A 433.

1903, April 8, $21^h\ 46^m$. Hour angle W $2^h\ 42^m$.

Star spectrum excellent; comparison spectrum excellent.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4427.420	−7.25	−7.45	4501.422	−7.73	−7.46
28.711		9.14	12.063		8.51
41.881		8.24	12.906	3.79	
42.510	7.42	7.36	22.853	4.38	
43.976	6.61	6.88	28.798	4.37	6.89
47.892	8.43	7.21	46.129	8.11	
56.030	8.01	7.27	48.024	5.34	
57.656	7.13	7.27	48.938	8.11	
68.663	5.03	5.90	54.211	6.06	
76.214	6.83	7.84			
82.376	7.09	7.16			
94.738	9.01	7.74			
97.046	7.93	6.47		(19)	(16)
No. of compar. lines	15	12	Means	−6.77	−7.42
ϵ	±1.53	±0.96	Mean	−7.10	
ϵ_0	±0.35	±0.25	V_a	+2.43	
Ti standards used	4427.266	4427.266	V_d	−0.21	
	4496.318	4481.438	Reduction to Sun	+2.22	
	4555.662	4527.490	Radial velocity	−4.9	

α BOÖTIS—B′ 499.

1903, May 6, $14^h\ 44^m$. Hour angle E $2^h\ 25^m$.

Star spectrum excellent; comparison spectrum slightly weak.

Line, Wave-Length in Sun	F.	A.	Line, Wave-Length in Sun	F.	A.
4427.420	+3.12	+2.51	4501.422	+2.46	+4.66
41.881		2.77	12.906	5.98	
42.510	5.94	4.72	26.644	5.23	3.91
43.976	6.55	5.40	28.798	6.16	4.70
47.892	3.71	3.37	34.953	2.84	
56.030	4.10	3.70	46.129	3.89	
57.656	3.90	4.10	48.024	4.48	
60.460		4.91	48.938	3.43	
68.663	6.04	4.70	54.211	1.71	
76.214	3.48	2.08	63.939	4.73	
82.376	5.02	4.42	71.275	3.15	
94.738		2.94		(20)	(15)
No. of compar. lines	18	10	Means	+4.30	+3.93
ϵ	±1.37	±1.00	Mean	+4.12	
ϵ_0	±0.31	±0.26	V_a	−9.36	
Ti standards used	4427.266	4427.266	V_d	+0.19	
	4501.445	4481.438	Reduction to Sun	−9.17	
	4572.156	4527.490	Radial velocity	−5.1	

β OPHIUCHI—B 378.

1902, August 7, $15^h\ 6^m$. Hour angle W $0^h\ 37^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+ 8.53	+ 8.53	4501.422		+ 8.33
42.510	10.80	10.12	12.906	+ 8.31	9.83
43.976		10.19	17.702		11.55
47.892	10.85	9.24	26.644	9.21	
56.030		11.17	27.518	10.53	
57.656	9.82	9.55	28.798	6.95	10.13
59.922		10.35	46.129	10.42	
68.663	10.67	7.52	48.024	9.76	
76.214	8.04	11.12	48.938	9.29	
82.376	9.43	8.36	54.211	8.49	
94.738		8.01			
97.046	8.94	10.34		(16)	(16)
No. of compar. lines	11	11	Means	+ 9.38	+ 9.65
ϵ	±1.13	±1.21	Mean	+9.52	
ϵ_0	±0.28	±0.30	V_a	−20.20	
Ti standards used	4427.266	4427.266	V_d	− 0.05	
	4481.438	4481.438	Reduction to Sun	−20.25	
	4555.662	4527.490	Radial velocity	−10.7	

β OPHIUCHI—A 451.

1903, April 30, $19^h\ 55^m$. Hour angle E $1^h\ 8^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−29.26	−30.21	4468.663	−29.46	−28.85
35.184	28.34		76.214	30.22	29.48
41.881		31.53	82.376	28.50	29.03
42.510	32.87	28.35	94.738	35.89	30.36
43.976		30.10	96.222 B	27.77	
47.892	31.14	28.58	97.046	27.67	29.61
56.030	29.94	31.36	4501.422	30.77	30.44
57.656	31.35	30.14	28.798	30.46	31.05
60.460	30.39	28.37		(15)	(15)
No. of compar. lines	12	11	Means	−30.27	−29.83
ϵ	±2.11	±1.05	Mean	−30.05	
ϵ_0	±0.54	±0.27	V_a	+18.46	
Ti standards used	4427.266	4427.266	V_d	+ 0.10	
	4481.438	4481.438	Reduction to Sun	+18.56	
	4527.490	4527.490	Radial velocity	− 11.5	

β OPHIUCHI — B′ 508.

1903, June 26, $15^{h}\ 32^{m}$. Hour angle E $1^{h}\ 42^{m}$.

Star spectrum good; comparison spectrum fair.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4427.420	−9.28	−7.45	4501.422		−6.06
43.976		6.55	12.063		7.44
47.892	5.93	6.00	12.906	−6.64	7.31
56.030	8.41	6.46	17.702		4.45
57.656	6.53	7.27	27.518	5.03	
59.922		6.45	28.798	6.62	6.22
68.663	3.62	4.29	46.129	7.12	
76.214	7.91	6.90	48.024	4.55	
82.376	6.15	6.36	48.938	7.67	
94.738	7.60	5.87	54.221	7.18	
97.046	9.27			(16)	(15)
No. of compar. lines	14	12	Means	−6.84	−6.34
ϵ	±1.58	±0.96	Mean	−6.59	
ϵ_0	±0.39	±0.25	V_a	−4.58	
Ti standards used	4427.266	4427.266	V_d	+0.15	
	4481.438	4481.438	Reduction to Sun	−4.43	
	4555.662	4527.490	Radial velocity	−11.0	

γ AQUILAE — B 398.

1902, August 27, $17^{h}\ 2^{m}$. Hour angle W $1^{h}\ 50^{m}$.

Star spectrum weak; comparison spectrum too strong.

4456.030	+12.38		4528.798	+13.97	
57.656	14.53		46.129	14.57	+14.51
68.663	15.10	+14.43	48.024	15.62	
76.214	12.19	11.19	48.938		10.09
82.376		10.04	54.211	12.25	
85.846		10.83	60.225		13.55
97.046	13.34	14.27	65.750		14.25
4500.451		14.39	74.899		13.44
01.422	12.32		4603.126		12.31
12.906	13.22	11.50	11.455		13.07
15.475	13.28				
27.518	14.57	14.50		(13)	(15)
No. of compar. lines	13	15	Means	+13.64	+12.82
ϵ	±1.17	±1.70	Mean	+13.23	
ϵ_0	±0.32	±0.44	V_a	−14.49	
Ti standards used	4455.485	4481.438	V_d	−0.16	
	4497.023[1]	4548.938	Reduction to Sun	−14.65	
	4555.662	4617.452	Radial velocity	−1.4	

[1] *Cr.*

γ *AQUILAE* — B 417.

1902, October 9, 14^h 12^m. Hour angle W 1^h 52^m.

Star spectrum slightly weak; comparison spectrum good.

Line, Wave Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+22.28		4501.422		+20.05
28.711		+21.53	12.906	+22.39	22.46
34.021	24.01		28.798	23.64	22.98
41.881		21.67	46.129		23.61
42.510	23.08	21.60	47.196	23.14	
47.892	23.12	22.99	48.938		21.75
56.030	22.61	21.33	56.202 B		22.18
68.663	26.64		60.225		21.77
76.214	22.24	22.91	63.939		21.42
82.376	23.15	23.15	65.750		21.28
97.046	23.80	23.60			
				(12)	(17)
No. of compar. lines	13	14	Means	+23.34	+22.13
ϵ	±1.19	±0.96	Mean	+22.74	
ϵ_1	±0.34	±0.23	V_a	−24.85	
Ti standards used	4427.266	4427.266	V_d	− 0.16	
	4481.438	4481.438	Reduction to Sun	−25.01	
	4544.864	4555.662	Radial velocity	− 2.3	

γ *AQUILAE* — B 509.

1903, June 26, 17^h 25^m. Hour angle E 1^h 50^m.

Star spectrum good; comparison spectrum good.

Line, Wave Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−11.85		4512.906	−13.56	12.76
29.366	12.58		17.702		12.34
42.510	9.99		26.644		13.58
43.976	12.02		28.798	11.32	12.05
47.892	12.40		46.129	12.99	12.33
56.030	14.06		48.024	13.71	12.99
57.656	11.50		48.938	12.20	12.85
68.663	8.33	−11.00	54.211	12.90	
76.214	13.80	12.66	63.939	12.02	10.51
82.376	14.58	13.31	71.275	15.68	14.76
97.046	11.80	11.94	77.356		14.54
4501.422	12.92	13.79			
12.063		−15.02			
				(20)	(16)
No. of compar. lines	20	13	Means	−12.51	−12.90
ϵ	±1.57	±1.28	Mean	−12.70	
ϵ_1	±0.34	±0.32	V_a	+10.70	
Ti standards used	4427.266	4465.975	V_d	+ 0.15	
	4496.318	4527.490	Reduction to Sun	+10.85	
	4590.126	4590.126	Radial velocity	− 1.8	

ε PEGASI—A 364.

1902, July 31, $19^h\ 15^m$. Hour angle W $0^h\ 14^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−3.52		4497.046	−4.53	−5.20
34.021	4.67		4500.451		2.27
42.510	2.57		01.422	2.93	4.26
45.641		−4.38	05.003		3.86
47.892	2.97		12.906	3.26	3.06
49.313		4.78	27.518		5.30
56.030	5.99		28.798	3.64	3.11
57.656	4.91		46.129	5.28	5.14
59.922	4.24		48.938	6.00	5.47
68.663	1.14	2.35	65.750		5.45
76.214	4.02	5.76	74.899		2.43
82.376		2.81	90.126		3.85
88.363 B	4.21				
				(16)	(17)
No. of compar. lines	17	14	Means	−3.99	−4.09
ϵ	±1.28	±1.22	Mean	−4.04	
ϵ_0	±0.32	±0.30	V_a	+10.17	
Ti standards used	4427.266	4449.313	V_d	−0.02	
	4481.438	4512.906	Reduction to Sun		+10.15
	4544.864	4590.126	Radial velocity		+6.1

ε PEGASI—B 379.

1902, August 7, $16^h\ 37^m$. Hour angle E $1^h\ 55^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+0.47		4512.906	−2.26	−2.46
42.510	+0.67	+0.74	15.475	−3.85	−2.46
47.892	−0.34	−1.08	28.978	−1.26	
56.030	−1.61		34.168		−1.39
57.656	−1.88		46.129	−2.37	−1.58
68.663	0.00	−0.13	48.024	−0.33	
76.214	−1.68	−1.41	48.938	−2.24	−0.73
82.376	−3.01	+0.27	63.939	−0.20	−0.85
97.046		−0.53	65.750		−2.43
97.842	−0.20		71.275	−0.98	−2.76
4501.422	−1.73	−0.47		(18)	(15)
No. of compar. lines	17	13	Means	−1.27	−1.15
ϵ	±1.24	±1.06	Mean	−1.21	
ϵ_0	±0.29	±0.27	V_a	+7.17	
Ti standards used	4427.266	4443.976	V_d	+0.16	
	4481.438	4512.906	Reduction to Sun		+7.33
	4544.864	4572.156	Radial velocity		+6.1

ε PEGASI — B 418.

1902, October 9, 16^h 14^m. Hour angle W 1^h 54^m.

Star spectrum a trifle faint; comparison spectrum fair.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4442.510		+27.88	4527.518	+27.82	+26.82
57.656	+26.64		28.798	26.29	
59.922	26.09		34.168		27.44
60.460		26.62	44.077 B		24.81
68.663	28.72	28.58	46.129	25.65	27.64
76.214	27.87	28.74	47.196	25.51	
90.900		25.36	48.024	27.23	
95.664 B		25.15	48.938	26.96	27.29
4501.422		24.98	65.750		26.14
12.906	24.85	25.52		(11)	(14)
No. of compar. lines	12	15	Means	+26.69	+26.64
ϵ	±1.16	±1.34	Mean	+26.66	
ϵ_0	±0.34	±0.36	V_a	−19.98	
Ti standards used	4457.600	4449.313	V_d	−0.16	
	4512.906	4512.906	Reduction to Sun	−20.14	
	4552.632	4555.662	Radial velocity	+ 6.5	

γ PISCIUM — B 381.

1902, August 7, 19^h 23^m. Hour angle E 0^h 40^m.

Star spectrum weak; comparison spectrum fair.

Line, Wave-Length in Sun	F.	A.	Line, Wave-Length in Sun	F.	A.
4427.420	−28.85		4497.046	−28.07	−29.54
42.510	25.25		4501.422	26.64	27.04
43.976		−29.22	08.455		28.40
47.892	26.90	27.30	12.906	26.58	
56.030	26.98	27.59	15.475	29.55	
57.656	30.94	29.67	26.644		27.69
68.663	27.24	29.26	28.798	28.34	27.75
76.214	25.80	29.95	46.129	26.91	
82.376	27.23	27.43	48.024	27.88	28.02
94.738	27.62	29.02	54.211		29.30
				(16)	(15)
No. of compar. lines	13	11	Means	−27.55	−28.48
ϵ	±1.41	±0.99	Mean	−28.02	
ϵ_0	±0.35	±0.26	V_a	+16.59	
Ti standards used	4427.266	4449.313	V_d	+ 0.06	
	4481.438	4501.445	Reduction to Sun	+16.65	
	4544.864	4552.632	Radial velocity	−11.4	

γ *PISCIUM*—B 415.

1902, October 8, $14^h\ 38^m$. Hour angle E $1^h\ 21^m$.

Star spectrum weak; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4441.881		+0.14	4526.644	+2.52	+2.98
42.510		+3.44	28.798	+6.42	+4.17
47.892		+0.88	46.129	+1.91	+4.35
56.030		−1.21	48.938		+1.38
57.656	+2.15		54.211	+1.32	
59.304	+1.82		58.827	+0 07	
68.663	+4.16	+2.15	65.750		−1.12
76.214	+2.34	+2.21	71.275		−0.98
82.376	+2.14	+2.94	72.156	−2.56	−2.16
94.738	+1.33				
97.046	+4.00	+3.87	81.634		+3.73
4501.422	+3.20	+4.06	90.126	−1.31	
4512.906	+3.12			(16)	(17)
No. of compar. lines	12	13	Means	+2.20	+1.81
ϵ	±1.94	±2.19	Mean	+2.00	
ϵ_0	±0.48	±0.53	V_a	−12.85	
Ti standard used	4457.600	4449.313	V_d	+ 0.12	
	4527.490	4527.490	Reduction to Sun	−12.73	
	4590.126	4590.126	Radial velocity	−10.7	

γ *PISCIUM*—B 436.

1902, October 30, $14^h\ 39^m$. Hour angle W $0^h\ 5^m$.

Star spectrum very good; comparison spectrum a trifle strong.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+10.50	+11.38	4468.663		+12.48
33.390		12.37	76.214	+11.12	10.79
34.021	11.36		82.376	12.18	11.11
35.851	10.00	8.86	94.738	9.54	9.54
42.510	13.03	12.49	97.046	11.14	10.94
43.976		11.27	4501.422	12.92	11.79
47.892	11.80	11.93	08.455		11.11
54.993 B	9.69		12.906	11.23	
56.030		11.57	15.475	11.62	
57.656	10.56	10.70	28.798	11.59	12.05
66.701	11.00			(16)	(16)
No. of compar. lines	12	12	Means	+11.20	+11.27
ϵ	±1.01	±1.00	Mean	+11.24	
ϵ_0	±0.25	±0.25	V_a	−21.94	
Ti standard used	4427.266	4427.266	V_d	− 0.01	
	4481.438	4481.438	Reduction to Sun	21.95	
	4527.490	4527.490	Radial velocity	−10.7	

SUPPLEMENTARY STARS.

ι AURIGAE — B 446.

1902, November 6, $15^h\ 30^m$. Hour angle, E $4^h\ 12^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+2.30		4497.046	+3.40	+4.40
29.366		+2.23	4501.422	2.86	3.40
42.510	5.80	5.06	12.906	2.86	3.52
47.892		4.04	18.198		4.45
56.030	4.17	2.00	26.644	1.32	
57.656	2.35		28.798	3.58	4.30
60.460		4.03	46.129	2.70	
68.663	8.25		48.024	4.81	
76.214	3.08	2.61			
82.376	3.55	3.75		(14)	(12)
No. of compar. lines	12	13	Means	+3.64	+3.66
ϵ	±1.73	±0.93	Mean	+3.65	
ϵ_0	±0.46	±0.27	V_a	+15.07	
Ti standards used	4427.266	4427.266	V_d	+ 0.26	
	4481.438	4481.438	Reduction to Sun	+15.33	
	4544.864	4527.490	Radial velocity	+19.0	

ε LEONIS — B 483.

1903, January 8, $21^h\ 56^m$. Hour angle W $1^h\ 36^m$.

Star spectrum good; comparison spectrum fair.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−9.96	−11.65	4494.738	−12.07	−10.94
35.851	14.20	12.03	97.046	10.67	
41.881		8.71	4501.422	9.33	8.33
42.510		9.38	08.455	10.04	10.31
43.976	8.17		15.475	9.83	
47.892	10.38	11.32	28.798	10.06	9.54
57.656	8.14	8.01	46.129		10.88
68.663	9.73	9.26	48.938		11.60
76.214	10.79	10.45	54.211		8.23
82.376	9.43	9.30		(14)	(16)
No. of compar. lines	12	11	Means	−10.20	−10.00
ϵ	±1.53	±1.32	Mean	−10.10	
ϵ_0	±0.41	±0.33	V_a	+15.57	
Ti standards used	4427.266	4427.266	V_d	− 0.13	
	4481.438	4481.438	Reduction to Sun	+15.44	
	4527.490	4555.662	Radial velocity	+ 5.3	

ϵ *LEONIS*—A 427.

1903, April 8, $16^h\ 11^m$. Hour angle W $1^h\ 45^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4399.903		+29.92	4476.214		+28.68
4427.420	+28.51	30.61	82.376	+26.22	27.43
28.711		28.24	97.046	29.67	30.67
41.881		27.34	4501.422	30.64	31.77
42.510		30.17	08.455	31.73	
43.976	29.28	30.10	15.475	30.48	
47.892	31.08	31.41	28.798	32.38	30.59
57.656	26.10	27.18	54.211	32.06	
60.460		28.18			
68.663	29.45	29.52		(13)	(15)
No. of compar. lines	13	11	Means	+29.69	+29.45
ϵ	±2.01	±1.51	Mean	+29.57	
ϵ_0	±0.56	±0.39	V_a	−24.79	
Ti standards used	4427.266	4399.935	V_d	− 0.14	
	4481.438	4457.600	Reduction to Sun		−24.93
	4552.632	4527.490	Radial velocity		+ 4.6

ϵ *LEONIS*—A 443.

1903, April 22, $14^h\ 48^m$. Hour angle W $1^h\ 14^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+32.85	+32.04	4482.376	+35.99	+35.59
42.510		33.28	94.738		32.35
43.976		36.03	97.046	37.07	
47.892	33.23	35.73	4501.422		33.44
56.030	35.19	35.13	08.455	32.66	34.85
57.656	33.50	33.77	15.475	33.07	
59.304	34.16		28.798	34.56	32.25
60.460		32.34	47.196	33.77	
68.663	34.22	35.23	54.211	34.63	
76.214	34.24	36.05		(14)	(14)
No. of compar. lines	14	11	Means	+34.22	+34.15
ϵ	±1.23	±1.53	Mean	+34.18	
ϵ_0	±0.33	±0.41	V_a	−27.67	
Ti standards used	4427.266	4427.266	V_d	− 0.10	
	4481.438	4481.438	Reduction to Sun		−27.77
	4555.662	4527.490	Radial velocity		+ 6.4

γ CEPHEI—B 428.

1902, October 16, $17^h\ 44^m$. Hour angle, W $1^h\ 50^m$.

Star spectrum weak; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4456.030		−49.05	4526.644	−46.57	
57.656	−46.55	49.51	27.518	46.23	
60.460		49.28	28.798	46.68	−46.95
68.663	48.85	49.32	46.129	43.99	47.69
76.214	49.71	49.19	48.024		48.66
82.376	47.70	48.50	48.938		47.59
97.046	48.01		54.211	50.96	
4501.422		49.76	65.750		49.32
12.906	51.03		71.275		48.61
17.702		46.80	72.156		48.20
				(11)	(15)
No. of compar. lines	9	11	Means	−47.87	−48.56
ϵ	±2.20	±0.93	Mean	−48.22	
ϵ_0	±0.66	±0.24	V_a	+7.32	
Ti standards used	4457.600	4455.485	V_d	−0.04	
	4512.906	4501.445	Reduction to Sun	+ 7.28	
	4552.632	4572.156	Radial velocity	−40.9	

γ CEPHEI—A 424.

1903, April 3, $15^h\ 41^m$ Hour angle W $10^h\ 54^m$

Star spectrum good; comparison spectrum a trifle strong.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4400.577 B	−31.89		4459.304	−30.53	
08.549	33.05		60.460		−32.67
27.420	32.31	−31.97	68.663	30.26	32.41
28.711		30.74	76.214	32.36	32.96
35.184	32.46		82.376	30.91	30.84
42.510		33.14	94.738	32.29	31.43
43.976		33.87	97.046		32.54
47.892	32.63	30.60	4528.798	32.91	32.65
56.030		33.17	47.196	31.00	
57.656	31.75	31.01	54.211	28.51	
				(14)	(14)
No. of compar. lines	13	10	Means	−31.63	−32.14
ϵ	±1.26	±1.05	Mean	−31.88	
ϵ_0	±0.34	±0.28	V_a	−9.23	
Ti standards used	4399.935	4427.266	V_d	−0.02	
	4481.438	4481.438	Reduction to Sun	− 9.25	
	4555.662	4527.490	Radial velocity	−41.1	

γ *CEPHEI*—B' 501.

1903, May 6, $17^h\ 42^m$. Hour angle E $8^h\ 50^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−38.06	−39.42	4482.376	−40.27	−39.00
28.711		38.59	94.738		37.56
42.510	37.12	38.20	97.046	38.47	39.01
43.976		38.67	4501.422	41.63	
47.892	39.64	39.50	17.702		38.16
56.030	39.16	39.43	22.853	39.91	
57.656	37.60	38.01	28.798	37.41	37.75
60.460		38.25	46.129	39.37	
68.663	36.30	37.84	48.938	38.10	
76.214	37.86	38.32	54.211	38.45	
				(15)	(15)
No. of compar. lines	13	12	Means	−38.62	−38.51
ϵ	±1.38	±0.64	Mean	−38.56	
ϵ_0	±0.36	±0.16	V_a	−3.06	
Ti standards used	4427.266	4427.266	V_d	+0.06	
	4481.438	4481.438	Reduction to Sun	−3.00	
	4555.662	4527.490	Radial velocity	−41.6	

SUMMARY.

The results of the above detailed reductions are summarized in the following table. The values obtained by the two observers are placed in parallel columns, followed by the difference between the two for each plate. In the last column is given the mean of both determinations, and, finally, at the foot of the summary for each star are found the results derived from all of the plates.

In the final determinations given, those for β *Leporis* and ι *Aurigae* are, of course, entitled to low weight, since but one plate has been measured in the case of each star. Among the other stars of the list, the result obtained for α *Crateris* is probably subject to the greatest uncertainty: the low altitude and photographic faintness of the star have made it a difficult object, and all of the plates secured have been too weak for the most accurate measurement. A few of the plates of γ *Aquilae* and γ *Piscium* are also somewhat weak, but in a less degree.

The excellent agreement of the values of the two observers for each plate of ϵ *Leonis*, but unusually large discordance in the

results given by the different plates, might lead one to suspect that the range of 1.8 km. may be real, but a much larger number of plates would be required before a definite conclusion could be reached on the subject. The range in the values obtained for *a Persei* we consider to be due to the character of its spectrum.

α *ARIETIS.*

Series and Number	Date	F.	A.	F.-A.	Mean
B 420	1902, October 9	−13.23	−13.40	+0.17	−13.3
B 430	October 29	−12.38	−13.99	+0.61	13.7
B 465	November 27	−14.01	−14.12	+0.11	14.1
	1902.84	−13.5	−13.9	+0.30	−13.7

α *PERSEI.*

B 382	1902, August 7	−2.14	−1.45	−0.69	−1.8
B 431	October 29	−1.63	−1.69	+0.06	1.7
B 458	November 19	−3.18	−2.79	−0.39	3.0
	1902.77	−2.3	−2.0	−0.34	−2.1

β *LEPORIS.*

B 449	1902, November 6	−12.20	−12.55	+0.35	−12.4
	1902.85	−12.2	−12.6	+0.35	−12.4

β *GEMINORUM.*

B 477	1902, December 31	+2.83	+3.77	−0.94	+3.3
A 398	1903, February 5	+3.31	+3.67	−0.36	3.5
A 426	April 8	+3.44	+3.57	−0.13	3.5
	1903.12	+3.2	+3.7	−0.48	+3.4

α *CRATERIS.*

B 491	1903, February 4	+46.09	+47.65	−1.56	+46.9
A 438	April 16	+47.28	+47.29	−0.01	47.3
A 444	April 22	+48.94	+47.12	+1.82	48.0
	1903.23	+47.4	+47.4	+0.08	+47.4

α BOÖTIS.

Series and Number	Date	F.	A.	F.-A.	Mean
A 373	1902, September 6	−4.19	−4.81	+0.62	−4.5
B 492	1903, February 4	−4.48	−4.38	−0.10	4.4
B 497	March 24	−5.51	−5.02	−0.49	5.3
A 433	April 8	−4.55	−5.20	+0.65	4.9
B′ 499	May 6	−4.87	−5.24	+0.37	5.1
	1903.12	−4.7	−4.9	+0.22	−4.8

β OPHIUCHI.

B 378	1902, August 7	−10.87	−10.60	−0.27	−10.7
A 451	1903, April 30	−11.71	−11.27	−0.44	11.5
B′ 508	June 26	−11.27	−10.77	−0.50	11.0
	1903.14	−11.3	−10.9	−0.40	−11.1

γ AQUILAE.

B 398	1902, August 27	−1.01	−1.83	+0.82	−1.4
B 417	October 19	−1.67	−2.88	+1.21	2.3
B′ 509	1903, June 26	−1.66	−2.05	+0.39	1.8
	1902.98	−1.4	−2.2	+0.81	−1.8

ε PEGASI.

A 364	1902, July 31	+6.16	+6.06	+0.10	+6.1
B 379	August 7	+6.06	+6.18	−0.12	6.1
B 418	October 9	+6.55	+6.50	+0.05	6.5
	1902.65	+6.2	+6.2	+0.01	+6.2

γ PISCIUM.

B 381	1902, August 7	−10.90	−11.83	+0.93	−11.4
B 415	October 8	−10.53	−10.92	+0.39	10.7
B 436	October 30	−10.75	−10.68	−0.07	10.7
	1902.73	−10.7	−11.1	+0.42	−10.9

SUPPLEMENTARY STARS.

ι AURIGAE.

Series and Number	Date	F.	A.	F.-A.	Mean
B 446	1902, November 16	+18.97	+18.99	−0.02	+19.0
	1902.85	+19.0	+19.0	−0.02	+19.0

ε LEONIS.

B 483	1903, January 8	+5.24	+5.44	−0.20	+5.3
A 427	April 8	+4.76	+4.52	+0.24	4.6
A 443	April 22	+6.45	+6.38	+0.07	6.4
	1903.20	+5.5	+5.4	+0.04	+5.5

γ CEPHEI.

B 428	1902, October 16	−40.59	−41.28	+0.69	−40.9
A 424	1903, April 3	−40.88	−41.39	+0.51	41.1
B′ 501	May 6	−41.62	−41.51	−0.11	41.6
	1903.13	−41.0	−41.4	+0.36	−41.2

An examination of the above results for systematic differences between the two observers indicates a slight tendency for F.−A. to be positive. A general mean for all of the stellar plates (37) gives F.−A. a little less than +0.10 kilometers. It is, however, extremely doubtful whether this is to be considered as other than accidental, as it is chiefly due to the abnormally large positive differences given by two somewhat inferior plates of *γ Aquilae*. Accordingly it would seem that the personality errors are pretty evenly balanced in this series of measures; particularly when the possibility is considered of errors due to the regular difference in the corrections applied for curvature.

No evidence is shown in the results given above of any dependence of the value obtained upon the position of the spectrograph in reference to the pier of the telescope. Of other instrumental causes which might give rise to errors, temperature changes may be regarded as eliminated, as a brief examination

of the Journal of Observations will show. Possible errors arising from changes in the electrical conditions of the comparison spectrum apparatus may also be disregarded, as this has remained without essential modification during the entire interval covered by the observations.

Spectrographic determinations of the radial velocities of four of the stars in the principal list, and of one of the supplementary list, have been published since Vogel and Scheiner obtained their results in 1889–90. These are collected in the following table. For convenience in comparison, the value we have found for each star in this paper is repeated immediately below its name.

Star	Observer	Velocity	Epoch	No. of Plates	Range	Reference
		km			km	
α Arietis	Campbell	−14.1	1896.8	4	0.6	Astrophysical Journal, 8, 150, 1898
−13.7	Adams	−13.7	1901.9	1	...	Astrophysical Journal, 15, 24, 1902
	Newall	−14.3	1902.8	3	2.8	*Monthly Notices*, 63, 298, 1903
α Persei	Campbell	− 2.4	1897.8	4	1.7	Astrophysical Journal, 8, 150, 1898
−2.1	Vogel	− 3.2	1901.0	13	3.3	Astrophysical Journal, 13, 322, 1901
	Newall	− 2.6	1902.8	14	5.7	*Monthly Notices*, 63, 298, 1903
α Boötis	Frost and Adams	− 4.3	1902.3	8	1.8	*Publications of the Yerkes Observatory*, 2, 35, 1903
−4.8	Newall	− 5.8	1903.4	5	2.7	*Monthly Notices*, 63, 298, 1903
ε Pegasi	Campbell	+ 5.7	1897.8	4	1.2	Astrophysical Journal, 8, 150, 1898
+6.2						
ε Leonis	Wright	+ 5.1	1899.4	7	2.7	Astrophysical Journal, 11, 414, 1900
+5.5	Adams	+ 4.0	1900.8	3	0.9	Astrophysical Journal, 15, 25, 1902

To us a surprising difference in the above comparison is that of 0.5 km between our earlier measures of eight plates of *α Boötis* and the five plates of this paper. We attach greater weight to the present series, on account of the improved method of insuring the uniform illumination of the collimator lens by the light from the spark. But the range in velocity observed in either series is larger than might be expected for a star having such well-defined lines.

There would appear to be a slight tendency toward a systematic difference between our results and those of other observers, in the direction of a larger positive, or smaller negative, value for our velocities. The amount is so small, however, and the quantity of material with which comparison may be made is at present so meager, that no certain inference can now be drawn. An

investigation of this and of many other matters of interest will be possible as the results of other participators in the co-operation are published. It is known that, owing to changes being made in their spectrographs, at least two of those planning to co-operate have been unable to carry out the program during the past year, but expect to do so during the present year. A general comparison of approximately contemporaneous results by all participators will therefore not be available until later, but we have thought that the prompt publication of our results would nevertheless be of service. We shall continue our observations this year, following the program as closely as possible.

YERKES OBSERVATORY,
August 27, 1903.

ON THE SPECTRA OF IMPERFECT GRATINGS.

By A. A. MICHELSON.

It will be convenient to consider separately gratings which act by opacity and those depending on retardation. The simplest case of the former may be represented by the expression

$$y = a \cos ks + b \sin ks \ ,^{1} \qquad (1)$$

in which y represents the reflection (or transmission) factor at any point s of a line perpendicular to the rulings, and $k = 2\pi/\sigma$, where σ is the constant grating-space.

If a plane wave-train of frequency $n/2\pi$ fall normally on such a grating, the effect in a direction making an angle θ with the normal will be

$$\int yds \cos (nt - \partial)$$

where

$$\partial = \frac{2\pi}{\lambda} s \sin \theta \ .$$

The intensity of the diffracted light is

$$I = \left[\int yds \cos \partial\right]^2 + \left[\int yds \sin \partial\right]^2 \qquad (2)$$

or

$$I = a^2 \left[\int \cos ks \cos \partial \, ds\right]^2 + b^2 \left[\int \sin ks \sin \partial \, ds\right]^2 .$$

The integrals, taken from $-\infty$ to ∞, have sensible values only at $\sin \theta = \pm \frac{\lambda}{\sigma}$, and the intensity in these directions is

$$I = a^2 + b^2 \ .$$

If the grating cannot be represented by a simple sine curve, let

$$y = \Sigma a_m \cos mks + \Sigma b_m \sin mks \ .$$

This value substituted in (2) gives

$$I_m = a_m^2 + b_m^2$$

in directions such that $\sin \theta = \pm \frac{m\lambda}{\sigma}$.

[1] To avoid negative values, a constant should be added, but this affects only the central image and not the lateral spectra.

As an illustration suppose $y = 1$ from $s = 0$ to $s = \frac{1}{2}\sigma$, and $y = 0$ from $s = \frac{1}{2}\sigma$ to $s = \sigma$.

The value of the coefficients in the series for y will then be

$$a_m = 0\ ,\quad b_0 = \frac{\pi}{2}\ ,\quad b_1 = 1\ ,\quad b_2 = 0\ ,\quad b_3 = \frac{1}{3}\ ,\quad \text{etc.}$$

The intensities of the spectra will be in the proportion of $\frac{\pi^2}{4}$ for the central image and 1, 0, $\frac{1}{3^2}$, 0, $\frac{1}{5^2}$, etc., for the spectra in their order, in agreement with the results obtained by Lord Rayleigh.[1]

In the case of a reflection grating the retardation is

$$\partial = \frac{2\pi}{\lambda}[s \sin\theta - y\,(1 + \cos\theta)]\ ,^*$$

and the effect in a direction θ will be

$$\int ds \cos\,(nt - \partial)\ . \tag{3}$$

The intensity will be

$$I = \left[\int ds \cos\partial\right]^2 + \left[\int ds \sin\partial\right]^2. \tag{4}$$

Putting

$$\cos\frac{2\pi}{\lambda}\left(y\,.\,2\cos^2\frac{\theta}{2}\right) = \Sigma a_m \cos mks\ ,$$

and

$$\sin\frac{2\pi}{\lambda}\left(y\,.\,2\cos^2\frac{\theta}{2}\right) = \Sigma b_n \sin mks\ ,$$

also

$$\frac{2\pi}{\lambda}\left(m\frac{\lambda}{\sigma} - \sin\theta\right) = p\ ,$$

and

$$\frac{2\pi}{\lambda}\left(m\frac{\lambda}{\sigma} + \sin\theta\right) = p'\ ,$$

$$\sqrt{I} = \sum a_m \frac{\sin ps}{p} + \sum a_m \frac{\sin p's}{p'} + \sum b_m \frac{\sin ps}{p} - \sum b_m \frac{\sin p's}{p'}\ .$$

[1] *Lord Rayleigh Scientific Papers*, Vol. III.

* For a transmission grating $\partial = \frac{2\pi}{\lambda}\,[sn \sin\theta + y\,(1 - n\cos\theta)]$.

The limits being infinite, I has finite values only at $p = 0$ and at $p' = 0$, or for $\sin\theta = m\lambda/\sigma$ and $\sin\theta = -m\lambda/\sigma$. For the former

$$I = (a_m + b_m)^2$$

and for the latter

$$I' = (a_m - b_m)^2 \ . \tag{5}$$

If $a_m = b_m$, all the negative spectra vanish. If of the positive spectra all are to vanish except the mth, then

$$\cos\frac{2\pi}{\lambda}\left(y \,.\, 2\cos^2\frac{\theta}{2}\right) = \cos mks \ ,$$

$$\sin\frac{2\pi}{\lambda}\left(y \,.\, 2\cos^2\frac{\theta}{2}\right) = \sin mks \ ,$$

which gives

$$y = (x + n\sigma)\tan\frac{1}{2}\theta \ . \tag{6}$$

The section of the grating surface is that represented in Fig. 1. It is essentially the arrangement of plates in a reflecting echelon.

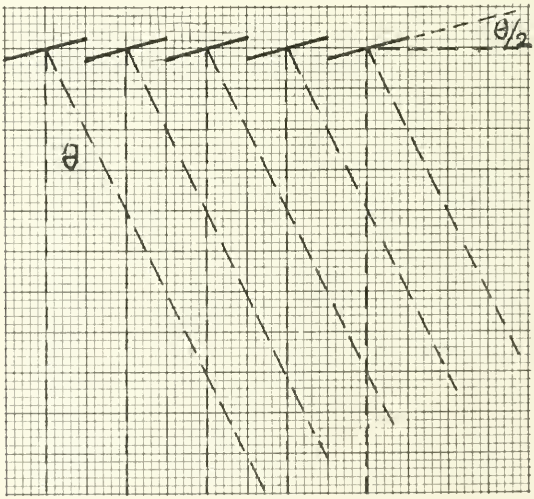


FIG. 1.

In actual gratings the ruled space is limited, and the spectrum of a homogeneous source will not be a line, but an image of finite width. It is proposed to find the distribution of light in this image for any given grating. This distribution can be affected only by the imperfections of the ruling (*e. g.* changes in the strength of the lines or in their spacing), but not by the character of the furrow. We may therefore let

$$y = \phi(s) \cos mks + \psi(s) \sin mks = f(s) \ .$$

By Fourier's formula

$$f(s) = \int_{-\infty}^{\infty} (C \cos asda + S \sin asda) \ ,$$

where $$C = \int_{-\infty}^{\infty} f(\lambda) \cos a\lambda d\lambda \ ,$$

and $$S = \int_{-\infty}^{\infty} f(\lambda) \sin a\lambda d\lambda \ . \qquad (7)$$

A single element of $f(s)$ is therefore

$$C \cos as + S \sin as \ , \qquad (7')$$

which gives a spectrum in a direction

$$\sin\theta = \pm ca \quad \text{whose intensity is} \quad I = C^2 + S^2 \ . \qquad (8)$$

If the spacing is exact, $\psi = C\phi$, whence

$$I = \left[\int \phi(\lambda) \cos p\lambda d\lambda\right]^2 + \left[\int \phi(\lambda) \sin p\lambda d\lambda\right]^2 ,$$

where $p = mk \pm a$. The spectrum images are always symmetrical.

If the spacing is not exact, but the strength of the rulings is constant, the usual case in actual gratings,

$$f(s) = \cos(ks - \omega) \ ,$$

$$\phi = \cos\omega \ , \quad \psi = \sin\omega \ , \quad C = \int \cos(\omega - p\lambda) d\lambda \ ,$$
$$S = \int \sin(\omega - p\lambda) d\lambda \ ;$$

hence

$$I = \left[\int \cos(\omega - p\lambda) d\lambda\right]^2 + \left[\int \sin(\omega - p\lambda) d\lambda\right]^2 . \qquad (9)$$

The spectral images are generally unsymmetrical.

One form of spacing error which doubtless occurs in practice is that caused by the sudden release of strain in some part of

the course, thus producing a "slip" in all the subsequent rulings. Such a case may be represented by putting $\omega = -a$ from $-l$ to 0 and $\omega = a$ from 0 to l. These values substituted in (9) give

$$C = \frac{1}{p}\left(\sin a - \sin (a - pl)\right) \ . \qquad (10)$$

The intensity curve which is given by squaring this expression is represented in Fig. 2 for $a = 0$, $\frac{1}{8}\pi$, $\frac{2}{8}\pi$, $\frac{3}{8}\pi$, and $\frac{4}{8}\pi$.

The minima are readily found by putting $C = 0$, which gives

$$pl = 2\left(a - \frac{2n+1}{2}\pi\right) \ . \qquad (11)$$

The maxima may be obtained by differentiating (10) for p and equating to zero. It can be shown, however, that the result is given with an error less than 0.002π from $a = \pi/2$ to $a = -\pi/2$ by the expression

$$p_1 l = \frac{3}{2}(a - n\pi) \ . \qquad (12)$$

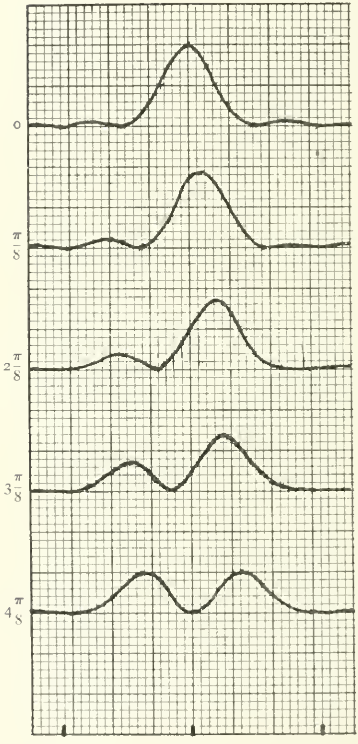


Fig. 2.

If h is the actual linear displacement of the rulings, $a = 2\pi h/\sigma$. Substituting this value in (12), restoring $a = k - p$ and remembering that $\sin\theta = \frac{\lambda}{2\pi}a$, we have

$$\sin\theta = \frac{\lambda}{\sigma} - \frac{3}{2}\frac{\lambda}{l}\frac{h}{\sigma} + \frac{3}{4}\frac{n\lambda}{l} \ .$$

For $h = 0$ and $n = 0$, $\sin\theta_0 = \lambda/\sigma$, whence

$$\sin\theta - \sin\theta_0 = \cos\theta\partial\theta = \frac{\lambda}{\sigma}\left(\frac{3h}{2l} - \frac{3}{4}\frac{n\sigma}{l}\right),$$

$$\frac{\partial\lambda}{\lambda} = \frac{\partial\theta}{\tan\theta} = \frac{3}{2}\frac{h}{l} - \frac{3}{4}\frac{n\sigma}{l}.$$

If the observations are made in the mth spectrum, σ must be replaced by σ/m. Putting also $2l = N\sigma$ we have

$$\frac{\partial\lambda}{\lambda} = \frac{3}{N}\left(\frac{h}{\sigma} - \frac{n}{2m}\right).$$

Treating (11) in a similar way,

$$\frac{\partial_1\lambda}{\lambda} = \frac{4}{N}\left(\frac{h}{\sigma} - \frac{2n+1}{4m}\right).$$

These expressions represent the percentage error due to setting on the maximum and the minimum respectively. The graphs of these errors are represented in Fig. 3 by the systems of straight lines.

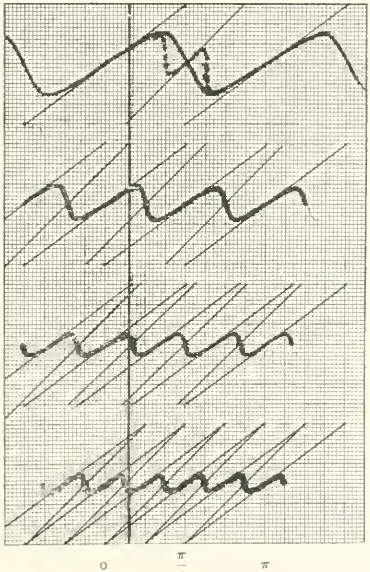


Fig. 3.

The actual phase which may be selected for measurement will, however, depend largely on the observer. When h is small, this will be the brightest part of the line, and the error will be represented by the lines of lesser inclination; while if h is nearly one-fourth of the grating space, the minimum would be selected, and for values between, the measurements would be uncertain, with a tendency to setting on the "center of gravity."

This surmise is amply verified by observation. For this

purpose a grating was mounted on a spectrometer, and a pair of plane-parallel glasses was placed in front of the collimator lens. One of the plates could be rotated through a measured angle, thus introducing in the corresponding half of the field any required retardation. The results are shown in Fig. 4.

The curves from this and other observations are repeated in Fig. 3. If a vertical line, for instance that at $a = 0.15\pi$, be drawn through the curves, the intersections will correspond to errors of 4.0, 3.5, 2.0 and -1.5 twentieths respectively. The constant error introduced in comparing the second with the

FIG. 4.

third spectrum will be 0.1, while between the third and fourth the error will amount to 0.17. If $a = \frac{1}{3}\pi$, corresponding to $h = \frac{1}{6}\sigma$, the error introduced in comparing the first spectrum with the second will be 0.5. The absolute error depends on the number of lines in the grating. If this be 100,000, these errors will be 1, 1.7, and 5 parts in a million — quantities of the same order as those given by Perot and Fabry,[1] indicating a systematic error in Rowland's table of wave-lengths.

If the ruling is affected by a periodic error, the spectral lines are accompanied by ghosts. The elementary explanation has long been known, but, so far as I know, no general theory has been proposed which furnishes a relation between the character of the periodic error and the intensity of the ghosts.

The result in its widest generality would be obtained by substituting for ϕ and ψ any periodic functions; but it will serve our purpose if ϕ is even and ψ is odd.

[1] *Comptes Rendus*, 133, 153, 1901.

Putting $\phi(s)=\Sigma a_q \cos qs$ and $\psi(s)=\Sigma b_q \sin qs$ in (7), we obtain

$$C=\int a_q \cos(q-p)\lambda d\lambda+\int b_q \sin(q-p)\lambda d\lambda\ ,$$
$$+\int a_q \cos(q+p)\lambda d\lambda-\int b_q \sin(q+p)\lambda d\lambda\ ,$$
$$S=0\ .$$

If the number of periods is large, the value of C will be appreciable only in the vicinity of $p=q$ and $p=-q$, and in these directions the intensity will be

$$I_q=C_q^2=(a_q+b_q)^2 \quad \text{and} \quad I_q'=(a_q-b_q)^2$$

or

$$\begin{aligned} I_q&=\left[\int_0^{2\pi}\phi(s)\cos qs\,ds+\int_0^{2\pi}\psi(s)\sin qs\,ds\right]^2 \\ I_q'&=\left[\int_0^{2\pi}\phi(s)\cos qs\,ds-\int_0^{2\pi}\psi(s)\cos qs\,ds\right]^2 \end{aligned} \qquad (13)$$

If $b_q=0$, the ghosts are symmetrical about the principal line. If $a_q=b_q$, the ghosts on the side of $p=-q$ vanish.

If the only error is that due to unequal spacing, let $\phi(s)=\cos\omega$ and $\psi(s)=\sin\omega$, then, if $q=\frac{2\pi m}{S}$, where S is the period of the error, and $m=1, 2, 3 \ldots$

$$a_q=\int_0^S \cos\omega\cos qs\,ds\ ,\quad b_q=\int_0^S \sin\omega\sin qs\,ds\ ,$$

$$I_q=\left[\int_0^S \cos(\omega-qs)\,ds\right]^2,\quad I_q'=\left[\int_0^S \cos(\omega+qs)\,ds\right]^2.$$

As an illustration take $\omega=hs$ from 0 to S; then

$$I_m=\left(\frac{\sin hS}{hS-2\pi m}\right)^2 \quad \text{and} \quad I_m'=\left(\frac{\sin hS}{hS+2\pi m}\right)^2.$$

The expression (4) represents the intensity of the reflected light, whether the surface be furrowed or not. For normal incidence, therefore, the intensity of the diffraction image is[1]

$$I=\left[\int ds\cos(2my\cos\theta-ms\sin\theta)\right]^2+$$
$$\left[\int ds\sin(2my\cos\theta-ms\sin\theta)\right]^2\ .$$

[1] A more general expression including both absorption and retardation is $I=C^2+S^2$, where

$$C=\int f(s)\cos(\omega-ps)ds \quad \text{and} \quad S=\int f(s)\sin(\omega-ps)ds\ .$$

But if we replace $2my \cos \theta$ by ω and $m \sin \theta$ by p, this is the same as (9), save that p is now single-valued. Hence all the preceding results except those relating to the form of the furrow may be applied to the case of smooth surfaces such as mirrors and prisms.

Ryerson Laboratory,
University of Chicago,
October 1903.

SOLAR PROMINENCES AND TERRESTRIAL MAGNETISM.

By A. L. Cortie, S.J.

Recent researches with regard to the relation which exists between Sun-spots and terrestrial magnetism, while confirming the general connection between the two classes of phenomena established by Mr. Ellis and Canon Spée, have emphasized the conclusion that this relation is not one of efficient cause and effect. Such is the outcome of the comparison of the magnetic storms recorded at Stonyhurst and the greater solar spots made by Father Sidgreaves, and the analysis of the Kew magnetograms for the period of eleven years recently concluded and published by Dr. Chree. The same conclusion was set forth in a former paper by the writer on "Minimum Sun-spots and Terrestrial Magnetism,"[1] which contained a detailed study of the individual Sun-spots and the magnetic curves for the period 1899–1901. There is an undoubted general connection, not a mere series of coincidences, between Sun-spots and magnetic storms, but the supposed direct action of Sun-spots is in individual cases so capricious and irregular that it is doubtful whether they can be regarded as even instrumental causes of the magnetic storms. The existence rather of a common cause is indicated by the observations, which acts sometimes on the Sun causing spots, sometimes on the magnets, and sometimes on both Sun and magnets.

But the question has been raised more than once, and recently in a paper on "The Relation between Solar Prominences and Terrestrial Magnetism" by Sir Norman and Dr. Lockyer,[2] whether solar prominences may not supply the places of spots in those cases in which great or active magnetic storms occur without the presence of any spot. The curves presented in the paper show, as was to be expected, that at times of Sun-spot maximum activity the disturbance is general, and extends even

[1] Astrophysical Journal, 16, 203–210, 1902. [2] *Proc. R. S.*, 71, 244, 1903.

to prominences in high solar latitudes, the curve of annual frequency for such prominences bearing a very striking similarity to that of great magnetic disturbances; that is, disturbances in which the range of the declination magnet is greater than 60′, as deduced from Mr. Ellis's observations. But we would submit that this fact does not show that solar prominences, and in particular those in high solar latitudes, possess any special virtue or efficacy in the causation of magnetic storms not possessed by Sun-spots, and does not warrant, of itself, the conclusion that prominences may cause magnetic storms in the absence of Sun-spots. Rather is it not only one manifestation of the increased solar external activity which is displayed in greater and more numerous spots, and increase in number, latitude, and extent of both prominences and coronal streamers?

Observations of the prominences and chromosphere were taken at Stonyhurst during the years 1880–92, and in the *Observatory* for March 1893 will be found the curve of profile area of the prominences, compared with the curve of Sun-spot surface deduced from the Greenwich records for the whole period of the observations. The accord of the two curves is very striking, and corroborates what was known before, that the activity of the prominences is in direct proportion to the activity of Sun-spots. Now, both Canon Spée and Mr. Ellis have shown the complete similarity of the curves for Sun-spot frequency and magnetic declination over long periods. Mr. Ellis, too, has demonstrated that the Sun-spot curve is also identical with the curve of frequency of magnetic storms. Hence it follows that the accord of the prominence curves and the magnetic curves is no proof of the causality of prominences with regard to magnetic storms. If it were possible to make a curve of isolated solar prominences and then show that such a curve, to the exclusion of the Sun-spot curve, corresponded to the curve of magnetic storm frequency, then the argument would have a greater force. But, except that they occur in latitude in which Sun-spots are not found, solar prominences are not phenomena isolated from spots and faculæ. Nay more, it is only one class of solar prominences that are even separated in latitude from Sun-spots, namely, the relatively

quiet, as distinguished from the eruptive, prominences. For, with regard to this latter class, they are almost entirely confined to the Sun-spot zones, and as a general rule accompany outbursts of spots or faculæ. To take one period as an instance, namely, that treated of by Professor Wolfer in the *Publicationen der Sternwarte des Eidg. Polytechnikums zu Zürich* (Band III). Of 315 metallic or eruptive prominences included in the lists discussed for the years 1893–95, 274, or 86 per cent., were connected with spots; 27, or 9 per cent., with faculæ; while only 14, or 5 per cent., were independent of spots and faculæ. Father Fenyi, too, in the publications from the Haynald Observatory at Kalocsa, calls attention to the same fact, though the converse is not true, as spots and faculæ have frequently no accompanying eruptive prominences.

Nevertheless, to quote from Sir Norman and Dr. Lockyer's paper: "The magnitude of magnetic storms appears to vary according to the particular position as to latitude of the prominence on the Sun's disk. The nearer the poles (either north or south) the prominence occurs, the greater the magnetic storm, and these are the regions where no spots exist." As a contribution to the elucidation of this point, it was determined to study in detail all the more noteworthy prominences of some selected period, and to see whether they exercised any effect upon the magnets independently of the spots and faculæ. Obviously a minimum period of solar activity is the best for this purpose, first because in a maximum period it is impossible to unravel the various outbursts of spots, faculæ, and prominences, and apportion to them their proper magnetic storms; and, secondly, because, if in a time of minimum Sun-spots an extraordinary prominence appears, and prominences are to be supposed, for the sake of argument, to be the cause of magnetic storms, it ought to be accompanied by at least an active movement of the magnets. It was intended to study the four years 1887–90, for which an excellent series of observations with detailed descriptions of the greater outbursts has been published by Father Fenyi at Kalocsa, but as the work of comparison is rather laborious, the results for only two years have been gathered together in the following table,

which contains details with regard to forty-eight prominences. The principle of selection has been to admit such as were especially noteworthy either on account of their eruptive nature, or the displacements of their spectrum lines, or were of a height equal to or greater than 100″. Their character as eruptive or non-eruptive is indicated in the third column, their maximum height or displacement and their heliographic co-ordinates being given in other columns of the table. In the columns devoted to the readings of the curves of the declination magnet, the first gives the maximum diurnal range for the date on which the prominence was observed, the second the intensity of the storm, if any, observed within three days before or after the prominence was seen, and the third the date on which the magnetic storm occurred. In Mr. Ellis's discussion of Sun-spot frequency and magnetic storms an active storm on the declination magnet is one in which the range exceeds 30′ and is less than 60′, while a great storm is one in which the range exceeds 60′. We have used the corresponding numbers, 3, 4, etc., as indications of intensity. In the last column are given the Greenwich numbers for such spot-groups as have been identified as connected with any of the prominences.

Of these forty-eight prominences twenty-nine were either immediately associated with spots and faculæ, or occurred in the latitudes frequented by Sun-spots. This number includes all the so-called metallic prominences. During the year 1887 there were only two noteworthy Sun-spot groups on the Sun; those numbered thirty-two and thirty-four in the Stonyhurst series. The first was born on the visible disk on May 14, and after five rotations disappeared, also while on the visible disk, on August 4, but in the next rotation there was an outburst in such close proximity to its position as evidently to have formed part of the same disturbance. This one disturbance covers the groups numbered 1978, 1987, 1992, 1998, 2006, and 2010 at Greenwich. Its mean longitude was 92° and its mean latitude −8°. At its first appearance it crossed the western limb on May 22, and the table shows that there was a fine metallic prominence seen on the limb, which also showed a displacement corresponding to no less

NOTEWORTHY PROMINENCES, 1887–88.

No.	Date	Character	Maximum Height	Maximum Displacement	Heliographic Longitude	Heliographic Latitude	Magnetic Declination: Diurnal Range	Storm: Intensity	Storm: Date	Greenwich Nos. Spots or Faculæ
	1887			km						
1	May 22	M.	49"	426	90°	− 8 W	10'.0	0	....	1978
2	June 6	H.	...	135	75	−12 E	7.0	0	....	1987 Faculæ
3	8	M.	24	...	48	− 1 E	3.1	0	....	1988
4	23	H.	137	...	30	+31 W	5.9	3	June 20	....
5	25	H.	20	70	180	− 7 E	7.5	0	....	1991?
6	26	M.	22	...	169	− 8 E	8.0	0	....	1991
7	27	H.	105	...	335	−11 W	8.0	0	....	....
8	July 1	H.	165	320	275	− 6 W	8.8	0	....	Faculæ?
9	9	M.	51	85	175	−13 W	15.8	0	....	1991
10	16	H.	113	...	265	−47 E	13.7	0	....	....
11	16	H.	122	...	85	+ 8 W	13.7	0	....	Faculæ?
12	18	M.	...	...	58	+ 2 W	20.7	0	....	1994
13	19	H.	125	...	45	+22 W	11.0	0	....	....
14	20	H.	119	...	210	−17 E	9.2	0	....	....
15	29	H.	246	...	270	+47 W	9.2	3	Aug. 1	....
16	30	M.	...	...	259	−12 W	9.0	3	Aug. 2	1995 Faculæ
17	30	M.	...	...	79	− 6 E	9.0	3	Aug. 2	1999
18	30	H.	92	284	280	−10 W	9.0	3	Aug. 2	1995
19	Aug. 12	M.	...	...	267	− 6 E	6.3	0	....	2001
20	25	M.	...	...	97	− 7 E	12.8	0	....	2006 Faculæ
21	29	M.	...	...	223	+ 1 W	31.3	3	Aug. 28	2005
22	29	H.	106	...	40	+21 E	31.3	3	Aug. 29	....
23	Sept. 15	M.	...	...	0	− 7 W	16.1	0	....	Faculæ
24	18	H.	...	184	320	−45 W	11.0	0	....	....
25	Oct. 12	H.	108	...	0	+46 W	16.9	0	....	....
	1888									
26	Jan. 10	H.	...	85	76	− 8 E	3.0	3	Jan. 7	2029
27	Feb. 1	H.	...	412	326	−10 W	1.0	0	....	....
28	7	H.	126	...	68	+18 E	1.0	0	....	....
29	17	M.	...	133	295	− 2 E	....	0	....	2035
30	March 6	H.	...	113	58	+11 E	9.0	3	March 9	Faculæ
31	7	H.	...	57	224	−49 W	15.6	3	March 9	....
32	14	M.	24	...	311	− 8 E	3.1	0	....	2039
33	20	M.	...	150	235	− 9 E	3.0	0	....	2041
34	April 6	H.	100	...	179	−17 W	13.3	3	April 4	Faculæ?
35	16	H.	101	...	238	−37 E	12.5	3	April 13	....
36	29	M.	...	...	246	− 8 W	11.0	0	....	2051
37	May 11	H.	...	102	266	− 6 E	0.0	3	May 7	2052?
38	June 7	H.	...	120	271	+58 E	11.2	4	June 3	....
39	July 13	H.	127	...	265	−22 W	9.5	0	....	2059 Faculæ
40	28	H.	...	145	314	− 7 E	15.5	0	....	2061
41	Aug. 22	H.	100	...	325	+20 E	6.5	0	....	....
42	Sept. 5	M.	151	...	147	−10 E	2.0	0	....	2070
43	6	M.	158	297	147	−10 E	2.0	0	....	2070
44	10	M.	38	...	272	− 8 W	4.0	0	....	2066
45	Oct. 13	H.	101	...	197	−47 W	14.9	0	....	....
46	25	H.	152	...	220	+34 E	1.5	0	....	....
47	Nov. 26	M.	27	...	159	− 9 E	16.1	0	....	2076
48	30	H.	120	...	104	−23 E	11.8	0	....	....

a velocity than 426 km per second in the line of sight. The maximum diurnal range on the day was 10′, and there was no disturbance of the magnets that could be classed as active. The other prominences of the table connected with this spot-group are No. 2, seen on its second entrance on the disk, again without magnetic storm, and No. 20, also without storm of any kind, the maximum diurnal range being 7′.0 and 12′.8 respectively. The first magnetic storm which occurs at any date near to that on which a prominence was seen was on June 20, a fine isolated prominence being observed on the twenty-third. If this is to be considered as a case of connection of isolated prominences and magnetic storm, the next big prominence, possibly connected with a facula, but having no connection with spots, beyond its occurrence in the spot-zones, is directly opposed to any such connection. It was a splendid prominence which changed its form very rapidly. In eleven minutes it rose from 60″ to 163″, with a corresponding line-of-sight displacement of the C line indicating a velocity of 320 km per second. In about seventeen minutes after attaining its maximum height it had practically disappeared. More than this, as the table shows, there were many fine prominences observed by Father Fenyi in July, and Canon Spée's tables deduced from observations at Rome and Palermo also show an increase in number of prominences during this month. Yet the magnets were very quiet, and remarkably so as the end of the month approached. However, at the end of the month, on the twenty-ninth and thirtieth, four fine prominences were observed, three connected with Sun-spot groups, one of which, that numbered eighteen, was active all day, and one an isolated hydrogen prominence in higher latitudes. The magnets were actively disturbed on August 1 and 2. Is the connection here, if any, with the spots or with the one isolated prominence? Again we have storms on the magnets on August 28 and 29, and on the twenty-ninth two fine prominences are observed, one connected with a Sun-spot, the other isolated. Which is to have the credit of the magnetic storm? On January 7 there was a magnetic storm, and on January 10 a fine prominence connected with group 2029 of the Greenwich

series, while the magnetic storm of March 9 may be credited to the isolated prominence of March 7, or to that connected with the faculæ of March 6. The most telling cases, however, in favor of isolated prominences affecting the magnets are those numbered 35 and 38 in the list. On April 16 the spotted area of the Sun was extremely small, and so too was the area covered by faculæ; therefore the storm of the thirteenth may possibly be connected with the prominence of the sixteenth. Similar remarks apply to the prominence of June 7, on which day there were no spots on the Sun, and a moderate area of faculæ, and the magnetic storm of June 3, allowing four days' grace, for a prominence which at the time of the storm must have been on the Sun's invisible hemisphere. But the most active prominence of the year was observed on September 5 and 6, in connection, however, with a spot-group of moderate size, No. 2070 in the Greenwich series, and which, according to the Stonyhurst records, was one of three distinct formations occurring in the same position. Its presence is also shown in the table in No. 47, connected with spot-group 2076. There is not the slightest corresponding movement of any sort on the magnets in connection with the big September prominences, the curves being straight lines, for several days before and after, and only a slight movement in connection with the November prominence. Finally on October 13 and 25, and November 30, 1888, there occur cases of isolated prominences higher than that of April 16, with which we connected the magnetic storm of April 13. The magnets, however, are quiet, the greatest range being $16'.1$ on November 26. It follows, therefore, that in the period discussed, which, though limited in extent, contains many fine prominences, no conclusion can be drawn as to the relationship of prominences and magnetic storms, seeing that in the cases of both eruptive and non-eruptive prominences, though some few seem to be connected with magnetic storms, others equally high and equally active are totally unconnected with any extraordinary movements of the needles.

Stonyhurst College Observatory,
September 1903.

THE SPECTRUM OF LIGHTNING.

By Philip Fox.

Spectra of lightning flashes were photographed on the nights of July 16, 17, August 3, and October 6 by means of an objective-prism spectroscope. The camera lens was of 35 mm aperture and 274 mm focal length. The 30° flint-glass prism was from a large spectroscope loaned to the Observatory by the Massachusetts Institute of Technology. Of the dozen plates showing spectra the best was obtained on August 3 at $9^h\ 10^m$ in the evening, and shows three flashes. They are reproduced herewith (Plate IX). The original negative is on a Cramer Isochromatic plate.

Vogel and Lohse,[1] and Schuster[2] identified certain lines in the lightning spectrum with lines of the spark spectrum of air. Following the suggestion of their work, the spectroscope was provided with a slit and collimator, and the air spectrum, obtained by passing a spark between silver terminals, was photographed. The agreement in position of the lines is shown in Fig. 1. The wave-lengths of the lines, with their identification, are given in the table, which also shows the wave-lengths of lines as determined by Vogel and Lohse, Schuster, and E. C. Pickering.[3] Lieutenant Herschel[4] also identified some of the lightning lines, the principal one probably being that at λ 5003 of the present determination.

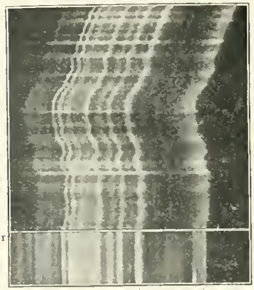


Fig. 1.—Spectrum of lightning and of air.

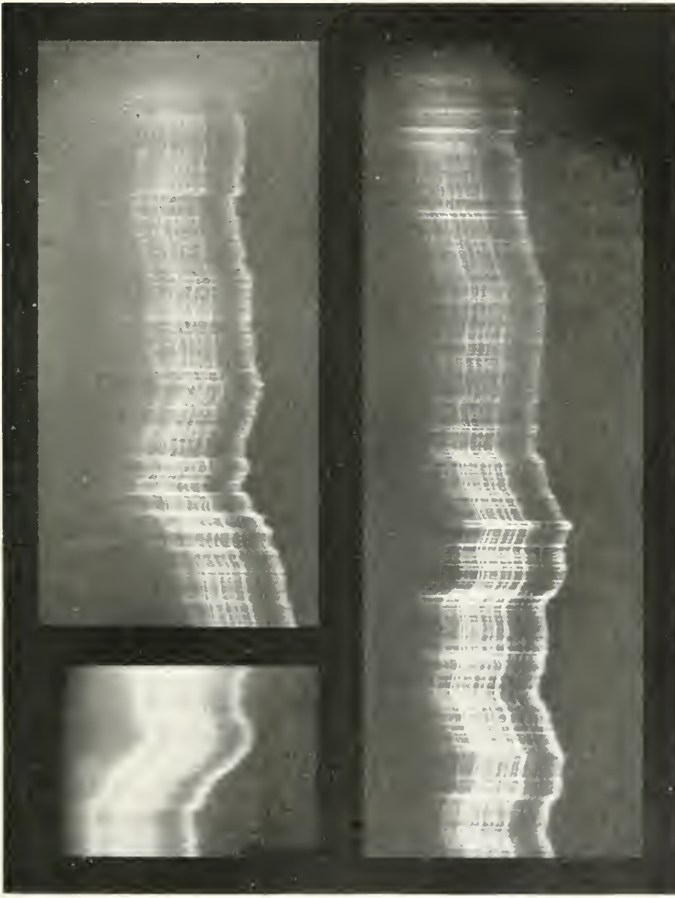
[1] Vogel, *Pogg. Ann.*, **143**, 653–654, 1871.

[2] *Phil. Mag.*, (5) **7**, 316–321, 1879.

[3] *Harvard College Observatory Circular* No. 62.

[4] *Proc. R. S.*, **17**, 61, 1869.

PLATE IX.



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1

SPECTRUM OF LIGHTNING FLASHES.

TABLE.

Measured Wave-Length		Character	Air Spectrum (Neovius)	Vogel	Schuster	Pickering
High Power	Low Power					
			3839.8			
			3843.1			
3848	3838	Faint, broad	3845.3			
			3850.6			
			3857.2			
	3890	Faint, double	3882.6			3881
3898	3915	at times	3912.2			
			3919.2			
			3945.3			
			3947.5			
3950	3943	Very faint	3954.6			
			3956.1			3956 } $H\epsilon$
3997	3997	Sharp	3995.2			3998 }
4041.5 s	4041,5 s	Sharp	4041.5			4046
			4072.4			
			4076.3			
4074	4077	Poor } coalesce at times	4097.3			
			4103.4			4102 $H\delta$
4106	4105	Fair	4105.2			
			4110.4			
			4152.0			4147
4143 v. edge		Broad,	4153.7			
4165 center	4154	double at	4176.7			
4183 r. edge	4183	times	4180.3			
			4185.8			4187
			4228.9			
4236	4238	Strong	4237.0			4222
			4242.7			4263
4349	4359	Broad, fair	4349.4			4341
			4415.0			
4439	4439	Strong	4417.3			
			4447.3			
4529	4535	Broad, hazy	4530.3			4519
			4596.6			
	4603. v.edge		to	4583 } band		
4630.7 s	4630.7 center	Broad, strong	4661.9	4673 }		
	4660. r. edge		maximum			
			4630.9			4643
4790	4786	Very faint	4788.5			4754
4858	4842	Very faint	4861.0	4860		4861 $H\beta$
5003.7 s	5003.7 s	Good	5002.7	5002	5002	4940
			5005.7			5022
5175	5156	Very faint	5180	5184	5181	5173
			includes			
			5453.8		5260	
5306 v. edge		Broad band,	5462.8			
5600 max.		red edge	5479.8	5341	5334	
5683 r. edge		very strong	5496.6			5595
			5535.2		5592	
			5667.1			
			5679.8		5681	

The letter s indicates the lines used as standards in deriving the formula for wave-lengths. The higher power used in the microscope magnified about fifteen times, the lower about three times.

In studying the spectra some curious facts were observed. The relative intensity of some lines with regard to their fellows is found to vary in different parts of the spectrum. In the first flash, the line at λ 4349 diminishes rapidly in relative intensity from the cloud to the ground. The line, a combination of λ 4074 and λ 4106, at the top of the flash is one of the heaviest lines, being much stronger than its two neighbors to the left, λ 4041.5 and λ 3997. Toward the ground, however, it has lost greatly in relative intensity, having been surpassed by both λ 4041.5 and λ 3997. These two have increased from very faint lines to rank among the strongest.

In comparing the first flash with the second, it is found that in the second these same changes do not exist. The only case of change in relative intensity is shown by the line at λ 4439, which increases slightly toward the ground.

The third flash, which was at the very edge of the plate, shows the line at λ 3848 very strong, while in the other flashes it is comparatively faint. This flash shows a line far to the violet of λ 3848.

This work has been conducted under the direction of Mr. Hale. The writer is indebted to Mr. Ellerman for assistance in the laboratory.

Yerkes Observatory,
October 12, 1903.

Minor Contributions and Notes.

PHOTOGRAPHIC SPECTRUM OF *NOVA GEMINORUM*.[1]

At the suggestion of Director Campbell, I obtained the spectrum of the *Nova* with the Crossley reflector, using the small slitless spectrograph. Six negatives were obtained on the night of April 2, with exposures ranging from 10 seconds to 19 minutes. Thirty seconds showed the stronger lines very faintly, and five minutes gave a good negative. The accompanying reproductions, Figs. 2 and 3, are chiefly from the negative having an exposure of ten minutes, although all the plates were made use of in determining the relative intensities. Fig. 2 shows the spectrum as a negative.

The plates used were ordinary commercial dry plates, sensitive only to blue and violet radiations. The recorded spectrum of *Nova Geminorum* consists of bright lines and bands superposed on a continuous spectrum, and extends from $H\beta$ to λ 335. The general appearance of the spectrum resembles somewhat the April 1901 spectrum of *Nova Persei* obtained by Campbell and Wright with the Mills spectrograph, in the region where the two instruments give comparable results. As no observations of the ultra-violet spectrum of *Nova Persei* were secured before September 1901, when the star had become a nebula, we do not know the early history of the lines at λ 339 and λ 346, photographed by Mr. Stebbins. These lines are certainly not yet developed in the case of *Nova Geminorum*, although there is a decided strengthening in the spectrum at about this point. There is very little similarity in the spectra of *Nova Persei* in September 1901 and *Nova Geminorum* in the region above $H\delta$. Lines occupying approximately the positions of $H\epsilon$ and $H\zeta$, as well as those at $\lambda\lambda$ 339 and 346, are the strongest lines in *Nova Persei*, but are indicated in *Nova Geminorum* only by slight strengthenings in the continuous spectrum. $H\beta$ and $H\delta$ are strong in the recent *Nova*, but very weak in the later spectrum of *Nova Persei*. The chief nebular line at λ 501 is not shown in the spectrum of *Nova Geminorum*, but was conspicuous in *Nova Persei*.

It is altogether probable that these differences between the spectrum of *Nova Geminorum* and the later spectrum of *Nova Persei* are

[1] From *Lick Observatory Bulletin* No. 37.

due to the different stages of development of the two stars. As *Nova Geminorum* assumes the nebular state, we may expect its ultra-violet spectrum to conform more and more to that shown by *Nova Persei.*

Following are the positions of the lines and maxima in the spectrum determined from three of the plates:

No. 8 5 m.	1903. April 2 No. 9 10 m.	No. 10 19 m.		Description
λ	λ	λ		
486	486	486	$H\beta$	Strong. Narrow
462	462	463		Very strong. Broad
445	446	446		Faint
434	434	434	$H\gamma$	Very strong
410	410	410	$H\delta$	" "
397	397	397	$H\epsilon$	Moderately strong
389	389	390	$H\zeta$	Faint
	384			Max. of band $H\eta$
374	374	372		" " $H\kappa$
350	350	352		" "
335	335	333		End of spectrum

Spectrograms were secured also on April 3, 5, 6, and 8. A comparison of those taken on the 2d and 8th show changes in the character of the spectrum, in the interval of six days. The most noticeable change is in the ultra-violet, where the continuous spectrum has become weaker and the bands at λλ 350, 374, and 384, more conspicuous in consequence. There are indications also of the development of the lines at λλ 339 and 346. Below $H\delta$ there seems to be little or no change in the continuous spectrum. $H\beta$ has become weaker, however, and there are traces of radiations in the region of the chief nebular line.

The spectrum was examined visually on all the nights when spectrograms were secured. The $H\alpha$ line was always very conspicuous. A brightening was observed in the yellow at about the position of the sodium lines.

In this connection it should be said that the dispersion with this spectrograph is so small, the linear distance between $H\beta$ and λ335 being only 3½ mm, that close lines cannot be separated with it.

Between April 1 and 8 the *Nova* decreased slightly in brightness. On the latter date it was recorded as fully as bright as the 8.6 magnitude star preceding, with which it was compared each night.

C. D. PERRINE.

APRIL 15, 1903.

PLATE X.

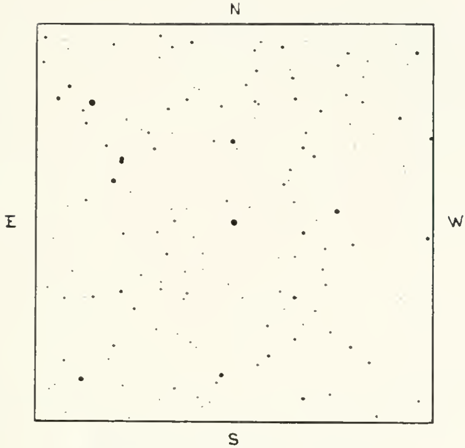

FIG. 1.— Region about *Nova Geminorum*.

FIG. 2.—The Photographic Spectrum of *Nova Geminorum* (Negative). April 2, 1903.

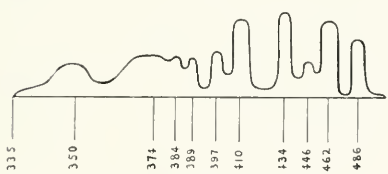

FIG. 3.—Intensity Curve.

THE SPECTRUM OF *NOVA GEMINORUM*.[1]

The first opportunity to observe the spectrum of *Nova Geminorum* was on the evening of April 1, when its magnitude was estimated at 8.5. Only visual observations were attempted, by means of Spectrograph I, consisting of a single light flint prism in connection with the collimator and camera sections of the Mills Spectrograph.[2] These visual observations and later photographs showed that the spectrum was intermediate between that of *Nova Persei* in April 1901 and in the following July. It consists mainly of isolated bright bands, many of which coincide approximately with the lines of hydrogen. The brightest band was coincident with $H\beta$. To the less refrangible side of it there was another strong band, and a stretch of faint continuous spectrum extending just beyond the position of the D lines. There were several faint maxima near the red end of this continuous spectrum. On another night we found that one of these coincided approximately with the D lines; and the bright $H\alpha$, overlooked on the first evening, was easily visible.

To the violet of the $H\beta$ band there appeared another band, fainter and broader, and farther up there was a faint band about the position of $H\gamma$. Nothing could be seen above this.

Following is the list of good photographs secured to date:

Number	Date, G. M. T.	Exposure	Kind of Plate
2717*A*........	April 2, 17^h 18^m	4^h 0^m	Cramer Crown
2818*D*........	5, 17 26	3 46	Cramer Iso. Inst.
2722*B*........	6, 17 21	3 55	Cramer Iso. Inst.
2731*A*........	8, 17 33	4 40	Cramer Iso. Inst.
2737*D*........	22, 17 12	3 0	Cramer Crown
2757*A*........	May 4, 17 9	2 36	Cramer Crown

The first photograph was secured on the evening of April 2. The auxiliary correcting lens brought all light in the $H\gamma$ region in focus on the slit, and the adjustments of the instrument were such that the camera was in fair focus from λ5000 to the ultra-violet. The plate substantiated our results of the previous night in the region common to both visual and photographic observation. Fig. 4 is a reproduction of this plate, and Fig. 5 is its intensity curve. Table I gives a description of the details of the bands. The wave-lengths are calculated by means of the Hartmann-Cornu formula, the constants of which are determined

[1] From *Lick Observatory Bulletin* No. 37. [2] See *L. O. Bulletin*, 1, No. 8, p. 46.

from certain lines in the comparison spectrum of iron. The wave-lengths are corrected for the orbital and the diurnal motions of the Earth. The effect of curvature of the lines is negligible. They are not corrected for the velocity in the line of sight of the star itself, for we have no means of determining that velocity with certainty. The spectrum seems at first sight to resemble that of *Nova Persei* in July 1901,[1] but there are great differences in the details. The several bands in the present case have not the uniformity of structure that characterized those of *Nova Persei*, nor are the maxima and minima by any means so sharp and definite. Indeed, it is quite possible that a number of the details shown in Fig. 5 and Table I are due to accidental arrangements of the silver grains.

More important still is the apparently total absence of certain bands which were present in the spectrum of *Nova Persei*. One of these is that corresponding to the nebular line at λ 3869, which in *Nova Persei* was considerably stronger than the $H\delta$ band. No trace of it appears on the present plate.

The band farthest to the violet has its maximum at λ 3987. Here again is a decided difference: in *Nova Persei* this band was very strong, and was displaced to the violet of the corresponding hydrogen line $H\epsilon$ more than the other bands corresponding with hydrogen lines; and it was shown[2] that the preponderating radiation was not $H\epsilon$, but a companion nebular line, previously unobserved. In *Nova Geminorum* this band is quite faint, and the maximum is 17 tenth-meters to the red of $H\epsilon$.

In *Nova Persei* the $H\gamma$ band was superimposed on another corresponding to the nebular line λ 4363. In *Nova Geminorum*, though the corresponding band shows greater evidences of complexity than any other in the spectrum, λ 4363 would fall almost wholly outside of it, so that no radiations corresponding to this line exist with an intensity at all comparable with those due to $H\gamma$.

There is in *Nova Geminorum* a band with maximum at λ 4625, which may perhaps be said to correspond with the λ 4643 band in *Nova Persei*, but there is none corresponding to the complex band λ 4669—λ 4746, which was apparently formed of two or more superposed bands. In *Nova Geminorum* the nebular line λ 4686 would fall just on the edge of the band that is present, and the helium line λ 4713 would lie entirely outside of it. No trace of the two nebular lines λ 4959 and λ 5007 can be seen on this plate. No doubt this is partially due to the lower sen-

[1] See *L. O. Bulletin*, 1, No. 8. [2] *L. O. Bulletin*, 1, No. 8, p. 54.

sitiveness of the plate at this region, and to the fact that much of the light is lost at the slit, for a later negative showed a very faint band at λ 5007 when the slit was properly focused for this light and an isochromatic plate used. It may be concluded, however, that with this exception the star showed no certain traces of the so-called "nebular" lines in the early part of April.

The positions of the bands are interesting. The strong band between $H\beta$ and $H\alpha$ has its maximum 18 tenth-meters to the violet of what we may call the corresponding band in the July spectrum of *Nova Persei*. The band corresponding to $H\epsilon$ has its maximum on the other hand 17 tenth-meters to the red of the position of the hydrogen line. Hartmann[1] states that on March 31 the middle of the $H\beta$ and $H\gamma$ bands were both displaced 8 Ångström units toward the red from the positions of the corresponding hydrogen lines. Our plate shows no such displacement; $H\beta$ appears only $1\frac{1}{2}$ and $H\gamma$ about $\frac{1}{2}$ of a tenth-meter too far to the red, and these amounts are hardly greater than the probable error. The discordance between Hartmann's results and ours may be explained, of course, by a real change in the spectrum between March 31 and April 2; but more probably it is due to the fact that Hartmann's plate was underexposed. The limits he assigns to the bands fall well within ours in each case, and in each of the bands the more intense part is toward the red of the middle. These two facts would certainly tend to make the middle of the band appear more to the red on his plate than on ours.

Messrs. Frost and Adams, discussing a plate taken March 28[2] describe $H\gamma$ as a very faint band merging into a brighter band which extends from λ 4347 to λ 4371. As indicated in Figs. 4 and 5, our plate shows some evidence of a composite character in what we have called the $H\gamma$ band, with much greater intensity near the red border than near the violet; but, as noted above, the middle of the whole band coincides very closely with the normal position for $H\gamma$, so that we have thought it best to speak of it as a single band showing considerable detail in structure. In any case, however, the limit λ 4371 falls outside the limits of the band on our plate, in spite of the fact that this part of the spectrum is doubtless much stronger on our plate than on theirs. There seems to be some evidence, therefore, of a perceptible change in the $H\gamma$ region between March 28 and April 2.

[1] *Astronomische Nachrichten*, 161, 324, 1903.

[2] ASTROPHYSICAL JOURNAL, 17, 304, 1903.

A second photograph of the same range of spectrum was taken April 22. It is reproduced in Fig. 6, with the corresponding intensity curve shown in Fig. 7. Table II gives a description of the details. As in the case of the earlier negative, many of the details plotted in

TABLE I.

April 2 2717 *A*	Description	
λ 3974	Beginning	Faint band
3987	Maximum	Faint band
3992	End	Faint band
	Faint continuous spectrum between these points	
4064	Slight increase in brightness	
4078	Beginning of stronger part of band	*Hδ* band
4081	Maximum	*Hδ* band
4083	Minimum	*Hδ* band
4094	Suspect a minimum	*Hδ* band
4098	Maximum	*Hδ* band
4100	Suspect a minimum	*Hδ* band
4102	Maximum	*Hδ* band
4103.0	Minimum rather sharp	*Hδ* band
4107	Maximum	*Hδ* band
4110.9	Minimum	*Hδ* band
4116	Maximum	*Hδ* band
4116.8	Minimum	*Hδ* band
4120	Maximum	*Hδ* band
4124	End of stronger part	*Hδ* band
4144	End of weaker part	
4317	Beginning of bright band	*Hγ* band
4327	Maximum	*Hγ* band
4331.7	Minimum	*Hγ* band
4334	Maximum	*Hγ* band
4336.8	Minimum	*Hγ* band
4340	Maximum, not well defined	*Hγ* band
4342.4	Minimum	*Hγ* band
4347	Maximum	*Hγ* band
4350.2	Minimum	*Hγ* band
4358	Maximum	*Hγ* band
4365	End of band	*Hγ* band
	Continuous spectrum between these points	
4449	Beginning	Faint band
4464	Maximum	Faint band
4492	End	Faint band
4570	Faint beginning of band	Strong band
4625	Principal maximum	Strong band
4646.2	Suspect narrow bright line or maximum	Strong band
4651.4	Suspect narrow bright line or maximum	Strong band
4658.9	Suspect narrow bright line or maximum	Strong band
4695	End of band	Strong band
4839	Beginning	*Hβ* band
4866	Maximum	*Hβ* band
4887	End	*Hβ* band

PLATE XI.

FIG. 4.—*Nova Geminorum*, enlarged ninefold. 1903, April 2.

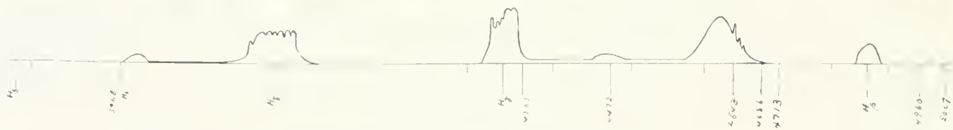

FIG. 5.—*Nova Geminorum*, Intensity Curve. 1903, April 2.

FIG. 6.—*Nova Geminorum*, enlarged ninefold. 1903, April 22.

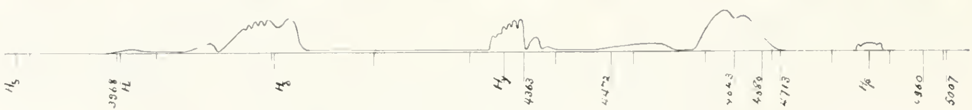

FIG. 7.—*Nova Geminorum*, Intensity Curve. 1903, April 22.

Fig. 7 and tabulated in Table II may be false effects due to chance grouping of the silver grains, though in each case the plate was measured under low magnification to avoid as far as possible errors of this kind.

TABLE II.

April 22 λ 2727 *D*	Description	
3959	Beginning of very faint continuous spectrum	
3982	Very faint maximum	
4021	Beginning of faint part of band	$H\delta$ band
4046	Maximum	
4052	Minimum	
4078	Suspect a minimum	
4080	Maximum	
4082.1	Minimum	
4084	Maximum	
4086.9	Minimum	
4090	Maximum	
4093.0	Minimum	
4095	Maximum	
4107	Very broad minimum	
4116	Broad maximum	
4126	End of brighter part	
4145	End of fainter part	
4322	Beginning of bright band	$H\gamma$ band
4326	Maximum	
4329.4	Minimum	
4334	Maximum	
4337.3	Minimum	
4341	Maximum	
4342.4	Minimum	
4346	Maximum	
4348.9	Minimum	
4352	Maximum	
4355.2	Minimum	
4358	Maximum	
4364	End of main part of band	
4378	Faint broad maximum	
4382	End of stronger part of auxiliary band	
4392	Suspect very faint maximum	
4454	Beginning	Very faint band in continuous spectrum, extending from λ 4392 to λ 4584
4536	Maximum	
4557	End	
4584	Beginning	Broad strong band
4630	Broad maximum	
4642	Broad minimum	
4656	Broad maximum	
4714	End	
4838	Beginning	$H\beta$ band
4843	Suspect a minimum	
4877	Suspect a minimum	
4884	End	

Even a casual examination showed that pronounced changes had taken place in some of the bands. In regard to the $H\beta$ band little can be said, for it is too weak on both plates to be sure of much structural detail. The middle of the strong band between $H\beta$ and $H\gamma$ had been moved about 17 tenth-meters to the red, making it more nearly coincide with the band observed at about this place in *Nova Persei* in the summer of 1901. It was certainly changed in detail also, for it showed two pronounced broad maxima on the later plate, as against one such maximum and two or three suspected narrow maxima on the earlier date. The faint band to the violet of this was also

TABLE III

April 5 2718 *D*	April 6 2722 *B*	April 8 2731 *A*	Description	
4838	4836	....	Beginning of band	$H\gamma$ band
4843	....	....	Dark line?	
4852	....	....	Slight maximum	
4859.7	4859.9	....	Minimum (or dark line?)	
4866	4865	....	Slight maximum	
4870	....	....	Slight minimum or dark line	
4874	....	....	Slight minimum or dark line	
4877	4878	....	Maximum of band. Exact position uncertain	
4886	4886	....	End of band	
....	4987	....	First trace of very faint band	λ 5007
....	5015	....	Minimum?	
....	5022	....	Beginning of second portion	
....	5030	....	End of band	
....	....	5409	First certain trace of continuous spectrum	
....	....	5480	Very slight maximum	
....	....	5493	Well defined minimum	
....	....	5516	Slight maximum suspected	
....	....	5527	Minimum?	
....	....	5557	Maximum of band	
....	....	5579	Slight minimum	
....	....	5589	Slight maximum	
....	....	5634	Faintest portion between bands	
....	....	5653	Dark line suspected. May be chance arrangement of silver grains	
....	....	5671	Maximum of band	
....	....	5678	Minimum	
....	....	5687	Maximum	
....	....	5700	End of brighter portion of band	
....	....	5708	End of band	
....	....	5727	Beginning of band	
....	....	5739	Maximum of band	
....	....	5746	Slight maximum?	
....	....	5758	Slight maximum?	
....	....	5766	Slight maximum?	
....	....	5767	Rather well marked minimum	
....	....	5777	End of band	

changed, becoming much broader and displaced much more toward the red. This band is so faint and broad that determinations of its limits and of the position of the maximum are uncertain. The faint band near $H\epsilon$ appeared relatively still fainter than on the earlier plate, with the maximum perhaps more toward the violet; that is, more nearly coincident with $H\epsilon$. The $H\delta$ band showed some changes in structure, as indicated in the intensity-diagrams, and also a faint companion band to the violet.

The most striking change, however, was in the $H\gamma$ band, which showed a fairly strong companion band to the red. We at once suspected that this was the first appearance of the nebular line λ 4363, but the measurements locate it too far to the red of the proper position of that line. Still it is not impossible that the whole complex band is due to radiations physically connected with the nebular line and the hydrogen $H\gamma$. No further discussion of this plate seems necessary, except to say that the $H\gamma$ band seems to have grown fainter with respect to the $H\delta$ and the λ 4643 bands.

A third plate of the same region was obtained on May 4. Unfortunately, the star was by this time so far west at dark that the exposure was necessarily rather short, so that the negative is weak in spite of a somewhat wider slit than was used in the other cases. So far as it goes, however, it seems to show that no perceptible change had occurred since April 22.

Table III gives the results for the plates taken in the $H\beta$ and the D regions. These plates are Nos. 2718*D*, 2722*B*, and 2731*A*. The comparison lines used are iron and helium. In the plates for April 6 and April 8 there is evidence of slight flexure due to the necessity for carrying the exposures to so large an hour angle. On all these plates the bands show little certain evidences of structure, and some of the details plotted in the intensity curves may well be due to a fortuitous grouping of silver grains. Plates 2718*D* and 2722*D* show also traces of the band at λ 4643, but as this band is well seen on other plates, no attempt has been made to study its details on these plates. No trace is seen on Plate 2731*A* of any band at D_3 or beyond.

A comparison of these plates with those of similar regions of *Nova Persei* taken in July and August 1901, is rendered somewhat difficult, as far as the more minute details are concerned, by the comparatively diffuse and uncertain character of the structural details seen in the *Nova Geminorum* plates. In the case of $H\beta$ the maximum intensity is very clearly at the red edge of the band (λ 4878), while in *Nova Persei*

it lies at the violet edge (λ 4852). A very slight maximum is, however, to be seen at about λ 4852 in the plate of April 5. In the region from λ 5400–λ 5800 the similarity in general detail is more marked, but the band at λ 5671 is relatively much brighter in *Nova Geminorum* than in *Nova Persei* on August 11, 1901. The most striking differences in this portion of the spectrum are the faintness of the band at λ 5007 and the apparent lack of any band at D_3. λ 5007 was relatively faint in *Nova Persei* in February and March 1901, but in July and August was very much brighter than any of the bands between it and D_3. In *Nova Geminorum* it appears as a mere trace on one plate only, and in neither plate is there a trace of the band at λ 4959. Too much reliance cannot be placed on the apparent absence of a band at D_3, on account both of the faintness of the star and of the fact that the curve of sensitiveness of the plates used changes very rapidly at this point.

Figs. 8 and 9 give a view of the region covered in Plate 2731*A* and the corresponding intensity curve. Fig. 10 shows the $H\beta$ band and the extremely faint trace of λ 5007. Figs. 11 and 12 give the intensity curves for Plates 2718*D* and 2722*B*.

It will be seen from Table III that in the plates for April 5 and April 6 there is a suspected dark line at approximately λ 4860. The mean corrected shift of this line, if real, is 1.7 t. m., corresponding to a velocity of −105 km per second. The observed appearance is too faint, however, to warrant placing much reliance upon the velocity thus obtained.

The spectrum was tested on April 4 for evidences of Zeeman effects, by rotating a Nicol prism in the eyepiece of the spectroscope. The forms of the broad bright bands appeared to remain unchanged in all positions of the prism. The same tests applied to *Nova Persei* in April 1901, gave similar results.

H. M. Reese.
H. D. Curtis.

May 9, 1903.

A LIST OF FIVE STARS WHOSE VELOCITIES IN THE LINE OF SIGHT ARE VARIABLE.[1]

The following five stars with variable radial velocities, discovered with the Mills spectrograph, are additional to the forty-two already

[1] *Lick Observatory Bulletin* No. 46.

PLATE XII.

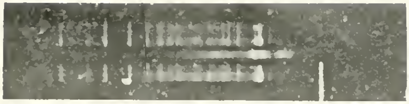

FIG. 8.—*Nova Geminorum*, Region near *D*, enlarged ninefold. 1903, April 8.

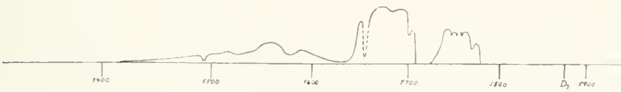

FIG. 9.—*Nova Geminorum*, Region near *D*, Intensity Curve. 1903, April 8.

FIG. 10.—*Nova Geminorum*, *Hβ* and λ 5007. 1903, April 8.

FIG. 11.

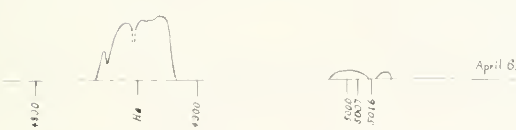

FIG. 12.

FIGS. 11 and 12. *Nova Geminorum*, Intensity Curves for *Hβ* Region. 1903, April 5 and 6.

announced. A number of the photographs and one of the measures on which these results depend are by Dr. Reese.

γ *Corvi* ($\alpha = 12^h\ 10^m.7$; $\delta = -16°\ 59'$).

Date	Velocity	Measured by
1902, December 30	+ 2 km	Curtis
1903, February 23	0	Curtis
May 6	−20	Curtis
May 11	− 6	Curtis
May 17	+ 4	Curtis

The variation was discovered from the third plate. The line at $H\gamma$ is very fair with proper exposure; those due to helium and magnesium are good.

η *Virginis* ($\alpha = 12^h\ 14^m.8$; $\delta = -0°\ 6'$).

Date	Velocity	Measured by
1903, May 17	+17 km	Curtis
May 24	+ 4	Curtis

$H\gamma$ is fair and $\lambda 4481$ good. There are in addition scattered iron lines, narrow and quite good.

α *Draconis* ($\alpha = 14^h\ 1^m.7$; $\delta = +64°\ 51'$).

Date	Velocity	Measured by
1902, June 16	± 0 km	Stebbins
	+ 2	Curtis
1903, April 29	−43	Curtis
May 24	−42	Curtis

The variable velocity was discovered from the second plate. The measures depend on two lines only, a rather broad $H\gamma$ and a strong magnesium.

ϵ *Herculis* ($\alpha = 16^h\ 56^m.5$; $\delta = +30°\ 4'$).

Date	Velocity	Measured by
1903, May 24	−70 km	Curtis
May 27	−34	Curtis

In this star $H\gamma$ is very broad; the magnesium line a λ 4481 is also rather broad. The measures depend on the magnesium line alone.

δ *Aquilae* ($\alpha = 19^h\ 20^m.5$; $\delta = +2°\ 55'$).

Date	Velocity	Measured by
1900, May 22	−25 km	Curtis
1902, July 31	−35	Reese
1903, May 12	− 2	Curtis
May 27	−32	Curtis

The iron lines are diffuse and broad; both $H\gamma$ and λ 4481 are broad and hard to measure. The first plate is a very poor one and the given velocity can be considered only as an approximation.

W. W. Campbell.
Heber D. Curtis.

June 11, 1903.

THE

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THE VARIABLE STAR 1921 *W AURIGAE.*

By J. A. PARKHURST.

THE variability of this star was discovered by Madame Ceraski from photographs taken by Blajko at the Moscow Observatory.[1] It was found to be 8.9 magnitude (evidently near maximum) in March and April 1898, but in October of the same year it was invisible in a "7-foot" telescope. Blajko[2] has published a summary of photographic and visual observations from 1895 to 1900, deducing from them a period of 0.75 year. In column 295 of this summary the *minimum* attributed to the writer should read *maximum*. Hartwig[3] has published a note on the place of the variable, but his conclusion in regard to the period, "somewhat greater than a year," is not in accord with the results of this paper. Esch[4] has published a note containing a few observations in February and March 1902, which are in good agreement with the light-curve shown in Fig. 2. Provisional results from the observations of 1898 to 1900 have been published by the writer,[5] with charts and approximate magnitudes of the comparison stars.

[1] *Astronomische Nachrichten*, **148**, 15, 1898.

[2] *Ibid.*, **153**, 295, 1900.

[3] *Ibid.*, **149**, 6, 1899.

[4] *Ibid.*, **160**, 335, 1902.

[5] ASTROPHYSICAL JOURNAL, **12**, 54, 1900; **14**, 171, 1901; *Astronomical Journal*, **20**, 6, 1899; *Popular Astronomy*, **7**, 43, 1899; **8**, 461, 1900.

POSITION OF THE VARIABLE.

The variable was connected on three nights with the star c, which is $B.D.+36^\circ 1141$, and whose place is:

	R. A.	Dec.	
	h m s		
From the Lund *A. G.* Catalogue	5 19 00.47	+36° 43′ 28″.7	(1875)
	5 20 41.84	+36 44 56.1	(1900)
Difference, *variable*−*c*	−0 33.28	−3 56.8	
Place of the *variable*	5 20 08.56	+36 48 52.9	(1900)

As the last decimal place should be discarded, the result can be stated:

	h m s		
1921 W Aurigae	5 20 08.6	+36° 48′ 53″	(1900)
	5 17 06.0	+36 46 11	(1855)

This place agrees closely with that given by Hartwig in the reference above cited.

INSTRUMENTS.

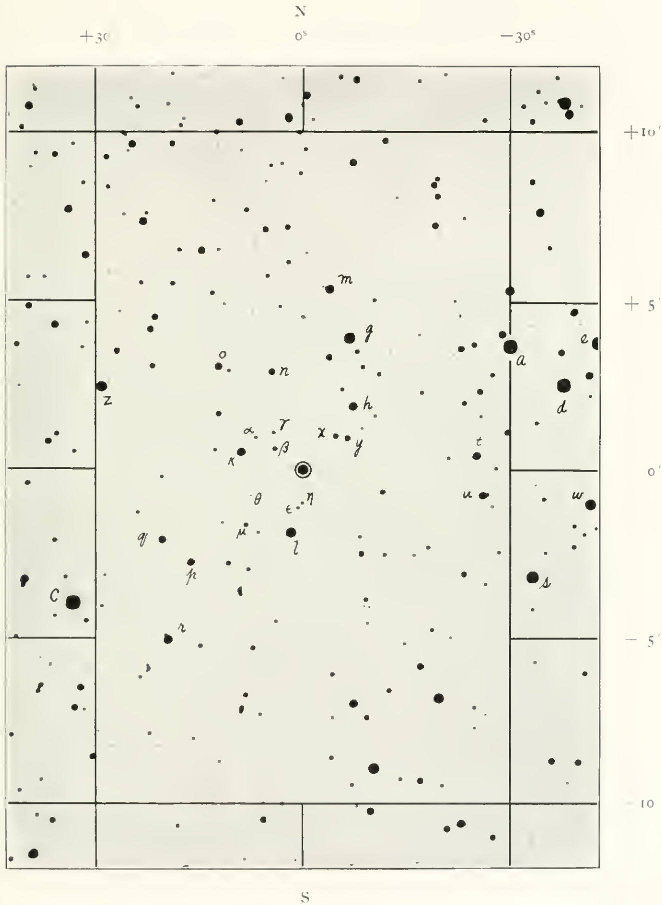
These are the same as described in the two preceding papers of this series,[1] a 157 mm (6.2-inch) reflector, a 305 mm (12-inch) refractor, and the 101 cm (40-inch) refractor, the equalizing wedge photometer used in the photometric work being attached to each of these instruments. In the photometric measures of the brighter standard stars with the 12-inch telescope an absorption glass was interposed in the cone of rays from the objective. The adopted value of the absorption, 1.70 magnitudes, depends upon a series of laboratory measures and also on measures of *Pleiades* stars, but further measures may change it a few hundredths of a magnitude.

THE CHART.

Plate XIII shows the field 18′ × 24′ around the variable, on a scale of 10″ to the millimeter. The negative was taken with the 24-inch reflector January 11, 1902, exposure from $6^h\ 0^m$ to $6^h\ 55^m$, Central Standard Time, and shows stars to the sixteenth magnitude distinctly; but as the fainter stars were not used in the comparisons, the print for the cut was made to show stars only

[1] "The Variable Star *X Cephei*," ASTROPHYSICAL JOURNAL, **17**, 48, 1903; "The Variable Star *V Lyrae*," *ibid.*, **18**, 33, 1903.

PLATE XIII.



1921 *W AURIGAE*

$(5^h\ 20^m\ 8^s.6;\ +36^\circ\ 48'\ 53'')$

to the fifteenth magnitude. The strong color of the variable is shown by the fact that the photographic magnitude is about 10.9, the diameter of the disk being between that of *g* and *l*. The light-curve shows that the variable was near maximum, probably about 9.5 magnitude visually at this time.

COMPARISON STARS.

The three standard stars on which are based the measures of the comparison stars are as follows:

TABLE I.

Star	*B. D.*				Magnitudes		
	No.	Mag.	Position for 1855		Potsdam		Harvard 45
			R. A.	Dec.	Color	Mag.	Mag.
			h m s				
A....	+37° 1160	7.2	5 12 42.0	+37° 31′.8	GW–	7.63	7.39
B....	+36 1122	6.8	5 14 59.4	+36 3.6	G–	6.60	6.74
C....	+36 1113	6.8	5 14 5.7	+36 15.1	GW	6.98	6.90
					Mean	7.07	7.01

Of the stars used for visual comparisons with the variable, the following are in the *Durchmusterung:*

TABLE II.

Star	No.	Mag.	1855	
			R. A.	Dec.
			h m s	
e	+36° 1133	9.5	5 16 22.6	+36° 49′.9
s	+36 1134	9.5	5 16 30.4	+36 42.9
a	+36 1138	9.5	5 16 35.8	+36 50.3
c........	+36 1141	8.9	5 17 40.0	+36 42.6
b........	+36 1145	9.2	5 18 13.9	+36 58.1
m′......	+36 1147	9.2	5 18 19.2	+36 53.2
o′.......	+37 1200	8.5	5 18 23.5	+37 6.2

The date for the remaining comparison stars are given in the following table:

TABLE III.

COMPARISON STARS FOR *W AURIGAE.*

(IN ORDER OF RIGHT ASCENSION.)

Star	Co-ordinates from *V*		Light Scale	Magnitude		Star	Co-ordinates from *V*		Light Scale	Magnitude	
	R. A.	Dec.		Meas-ured	From Curve		R. A.	Dec.		Meas-ured	From Curve
	s		steps				s		steps		
f....	−47	+5′ 42″	....	.....	.....	*l*...	+ 2	− 1′ 51″	25.4	10.94	.....
e....	−43	+3 48	33.8	.....	10.0	*β*..	+ 4	+ 0 37	9.1	13.58	.....
w...	−42	−0 57	20.4	.....	11.93	*γ*..	+ 4	+ 1 6	4.9	14.25	.....
d...	−38	+2 28	....	.....	.....	*n*..	+ 4	+ 2 58	20.3	.....	11.94
s....	−34	−3 5	31.3	.....	10.35	*α*...	+ 7	+ 0 56	9.8	13.65	
a...	−30	+3 19	33.3	10.35	.....	*θ*...	+ 8	− 0 47	6.7	14.13	.....
t....	−26	+0 26	19.3	.....	12.10	*μ*...	+ 8	− 1 39	11.3	13.39	.....
u...	−26	−0 43	18.3	.....	12.26	*k*...	+ 9	+ 0 30	25.9	11.17	.....
h...	− 7	+1 46	21.9	11.60	.....	*o*...	+12	+ 2 58	....	.....	.....
g...	− 7	+3 50	29.2	10.79	.....	*p*...	+16	− 2 37	22.0	.....	11.73
y....	− 7	+0 56	17.1	12.42	.....	*r*...	+19	− 4 59	20.9	.....	11.89
x...	− 5	+0 56	15.5	12.63	....	*q*...	+20	− 2 1	22.3	.....	11.67
m...	− 4	+5 20	22.0	.....	11.62	*z*...	+29	+ 2 48	28.9	.....	10.73
η....	− 0	−0 56	0.0	14.12	.....	*c*...	+33	− 3 57	37.2	9.58	.....
ε....	+ 1	−1 8	2.3	14.34	.....	*m′*.	+73	+ 7 0	38.4	.....	9.37
						o′..	+77	+20 0	42.0	.....	8.70

TABLE IV.

PHOTOMETRIC MEASURES OF COMPARISON STARS.

1902, January 24; 12-inch. Wedge V, seeing good, full Moon.

Star	Scale Readings						Mean Scale Reading	Magnitude
	First			Second				
Aa.....	23.7	24.2	23.3	23.8	23.0	23.0	23.50	7.63
Ca.....	15.3	15.0	16.0	15.0	15.8	15.8	15.48	6.98
Ba....	14.5	13.2	13.8	13.2	11.2	12.2	13.02	6.60
a......	34.1	35.0	35.0	35.2	33.4	35.0	34.45	10.66
g......	37.8	39.2	38.2	34.8	34.2	34.0	36.37	10.87
h......	43.3	42.0	41.3	42.2	42.2	43.2	42.37	11.54
k......	38.2	38.9	39.1	39.8	39.1	39.9	39.17	11.18
l......	38.2	38.2	37.0	40.2	39.7	40.8	39.02	11.17
v......	29.5	28.8	28.7	....	....	....	29.00	9.95

Wedge constant, 0.110 mag.

TABLE IV.—*Continued.*

1902, February 7; 12 inch. Wedge V, seeing good.

Star	Scale Readings						Mean Scale Reading	Magnitude
	First			Second				
Aa.....	18.8	17.5	17.8	....	....	....	18.37	7.63
Ca.....	12.0	12.8	13.2	....	....	....	12.67	6.98
Ba	9.2	10.7	10.2	....	....	....	10.03	6.60
a......	27.7	27.4	26.2	....	....	....	27.10	10.25
g......	32.1	32.2	32.2	....	....	....	32.17	10.80
h......	37.0	39.4	39.0	....	....	....	38.47	11.50
k......	34.5	33.7	34.0	....	....	....	34.07	11.01
l.....	32.1	31.1	31.0	....	....	....	31.40	10.72

Wedge constant, 0.110 mag.

1903, January 1; 12-inch. Wedge V, seeing good.

Star	First			Second			Mean Scale Reading	Magnitude
A......	14.7	12.0	12.8	....	....	....	13.17	7.63
Ba	19.0	21.2	19.8	....	....	....	20.00	6.60
C	7.3	4.1	4.7	....	....	....	6.16	6.98
Ca	21.4	23.1	22.0					
a......	37.8	38.0	38.8	37.2	37.3	37.7	37.80	10.35
g......	43.0	44.5	43.1	38.8	42.8	41.7	42.32	10.85
h......	52.9	49.4	51.4	50.1	49.9	50.0	50.62	11.76
k......	47.2	48.0	47.8	45.9	46.1	45.3	46.72	11.33
l......	44.8	45.8	45.3	43.0	42.3	42.8	44.00	11.03

Wedge constant, 0.110 mag.

1903, March 21; 6-inch. Wedge V, seeing fair.

Star	First			Second			Mean Scale Reading	Magnitude
B.....	11.5	11.3	11.7	12.4	12.5	12.3	11.95	6.60
C......	15.8	16.2	15.9	16.1	15.3	16.3	15.94	6.98
A......	22.2	22.0	22.3	20.6	21.7	21.7	21.75	7.63
a......	47.7	47.8	48.2	45.7	46.5	45.7	46.94	10.41

Wedge constant, 0.110 mag.

1903, March 22, 6-inch. Wedge V, seeing good.

Star	First			Second			Mean Scale Reading	Magnitude
B.....	14.0	13.8	13.4	13.6	15.0	14.7	14.08	6.60
C.....	17.0	15.7	16.2	17.7	18.5	18.2	17.22	6.98
A......	23.2	22.6	23.2	21.5	22.7	21.5	22.45	7.63
a......	45.5	45.5	44.7	43.3	43.7	43.0	44.28	9.97
g......	49.6	49.4	49.8	51.2	50.9	51.2	50.35	10.64
c......	40.8	39.0	39.7	40.0	40.9	40.4	40.78	9.58

Wedge constant, 0.110 mag.

TABLE IV.—*Continued.*

RESULTING MAGNITUDES, FROM MEASURES WITH 6- AND 12-INCH.

	1902 January 24	1902 February 7	1903 March 1	1903 March 21	1903 March 22	Means
c	.....	.....	.....	.....	9.58	9.58
a	10.66	10.25	10.35	10.41	9.97	10.35
h	11.54	11.50	11.76	.....	.....	11.60
g	10.87	10.80	10.85	.....	10.64	10.79
k	11.18	11.01	11.33	.....	.... .	11.17
l	11.17	10.72	11.03	.....	.....	10.94

1900, October 23; 40-inch. Wedge II, seeing fair.

Star	Scale Readings						Mean Scale Reading	Magnitude
	First			Second				
k	15.9	16.7	17.2	19.0	17.9	16.8	17.25	11.17
h	21.0	19.9	21.8	21.9	22.7	22.0	21.55	11.60
g	13.2	15.0	15.9	14.0	15.5	14.0	14.60	10.79
l	17.1	17.8	17.0	19.8	19.2	19.9	18.47	10.94
α	40.2	41.1	39.9	37.3	37.9	37.5	38.98	13.63
γ	46.7	46.4	42.8	39.3	41.8	41.1	43.02	14.21
β	40.8	39.5	38.0	38.9	38.2	37.0	38.73	13.60
θ	42.7	40.9	40.0	42.2	44.1	42.8	42.12	14.06
μ	38.2	37.1	36.1	34.5	35.0	35.0	35.98	13.24
ε	43.8	46.0	44.2	45.9	46.4	45.0	45.22	14.44
η	42.5	43.7	43.4	41.8	43.5	41.0	42.65	14.11
x	30.0	29.4	30.1	30.0	31.8	32.1	30.58	12.54
y	25.2	28.8	25.8	28.0	27.0	26.0	26.80	12.05
δ	32.9	31.0	31.0	29.9	32.0	31.2	31.33	12.64
v	34.2	33.0	31.5	34.0	31.9	33.9	33.09	12.79

Wedge constant, 0.130 mag.

1902, February 4; 40-inch. Wedge V, seeing good.

k	10.0	10.0	9.2	11.1	10.9	11.8	10.50	11.17
l	10.8	11.3	11.1	10.2	12.2	11.3	11.15	10.94
ha	26.6	28.3	26.1	....	....	....	27.00	11.60
ga	18.3	19.9	18.9	....	....	....	19.03	10.79
μ	30.3	31.0	31.0	....	....	....	30.77	13.47
θ	37.7	38.7	37.3	...	....	...	37.90	14.26
ε	37.8	38.4	38.6	....	....	....	38.27	14.30
β	30.5	31.6	31.3	....	....	....	31.13	13.51
γ	37.3	38.0	38.3	....	....	....	37.87	14.26
α	33.6	32.6	32.3	....	....	....	32.83	13.70
x	20.9	22.0	23.0	....	....	....	21.97	12.51
y	22.0	21.0	20.8	....	....	....	21.27	12.43

Wedge constant, 0.110 mag.

TABLE IV.—*Continued.*

1902, February 6; 40-inch. Wedge V, seeing fair.

Star	Scale Readings						Mean Scale Reading	Magnitude
	First			Second				
h_a.....	35.2	35.0	35.1	33.8	34.0	33.0	34.35	**11.60**
k_a.....	27.5	26.3	26.0	....	....	....	26.60	**11.17**
l_a . . .	29.4	29.2	29.3	28.0	27.1	27.2	28.37	**10.94**
y.....	27.8	29.0	28.2	....	....	....	28.33	12.78
x......	29.1	28.6	29.1	....	....	....	28.93	12.85
β......	35.2	35.8	36.8	....	....	....	35.93	13.62
γ......	42.0	42.0	42.3	....	....	....	42.10	14.30
α......	36.5	36.0	35.5	....	....	....	36.00	13.63
θ......	40.5	40.0	39.5	....	....	....	40.00	14.07
μ......	34.0	34.2	35.2	....	....	....	34.47	13.45
ϵ......	41.2	43.0	41.8	....	...	....	42.00	14.29

Wedge constant, 0.110 mag.

RESULTING MAGNITUDES, FROM MEASURES WITH 40-INCH.

	1900 October 23	1902 February 4	1902 February 6	Means
x......	12.55	12.51	12.85	12.63
y......	12.05	12.43	12.78	12.42
α......	13.63	13.70	13.63	13.65
β......	13.60	13.51	13.62	13.58
γ......	14.20	14.26	14.30	14.25
δ.....	12.65	.....	.....	12.65
ϵ......	14.44	14.30	14.29	14.34
θ......	14.06	14.26	14.14	14.13
η	14.12	.....	.....	14.12
μ......	13.26	12.47	13.45	13.39

The photometric measures of the comparison stars are given in Table IV, which is arranged similarly to the corresponding table in the two previous papers. The stars used as standards are given first with their magnitudes in *bold-faced* type. The magnitudes for A, B, and C are taken from Table I, using the Potsdam values; to reduce to the Harvard scale subtract 0.06 from the numerical magnitude. The values for the fainter standards, lettered from a to l, are taken from the part of Table

TABLE V.

1921 W Aurigae.

(Comparisons of the Variable by Argelander's Method.)

No.	Date				Ocular	Aperture	Comparisons
	Month	Day	Hour	Julian Day			
	1898		C.S.T.	G. M. T. 2410000+			
1	Dec.	10	6	4634.5	40 80	6 6	*o′* 4 *v*, *c* 2 *v*, *v* 1 *a* *c* 1 *v*, *v* 1 *a*
2		12	18	4637.0	40	6	*c* 0-1 *v*, *v* 2 *a*
3		13	7	4637.5	40	6	*m′* 2 *v*, *v c*, *v* 1 *b*
4		15	8	4639.6	40	6	*m* 1 *v*, *v* 2 *b*, *v* 0-1 *c*
5		17	7 8	4641.5 4641.6	40 80	6 6	*m′* 1 *v*, *v c*, *v* 3-4 *a* *v* 1 *c*
6		22	6	4646.5	40	6	*v* 2 *b*, *m′* 0-1 *v*, *v* 1 *c*, *v* 4 *a*
7		26	8	4650.6	40	6	*v* 2 *c*, *v m′*, *o* 5 *v*
8		28	8	4652.6	80	6	*c* 0-1 *v*
9		30	7	4654.5	40	6	*c* 1 *v*
	1899						
10	Jan.	5	6	4660.5	40	6	*v* 0-1 *c*
11		10	7	4665.5	150 40	6 6	*c* 2-3 *v*, *v* 3-4 *a* *c* 1-2 *v*, *v* 3 *a*
13		18	7	4673.5	150 40	6 6	*c* 4 *v*, *v* 1 *a* *c* 3 *v*, *v* 0-1 *a*
14		24	7	4679.5	40 150	6 6	*c* 4 *v*, *a* 2 *v* *c* 4-5 *v*, *a* 2 *v*, *v s*, *v* 3-4 *g*
15		28	7	4683.5	150	6	*c* 6 *v*, *a* 2 *v*, *s* 1 *v*, *v* 3 *g*
16	Feb.	1	7	4687.5	150	6	*c* 4 *v*, *e* 4 *v*, *s* 2 *v*, *v* 2 *g*, *v* 3 *l*
17		15	7	4701.5	150	6	*s* 5 *v*, *g* 2-3 *v*, *l* 1 *v*, *v* 0-1 *k*, *v* 4-5 *h*
18		28	8	4714.6	150	6	*h* 1-2 *v*, *v* 2 *n*, *v* 3 *y*
19	Mar.	6	8	4720.6	150	6	*h* 2 *v*, *v* 2 *y*
20		13	8	4727.6	150	6	*n* 2-3 *v*, *v* 2 *y*, *h* 4 *v*
21		28	8	4742.6	150	6	*y* 2-3 *v*
22	Apr.	4	8	4749.6	200	6	*y* 1-2 *x*, *x* 2 *v*
23		12	8	4757.6	150	6	*v* not seen
24		28	8	4773.6	150	6	*v* not seen
25	May	4	8	4779.6	150	6	*v* not seen
26	Oct.	30	8	4958.6	40 150	6 6	*a* 1 *v*, *v* 2 *s*, *v* 2-3 *g* *a* 0-1 *v*, *v* 2 *s*, *v* 3 *g*
27	Nov.	4	8	4963.6	150	6	*a* 1 *v*, *v* 1 *s*, *v* 3 *g*
28		20	7	4979.5	150	6	*g* 3 *v*, *v* 2 *l*
29		26	7	4985.5	150	6	*g* 5 *v*, *l* 0-1 *v*, *v* 1-2 *h*
30	Dec.	5	7	4994.5	150	6	*l* 3 *v*, *h* 1 *v*, *v* 1 *n*, *v* 4 *y*
31		19	7	5008.5	150	6	*h* 2-3 *v*, *v* 1 *n*
32		28	7	5017.5	150	6	*h* 6 *v* ±, *v* 0-1 *n*, *v y*, *v* 1 *x*
	1900						
33	Jan.	4	7	5024.5	200	6	*y* 1-2 *v*, *v* 1 *x*
34		8	6	5028.5	350	40	*y* 2 *v*, *v* 1 *x*
35		25	8	5045.6	175	12	*x* 5-6 *v*
36		26	10	5046.7	350	40	*x* 2 3 *v*, *v* 2-3 β β α, α 4 γ, γ 4 ε, ε 2 η
37	Feb.	4	11	5055.7	350	40	*x* 3 4 *v*, *v* 3-4 α *x* 5 α, α 2 β, β 6 γ, γ 1 ε, ε 2 η

TABLE V.

1921 W Aurigae.

(Reduction of Observations.)

No.	Details in Steps	Means: Steps	Means: Magnitudes	Seeing	Remarks
1	38.9, 35.2, 34.3 / 36.2, 34.3	35.7	9.74	good	
2	36.7, 35.3	36.0	9.68	fair, faint twilight	
3	36.4, 37.2, 36.6	36.7	9.59	good	
4	37.4, 37.6, 37.7	37.5	9.46	good	
5	37.4, 37.2, 36.8 / 38.2	37.6	9.64	good, Moon	
6	37.6, 37.9, 38.2, 37.3	37.2	9.50	good, Moon	
7	39.2, 38.4, 37.9	38.5	9.32	good, Moon	
8	36.7	36.7	9.59	poor	
9	36.2	36.2	9.67	fair	
10	37.7	37.7	9.43	fair to poor	
11	34.7, 36.8 / 35.7, 36.3	35.6	9.75	good	
13	33.7, 34.3 / 34.2, 33.8	34.0	9.97	good	
14	33.2, 31.3 / 32.7, 31.3, 31.3, 32.7	32.0	10.27	good, Moon	
15	31.2, 31.3, 30.3, 32.2	31.2	10.38	good	
16	29.3, 29.8, 29.3, 31.2	29.8	10.60	good	
17	26.3, 26.7, 24.4, 26.4, 26.4	26.0	11.13	good, Moon	
18	20.4, 22.3, 20.1	20.9	11.88	good	
19	19.9, 19.1	19.5	12.07	good	
20	17.8, 19.1, 17.9	18.2	12.27	good	
21	14.6	14.6	12.79	good	
22	13.5	13.5	12.94	fair to good	
23			<12.5	fair	limit *x* or *y*
24			<12.0	fair	limit *n*
25			<12.0	twilight	limit *n* and *o*
26	32.3, 33.3, 31.7 / 32.8, 33.3, 32.2	32.6	10.18	low, fair	
27	32.3, 32.3, 32.2	32.2	10.24	low, fair	
28	26.2, 28.4	27.3	10.96	poor	
29	24.2, 24.9, 22.4	23.8	11.47	fair	
30	22.4, 20.9, 21.3, 21.1	21.4	11.79	good	
31	19.4, 21.3	20.3	11.95	fair, Moon rising	
32	15.9, 20.8, 17.1, 16.5	17.5	12.37	good	
33	15.6, 16.5	15.8	12.58	small Moon	
34	15.1, 16.5, 15.8	15.8	12.61	Moon	
35	10.0	10.0	13.47	good	
36	13.5, 11.6	12.5	13.09		
37	13.5, 13.3	13.4	12.96		

TABLE V—*Continued.*

No.	Date				Ocular	Aperture	Comparisons
	Month	Day	Hour	Julian Day			
	1900		C.S.T.	G. M. T. 2410000+			
38	Feb.	18	10	5069.7	350	40	$v\ \alpha$, $v\ \beta$, v 5-6 γ, v 6 η x 6-8 v, v 1-2 α (2d comparison)
39		22	9	5073.6	350	40	α 2 v, β 2 v, v 3 γ, v 5 η α 1 v, β 1 v, v 4 γ, μ 2 β, β 3 θ
40		24	10	5075.7	175	12	v not seen
41	Mar.	1	8	5080.6	350	40	α 1-2 v, β 1-2 v, v 4 γ, v 1 θ, v 5 η
42		21	9	5100.6	275	12	v 1 β, μ 4 v, v 1 θ
43		22	9	5101.6	350	40	μ 3 v, v 1 θ, v 3 γ, β 1 v
44	Apr.	4	9	5114.6	275	12	μ 2 v, v 2 β
45		18	8	5128.6	275	12	y 2 v, v 1 x, v 2 μ
46		19	8	5129.6	275	12	v 2 μ, x 1 v, y 2-3 v, v 3 β
47		27	10	5137.7	350	40	$v\ y$, v 2 x, n 3 v
48		29	9	5139.6	150	6	v not seen
49	Aug.	30	19	5263.0	237	40	v 2 k, v 3 l
49*a*	Oct.	23	12	5316.8	237	40	
	1901						
50	Feb.	9	7	5425.5	150	6	v not seen
51		10	7	5426.5	150	6	v not seen
	1902						
52	Jan.	11	7	5761.5	...	24	
53		24	7	5774.6	67	12	
54	Feb.	6	10	5787.7	237	40	
	1903						
55	Jan.	1	8	6116.6	67	12	$v\ n$, h 5 v, v 4 x
56	Mar.	21	11	6195.7	80	6	v not seen

IV headed "Resulting Magnitudes from Measures with 6- and 12- Inch." The subscript $_a$ indicates that the star was measured through the absorption glass, before mentioned, which cuts down the light 1.70 magnitudes (equals 15.45 scale-divisions with wedge V). For the fainter stars, from x down, the average deviation of the separate night's result from the mean of the three is 0.09 magnitude. If the star y, for which the deviation is anomalous, be omitted, the average deviation becomes 0.07 magnitude.

MAGNITUDE-CURVE.

This is given in Fig. 1, platted with photometric magnitudes as abscissæ and positions in the light-scale as ordinates; giving the points shown by the round dots. In this case the "curve"

TABLE V—*Continued.*

No.	Details in Steps	Means		Seeing	Remarks
		Steps	Magnitudes		
38	9.8, 9.1, 9.4, 6.0; 8.5, 8.3	8.5	13.67	fair	
39	7.8, 7.1, 7.9. 5.0; 8.8, 8.1, 8.9	7.6	13.82	good	
40		<11.5	<13.25	fair	limit 4 < x
41	8.3, 7.6, 8.9, 7.6	7.5	13.83	good	
42	10.1, 7.3, 7.7	8.3	13.70	good	limit θ or 1 < β
43	8.3, 7.7, 7.9	7.9	13.77	good	limit 4 < η
44	8.3, 11.1	9.7	13.50	good, Moon	
45	15.1, 16.5, 13.3	14.9	12.76	fair, thick	
46	13.3, 14.5, 14.6, 12.1	13.6	12.94	good	limit β
47	17.1, 17.5, 17.3	17.3	12.39	fair	
48			<12.0		limit n
49	27.9, 28.4	28.1	10.83	twilight	
49*a*			12.79	fair	photometer
50		<20.4	<11.8	fair	limit 1-2 < h
51		<19.9	<12.0	fair	limit n
52			<10.9		photo. 55 m. exp.
53			9.95	good, Moon	photometer
54			10.22	fair to good	photometer
55	20.3, 16.9, 19.5	18.9	12.18	good	
56		<21.9	<11.6	good, low	limit h

is a straight line. The average distance of the dots from the line is 0.10 magnitude; omitting η, the distance is 0.08. The value of one step is 0.14 magnitude.

VISUAL OBSERVATIONS.

The visual comparisons of the variable by Argelander's method are given in detail in Table V. The light-scale was formed from them in the usual manner, giving the quantities in the fourth column of Table III. Table V also contains the results of three sets of photometric measures of the variable (observations No. 49_a, 53, and 54) and the magnitude determination from the photograph (No. 52).

LIGHT-CURVE.

In Fig. 2 the observed magnitudes from 1898 to 1903 are platted and the light-curve drawn through them; the parts of the curve covered by the observations being drawn full, while

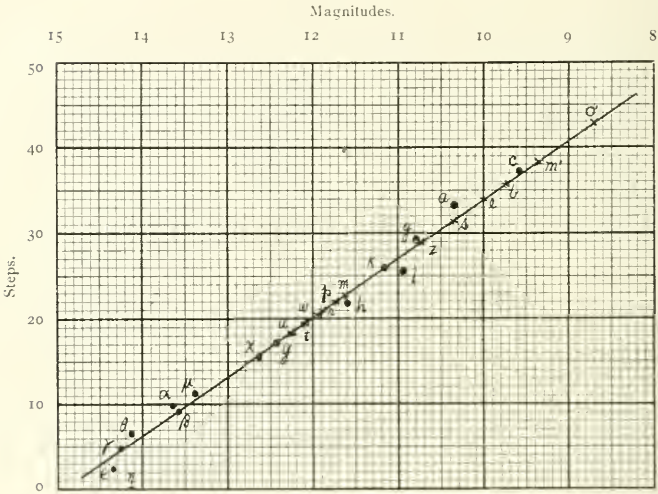


FIG. 1.— Magnitude Curve of *W Aurigae.*

the parts assumed, in absence of sufficiently numerous observations, are drawn with a broken line. The variations seem best represented by the elements of maximum —

1898, December 24 (J. D. 2414648) + 276 E (M-m = 113)

Magnitudes: Max. 9.3, Min. 13.8.

In Table VI are collected the dates of maximum and minimum, together with a comparison with the observations given in Blajko's summary previously quoted, from the *Nachrichten.*

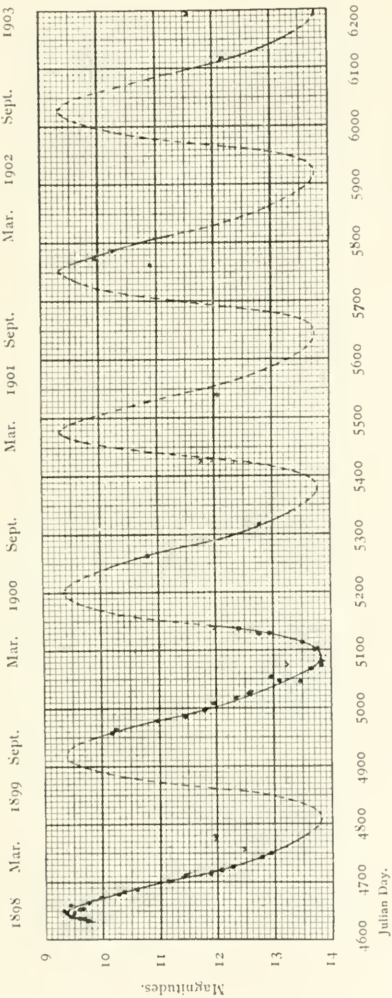


FIG. 2.—Light-Curve of *W Aurigae.*

TABLE VI.

MAXIMA.

Epoch	Calculated		Observations Blajko's Summary		Blajko's Maxima
	Julian Day	Calendar	Date	Magnitude	
	2410000+				
−5	3268	1895, Mar. 15	1895, Feb. 15 1895, Mar. 4	10.1 ph 9.3 ph	March or April
−4	3544	1895, Dec. 16	1895, Dec. 16 1895, Dec. 21	8.8 ph 8.8 ph	December
−3	3820	1896, Sept. 17			
−2	4096	1897, June 20			
−1	4372	1898, Mar. 23	1898, Apr. 10	9.2 vis	
0	4648	1898, Dec. 24	1898, October 1899, Feb. 21 1899, Mar. 31	<12 vis 11 vis <12 vis	December
1	4924	1899, Sept. 26	1899, October 1899, Dec. 28	10 to 11 vis 12 ± ph	
2	5200	1900, June 29	1900, April	12 vis	Second part of June
3	5476	1901, Apr. 1			March
4	5752	1902, Jan. 2			
5	6028	1902, Oct. 5			

MINIMA.

Epoch	Calculated		Observations Blajko's Summary	
	Julian Day	Calendar	Date	Magnitude
	2410000+			
−4	3431	1895, Aug. 25		
−3	3707	1896, May 27		
−2	3983	1897, Feb. 27	1897, Mar. 26	<12 ph
−1	4259	1897, Nov. 30	1898, Jan. 16	<12 ph
0	4535	1898, Sept. 2	1898, October	<12 ph
1	4811	1899, June 5	1899, Apr. 4 1899, Apr. 6	<12 ph <12 ph
2	5087	1900, Mar. 8		
3	5363	1900, Dec. 9		
4	5639	1901, Sept. 11		
5	5915	1902, June 14		
6	6191	1903, Mar. 17		

In the column of "Observations," "ph" stands for "photographic" and "vis" for "visual." It will be seen that the above elements satisfy the observation between 1895 and 1903. The

calculated dates of maxima and minima are well represented by the light-curve, though the observations are not sufficiently numerous to enable us to detect variations of less than fifteen or twenty days with certainty. The magnitudes on the photographs near the maximum at epoch -4 do not agree very closely with my photograph taken near maximum at epoch $+4$ (indicated on the light curve, Fig. 2, by the small cross), but doubtless a large part of the divergence can be accounted for by the use of different photographic plates.

YERKES OBSERVATORY,
September 1903.

ON CERTAIN METHODS OF ECONOMIZING THE LIGHT IN SPECTRUM ANALYSIS.

By W. J. Humphreys.

No matter what the source of light nor what kind of spectrograph is used to analyze it, there are distinct advantages in obtaining its spectra as brilliant as possible. In this way lines are found that otherwise would escape detection, and photographs, which of course integrate the light during the time of exposure, are obtained more quickly, and therefore under more nearly uniform conditions. Moreover, such economy is desirable in obtaining spectrograms of very volatile substances, small amounts of any material, arcs under pressure, and other temporary sources of light. Again, it renders practical the use of instruments of correspondingly larger dispersive powers — an advantage of the utmost value in nearly every line of spectrum work.

The above are some of the reasons why a spectrograph should be equipped with light-saving devices, and in what follows I shall briefly describe a few of these, most of which have been fully tested experimentally.

I. ILLUMINATION OF THE SLIT.

Assuming that the source of light is not under the experimenter's control, or if so, that its intensity has been pushed to the practicable limit — in short, with a fixed source, — probably the most obvious method of increasing the brilliancy of its spectrum, as produced by a given instrument with a definite adjustment, is that of increasing the illumination of the slit.

If the source is so far away that it appears as a point, a star for instance, or line, like the disappearing crescent of the Sun at a total solar eclipse, then it is quite feasible, as is generally known, to discard the slit entirely and use an objective-prism or grating; but for obvious practical reasons it is the custom, when working with large instruments, to focus the light with convex

refractors or concave reflectors upon a suitable slit, though such a procedure is necessarily beset with difficulties. Large refractors have to be made of glass, and are therefore more or less opaque to the ultra-violet, while the light that gets through is always in some measure dispersed by chromatic aberration. Reflectors, on the other hand, are free from chromatic aberration, but even though perfectly figured, if they consist of speculum metal, return scarcely two-thirds of the incident visible light and barely two-fifths of the ultra-violet; while, if silver on glass is used, nearly all the ultra-violet is lost.[1] With such sources the only means, appliable to all types of spectrographs, that has occurred to me of increasing the illumination of the slit, when the size of the apparatus is fixed, is by the use of properly selected reflectors, silver for the visible light and magnalium, wherever practicable, for the ultra-violet. A special method adapted to concave gratings will be described further on.

When a flame, electric arc, or other near-by source is used, the problem is radically different from the preceding one, where it was supposed to be far off. Here, by merely placing the source in front of the slit, a fair amount of light is caused to reach the grating or other analyzer—an amount which, if the source is uniformly brilliant throughout, is independent of its distance so long as the cone of light passing through the slit is large enough to cover the analyzing surface; and to the end that this cone shall be of ample size it is customary to use an ordinary condensing lens, commonly consisting of quartz for gratings to secure the ultra-violet, with the slit and source at its conjugate foci. When the light is intense and constant, this method is sufficient for most purposes. It also has the great advantage, when prisms or flat gratings are used, of giving the distribution in the arc or other source, of the substances producing the spectra. It has, however, besides the loss of some light by reflection, the disadvantage of chromatic aberration, by virtue of which the relative intensities of the colors are not the same at the analyzer that they are in the source.

Of course, the sharper the image on the slit, as produced by

[1] HAGEN and RUBENS, *Annalen der Physik*, 8, 1–21, 1902.

any one color, the greater the amount of that kind of light that reaches the analyzer; but so long as the slit is kept in one of the foci, and the cone of light from it fully covers the analyzing surface, there is but little to be gained by altering the position, size, or focal length of the condensing lens. In the case of concave gratings, however, and in general where the distribution of the light in the source is not under consideration, it is easy to increase greatly the brilliancy of the spectrum by the use of suitable concave reflectors.

a) As the most general type of such reflector it is convenient to consider a portion of an ellipsoid of revolution. Let this be adjusted so that the center of the source of light, say sparks between metallic points, shall be in one focus, and the slit, which should be parallel to the line joining the spark terminals, in the other focus of the ellipsoid, whose inner surface is supposed to be a highly polished good reflector. The line joining the foci should, of course, if prolonged, intersect the center of the prism or grating. By this means the light that fills a large solid angle whose vertex is at the source, is converged upon the slit, and, by adapting the dimensions of the ellipsoid to the spectrograph in use, much of it is made to reach the analyzer. This method evidently would avoid chromatic aberration, as well as utilize a larger amount of light than could be secured by means of a lens.

b) If the foci of the ellipsoid are separated until the figure becomes a paraboloid of revolution, with the source centered in the focus, the rays of reflected light become approximately parallel and may be condensed upon the slit with a suitable lens; or by placing the slit in the focus and close to the source, but between it and the parabolic reflector, the parallel rays thus obtained would be adapted to an objective concave grating or other analyzer requiring the use of a collimator. Of course, the disturbing light from the source would have to be screened off from the instrument, and this could be done by a small opaque object suitably placed; or, among other ways, the source might be put quite to one side and, by the aid of a condenser and small optical flat mirror, a brilliant image formed on the slit.

But the ellipsoid and paraboloid, while simple enough in

theory, are necessarily difficult to make, and therefore expensive; still they are mentioned in this connection because of their decided value, if properly constructed, and, further, they naturally belong with a third method described below, which is exceedingly easy of construction, inexpensive, and efficient.

c) Again, considering the ellipsoid of revolution, let its foci be brought closer together, till they finally coincide, and the surface becomes spherical. In this case the source, which is at the center of the sphere, and its image are superimposed, and the condensing lens in front of the slit receives both the direct and the reflected light. Evidently the source could be placed

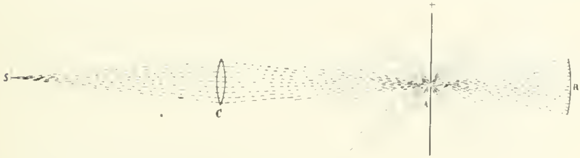


FIG. 1.

slightly to one side, and its reflected image alone received upon the slit. The particular advantage, that might occasionally be desirable, of this arrangement is the avoidance of chromatic aberration.

The method I have most frequently used is shown in Fig. 1, in which A is the source, R a spherical reflector, C a condensing lens, and S the slit.

In this way two distinct images are formed on the slit, one an inverted image produced by the direct light, the other an erect image given by the reflected light; and evidently they may be superimposed. But since the reflected light shines back through its own source, it must necessarily suffer greater or less absorption, so that the resulting brilliancy of the superimposed images is less than the sum of those of the direct and reflected images separately. Nevertheless, the combination gives spectra distinctly more brilliant than those produced by the direct light alone, and therefore much of the reflected light must somehow get through the source, and does so probably because the lumi-

nous vapors are not sufficiently dense or extensive to form a practically continuous intercepting layer.

To demonstrate photographically the efficiency of the apparatus, I have taken advantage of the fact that silver is a poor reflector of ultra-violet light, but an excellent one of longer wave-lengths. The first three negatives shown in the plate were taken in the region of $\lambda 3200$ of the third order, which coincides with $\lambda 4800$ of the second order. The outer portions of I are due wholly to light reflected from a silver surface, while the middle strip was produced by light directly from the arc made to pass through a silver film, on a quartz disk of sufficient thickness to be nearly opaque to all luminous rays. The position of maximum transmission naturally coincides with that of minimum reflection, approximately $\lambda 3200$, and therefore the heavy lines on the outer portions of I belong to regions of longer wave-length.

The outer portions of II are also due to reflected light alone, but the middle strip was produced by undisturbed light directly from the arc. It will be noticed that, while the times of exposure were as nearly as possible the same and the source kept constant, the ultra-violet lines have very unequal intensities in the direct and reflected portions, and that those of longer wave-lengths differ but little from each other. Finally, the middle strip of III is due to direct light, while its outer portions were produced by the joint effect of the superimposed images, direct and reflected. The arc was practically constant and the times of exposure the same. Here it is seen that those ultra-violet lines which are scarcely at all reflected have nearly equal intensities in the outer and middle portions, while those of longer wave-lengths that are strongly reflected are much heavier in the outer parts. The conclusion, therefore, is that the vapors of the arc do not absorb all the reflected light, and that the method described does materially increase the brilliancy of arc-, and presumably of certain other spectra.

Incidentally it is seen that a silver reflector, or screen, can be used as an aid to differentiate between ultra-violet lines and those of longer wave-length.

In most of my work I have used silver-on-glass reflectors, but,

PLATE XIV.

VIII.

VII.

VI.

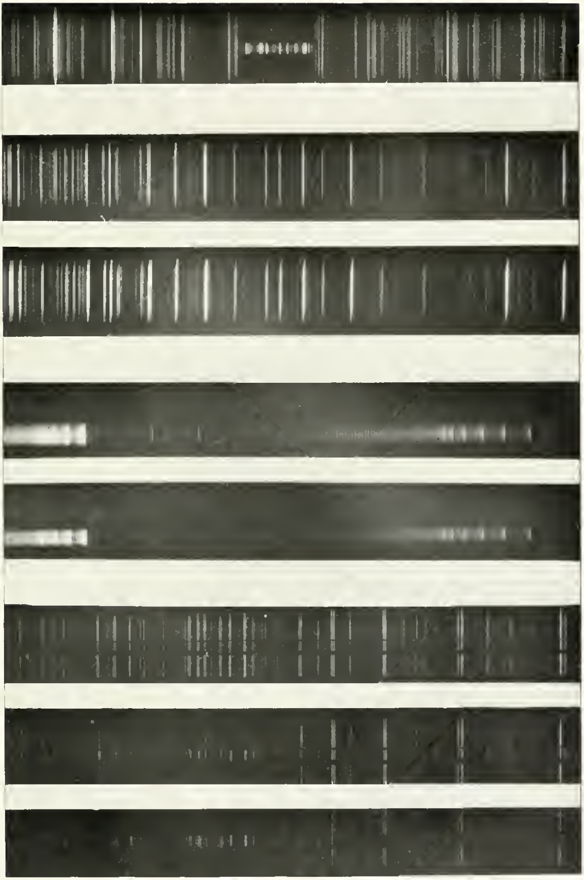
V.

IV.

III.

II.

I.



while this is the best known reflector of visible light, it is, as shown by the plate, a very bad one for the ultra-violet, and therefore for this region some other material should be used, probably magnalium for the best results.

d) Of the many possible optical trains that might be used to illuminate the slit, there is one, other than those above described, with which I have worked that, for small images, yields good results with the concave grating. This consists of a spherical condensing lens used in the ordinary way, either with or without a reflector behind the source, in conjunction with a double concave cylindric lens of short focal length, placed close to the slit and with its axis at right angles to the slit's direction.

By focusing the light on the slit with the condensing lens, and then introducing the cylindric lens as above described, the image will be elongated in the direction of the slit, while its width at the center will remain unaltered; and the pencil of light that gets through, instead of covering and surrounding the grating with a circle, will cover it with a surface roughly rectangular in shape, the length of which is equal to the diameter of the original circle, while its width is much less.

The cylindric lens does not appreciably alter the amount of light that gets through the slit, but places a much larger percentage of it on the ruled surface, so that the final effect is to lengthen out the spectrum lines, owing to the elongated image, and, on account of the astigmatism of the grating, to increase their brilliancy.

These effects are shown in negatives IV and V, the cylindric lens being used in the latter case. The source was a round hole, about one millimeter in diameter, in a sheet of metal placed close to a large, and therefore approximately uniform, electric arc. The times of exposure were as nearly as possible the same, and the plates were developed together and for the same length of time. Many eye observations were taken, and several other comparison plates secured in the first three orders of the spectrum, and the results were everywhere the same—a slight lengthening of the lines, and a marked increase in their brilliancy.

II. VIRTUAL INCREASE OF THE SLIT-LENGTH.

One of the peculiarities of the concave grating is its astigmatism, by virtue of which a point source in the slit produces a line image on the focal curve. When the slit is parallel to the rulings on the grating, the several line-images due to its consecutive points are superimposed, and a relatively intense line is the result. The longer the slit, within certain limits, the more intense the resulting lines, but when it is increased beyond a definite length, which depends on the grating and its position, the lines produced by light coming from one end of it fail to overlap those due to light from the other end, and clearly increasing the slit-length beyond this does no good. However, there are many sources of light, such as end-on Plücker tubes, sparks between close terminals, and others, whose images as produced by an ordinary condensing lens limit the slit to much less than its maximum efficient length. Besides, the image may always be rendered small by means of a short-focus condensing lens, and in many cases the only objection to this is the loss of some light due to greater chromatic aberration, and the increased difficulty of keeping the image properly placed on the slit.

In practically all cases, then, the circle of light that covers the grating may be, and usually is, much greater than the area of the ruled surface, and thus only a small portion of the light that gets through the slit is of any service.

Clearly, however, if this waste light could be made to fall on the ruled surface in the direction which it would have if it came from some point close to and in line with the slit, the astigmatism would cause the resulting spectrum lines to overlap those due to the direct light from the slit, and the brilliancy of the spectrum would be increased.

a) As already explained, excellent results of this kind can be obtained with a concave cylindrical lens placed before the slit, but similar effects are possible with suitable apparatus properly located between the slit and grating. One of these, which, owing to the difficulty of its construction, will probably have only a theoretical interest, is a curved convex cylindrical lens—a portion of a ring—of proper focal length, and of such shape and

position that it will be normally intersected by each plane determined by the slit and a line on the grating.

b) There is another method, however, efficient, simple to construct, and easy to use. This consists of a pair of optical flats close to the slit, but between it and the grating, with their ends properly separated and the pair so adjusted that the plane fixed by the slit and the central ruling on the grating shall be normal to their reflecting surfaces. Chromatic aberration is thus avoided.

The method is shown schematically in Fig. 2, which also gives a geometric means of determining the proper inclination of

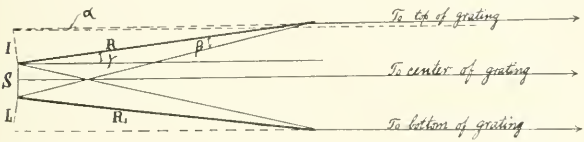


FIG. 2.

the reflectors to each other for any given slit-length and grating distance. For the sake of keeping the lines well separated in the drawing the angles are greatly exaggerated.

In the figure S is the slit, R, R_1 the two reflectors, and I, I_1 the slit's images in these reflectors. For a given position of the grating let a ray from the bottom of the slit be reflected from a point near the end of the upper reflector to the top of the grating. Other rays from the same point in the slit will go directly to the grating, others miss it entirely, but still others will be reflected from nearer points on R to parts of the grating below the top. The same thing holds for every other point in the slit, except that the higher it is, the nearer will be the parts of R that reflect its light to the grating. Similarly for the lower reflector.

To determine the proper setting of the reflectors, draw a line from the upper end of the image I, which should be of the same length as S, past the far end of R to the top of the grating, and another from the same point parallel to the line that joins the center of S with the middle point of the ruled surface. Let the

angle so formed be α, then if l is the length of the rulings on the grating, and L the distance from the slit to the grating,

$$\tan \alpha = \frac{l - 3\,S}{2\,L} \text{ nearly.}$$

Let the angle between R and the above ray to the top of the grating be β, and the length of the reflector a, then

$$\tan \beta = \frac{S}{a} \text{ nearly.}$$

Finally call the semi-angle between R and R_1 γ, then $\gamma = \alpha + \beta$, a determinate quantity.

The following table gives some of the numerical values of these constants with which I have worked.

Length of reflectors, 12.5 cm; focal length of grating, 6.4 m; length of rulings on the grating, 5 cm; distance of grating from the slit, a variable quantity, but as a somewhat extreme and unfavorable case say 4 m, which puts λ4400, third order, at the center of the camera.

TABLE OF CONSTANTS.

Length of Slit	Distance Between Reflectors at Slit	Distance Between Reflectors at End from Slit	Required Radius of Circle of Light at Grating
1 mm	1 mm	4.47 mm	14 cm
1.5	1.5	5.92	17
2	2	7.38	22

From this it evidently is not necessary to use even a large condensing lens, or cone of light of wide angle; and, further, it clearly is easy to obtain a slit and its reflections practically three times the length of the slit alone, all sending light to the grating in such directions that it will produce superimposed, and therefore relatively brilliant, spectrum lines. Besides, the angle of incidence is so large that the images are nearly as bright as the slit itself. The reflectors with which I have worked consist of silver chemically deposited on glass optical flats, though this, in spite of the large angle of incidence, is probably not the best material for the ultra-violet.

In those cases where the image is practically a point and the

angle of the cone of light very large, better results might be obtained with suitable cylindrico-parabolic reflectors with their common foci at the center of the slit, but under all ordinary circumstances the flats are best and easiest to use.

I have found it convenient to fasten the reflectors respectively to an upper and a lower piece of brass, and to provide these with two sets of screws, one of which adjusts the reflectors relatively to each other, while the other, by resting on a small platform, adjusts the combination to the slit. The successful use of this attachment, instead of being difficult, as one might suppose, is very easy. None of the adjustments is difficult, and once the device is properly set, but little additional attention is required other than keeping the image at the proper place on the slit.

The general character of the results is shown in negatives VI and VII. Since these were taken only for the purpose of illustrating the principle, a very short slit, of about one millimeter, was used instead of a small image. It will be noticed that the lines of VI are triple, the outer components being broad and hazy; and that, if all three were made equally sharp and brought together, they would overlap through most of their length and produce a decidedly more intense line. This effect of tripling the lines and blurring the lateral ones was secured by tipping the mirrors, and thus putting the images out of line with the slit and also out of line with the rulings on the grating. Negative VII was taken when the mirrors were properly adjusted, and though the three sets of lines are here superimposed, it is free from noticeable defects of any kind.

III. SHAPE OF THE GRATING.

The general theory of reflecting gratings leaves great freedom of choice as to the shape of the surface, but the difficulties of construction, convenience of use, and other considerations so narrow the limits that, to the best of my knowledge, only flat and spherical concave surfaces are used.

If the light is sufficiently intense, the astigmatism of the concave grating as commonly mounted will do no harm, but in the case of feeble sources, or even faint lines in a bright source, this

will cause many parts of the spectrum to be overlooked that could easily be seen if all the energy was concentrated to closer limits. It would therefore be very desirable, while retaining normal spectra, to reduce the astigmatism to a minimum, and consequently it seems worth while to determine what surface will do this, and, if the theoretical one is difficult of construction, to find the nearest practical approach to it.

Let the slit, grating, and camera be, as in the Rowland mounting, on the vertices of a right-angled triangle, as shown in Fig. 3, where S is the slit, G the grating, and F the position of the camera or viewing telescope. Let the plane determined by $S\ G\ F$ be horizontal, and the slit, which should be parallel to the rulings on the grating, vertical. Let the surface of the grating, concave as viewed from F, be a portion of a torus, and let the rulings be at right angles to the equator, and be divided into equal upper and lower halves by it. Further, let the radius of horizontal curvature of the grating be equal to $G\,F$, and its radius of vertical curvature equal to any desired value. Also let the rulings be equally spaced along a horizontal chord, so that the grating as completed and mounted will differ from the usual spherical concave grating only in having a different radius of vertical curvature. With these conditions it will be found, on making somewhat tedious but simple enough substitutions in Runge's formulæ,[1] *i. e*, by using the equations of a torus instead of those of a sphere, that the two concave gratings, spherical and toroidal, scarcely differ except in the matter of astigmatism.

The problem then reduces to that of finding the vertical curvature of a grating whose horizontal curvature is given, that will produce the required amount of concentration.

Let this requirement be that a point in the slit shall give a point image. To do this evidently the vertical curvature must be such that the sum of the distances from the given place in the slit to any point on a fixed ruling, say the middle one, and from that same point to the focus shall be a constant; that is, this particular ruling must be on the surface of an ellipsoid of revolution whose foci are at S and F.

[1] KAYSER'S *Handbuch der Spectroscopie*, I, p. 452 ff.

Let $A\ G\ B\ C$ be the horizontal section of this ellipsoid. Then to determine the proper radius of vertical curvature of the torus, cut this ellipsoid by a vertical plane passing through F and G; that is, by a plane passing through one focus and the end of a latus-rectum through the other focus and normal to the plane determined by this latus-rectum and the major axis.

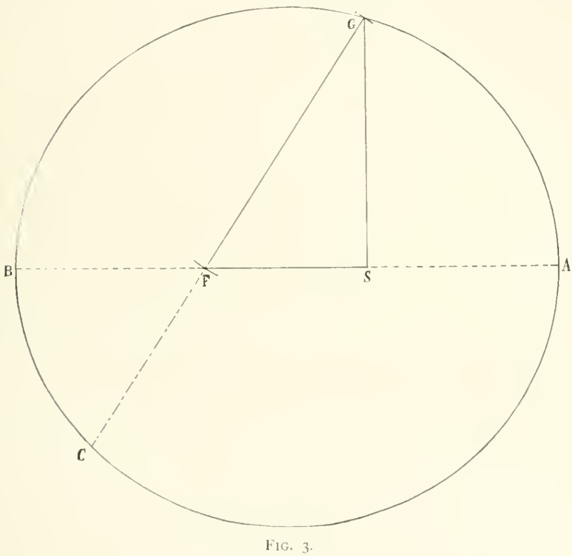


FIG. 3.

The curve thus cut out of the ellipsoid is an ellipse, and on revolving it about a vertical axis passing through F and parallel to its minor axis the required torus, a part of whose surface about G forms the grating, is generated.

Such an elliptical toroidal surface, however, is quite beyond the power of ordinary skill to form, but any uniform curve that closely fits this ellipse at the end of its major axis would give very nearly the same results, and doubtless among these the circle is the simplest; and, moreover, at the end of the major axis,

the place required, a circle whose radius is equal to the semi-latus-rectum has a very high order of contact with the ellipse, so that the circle is not only the simplest curve, but also one of closest approach to the ideally correct one.

To generate the required circular torus, revolve a circle whose radius is equal to the semi-latus-rectum of the elliptical section, as above described, at right angles to its plane, about a vertical axis passing through F, and have it so situated that the equator of the torus shall pass through G.

Of course, such a surface could give zero astigmatism at only one place, at F, where SF is the distance between the foci of the ellipsoid of revolution. It will thus be necessary, after determining the radius of horizontal curvature GF, and the ruling to be used, to decide in what part of which spectrum the maximum intensity is desired. This will fix the value of SF, and through it, as shown above, the radius of vertical curvature, which in all cases will be equal to SG.

This same value for the vertical curvature, with the condition that a point source at S shall give a point image at F, is readily obtained from Mitchell's[1] and also from Runge's[2] general equation for astigmatism, but possibly the above direct and simple discussion of the problem is not superfluous, since it shows that the figure, as determined by these equations, is only an approximation, though an exceedingly close one, to the ideally correct surface.

Mitchell's equation is

$$C = -Z + Z\sqrt{r\left(\frac{\cos\mu + \cos\gamma}{\rho} - \frac{1}{R}\right)}\ ,$$

in which C is the half-length of a spectrum line due to a point source in the slit, Z the half-length of a ruling, r the radius of horizontal curvature, R the distance of the slit from the grating, ρ the radius of vertical curvature as I use it, or radius of the sphere as Mitchell uses it, γ the angle of incidence, and μ the angle of diffraction.

[1] ASTROPHYSICAL JOURNAL, 8, 107, 1898.

[2] KAYSER'S *Handbuch der Spectroscopie*, I, 464.

When the Rowland mounting is used, $R = r \cos \gamma$, and at the center of the camera $\mu = 0$; and therefore $C = 0$ when

$$r\left(\frac{1 + \cos \gamma}{\rho} - \frac{1}{r \cos \gamma}\right) = 1 ,$$

that is, when

$$\rho = r \cos \gamma = R.$$

Runge's expression for the entire length of a spectrum line is

$$\frac{r'}{r} s + \left[\rho - a' + \frac{r'}{r}(\rho - a)\right]\frac{l}{\rho} ,$$

where s = length of slit, l = length of ruling, ρ = radius of curvature, vertical as I use it, r = distance of slit to grating, r' = distance of camera from grating, a = the x co-ordinate of the slit when the center of the grating is taken as the origin, a' = the corresponding x co-ordinate of the spectrum line.

When the Rowland mounting is used, and the x axis is the line joining the grating with the camera, we have, where γ is the angle of incidence,

$$a' = r', \; a = r \cos \gamma, \text{ and } r' \cos \gamma = r;$$

and therefore in this case for point sources, that is with $s = 0$, the length of the spectrum lines becomes

$$\left(\rho - r' + \frac{r'}{r}\rho - r\right)\frac{l}{\rho} \; ;$$

and this vanishes when $\rho = r$.

Evidently, then, zero astigmatism may be obtained by having the radii of horizontal and vertical curvature of the grating equal respectively to its distances from the camera and the slit.

Finally, it would be extremely desirable so to set the cutting diamond point that the grating shall give its most brilliant spectrum at that place where the astigmatism is least.

Possibly a satisfactory approach to the above surface may lie beyond the skill of practical opticians, but, whether for the present this is true or not, it seemed to me worth while to discuss one of its valuable properties when used for a grating.

IV. CAMERA ATTACHMENTS.

Prism and plane-grating spectrographs, when illuminated by a line source, produce line images, while the spherical concave

grating, as usually mounted, gives lines, whether the source is also a line or only a point.

Therefore, in all these cases the energy in any given wave-length is spread out over a line of greater or less magnitude, and, if it happens to be very feeble, may not be detected at all. Clearly, then, it would be desirable to concentrate this energy within narrower limits, and for photographic purposes this is quite possible, since for a given wave-length the actinic effect depends rather on the quantity of light that reaches a given point than on small differences in direction.

In what follows I shall assume that the spectrum lines are straight, as they are when due to gratings, either plane or concave, and as they may be in prism instruments, provided the slit is properly shaped. Further, though the principles are exactly the same for all, I shall describe the methods as applied to a large concave grating spectrograph, the instrument with which I have worked.

a) One method of thus concentrating the spectrum lines is to place in front of the photographic plate a short-focus cylindric lens, whose axis is parallel to the focal curve. By suitably adjusting the distance between the plate and lens the normally long lines are reduced to very short ones, and the intensity greatly increased. This method, however, is open to several objections. The lines probably will not have exactly the same intervals between them that they would have without the lens, and, besides, they are not so clearly defined. If the lens is long, it must necessarily be made of glass, and therefore not be applicable to much of the ultra-violet. Still, even with all these objections, the cylindric lens is of service in hunting for very faint lines, especially if they happen to be somewhat hazy.

b) Another, and in most cases much better method is to place good reflectors in front of, but close to, the plate. This arrangement is shown in Fig. 4, where P is the photographic plate, R_1, R_2 the reflectors, and G the grating. Evidently some of the light will reach the plate directly, while two other portions will be reflected to the same place, one from R_1, the other from R_2, and the three acting together will correspondingly increase

the photographic effect. The reflected rays clearly pass over somewhat longer paths than do the direct ones, but this difference, one millimeter at most, is not enough, when long-focus gratings are used, materially to affect the definition.

To secure the best results the surfaces of the reflectors must be normal to a plane containing the spectrum line in question and passing through the center of the grating. Therefore, for the middle of the camera of a concave grating the reflectors should not be optical flats, but each should be a portion of a right cone whose vertex is on the tangent to the middle of the central ruling. For a short distance this very closely coincides

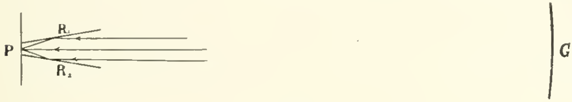


Fig. 4.

with a flat, and negative VIII was secured with such optical plates. A mere glance at this negative will show that the reflectors greatly increase the photographic effect, while a careful examination of it will reveal but little, if any, injury to the definition.

Much greater concentration could be had by using surfaces whose vertical sections through the axis give parabolas instead of straight lines as furnished by the cone. The photographic plate should, of course, in this case be placed near the latus-rectum. Each reflecting surface, according to this plan, would be part of a parabolic toroid.

Accurately to figure either of these surfaces, the conical or parabolic toroidal, would be difficult, but even for the best results accuracy of shape is not essential. It is only necessary that the reflected light should fall somewhere on a short straight line, and therefore it is sufficient if the horizontal sections of the reflectors are true circles. The vertical sections in the one case need not give perfectly straight lines, nor those of the other exact parabolas. With these conditions the surfaces would be very much more easily figured, and yet quite as efficient in every way.

Evidently in many cases two or more of these methods may be combined, with the result of correspondingly multiplying the final brilliancy of the spectrum lines.

While I have seen no reference to the actual use of any of these modifications or attachments, still from their very simplicity it would be surprising if no such application has been made of them. Indeed, Rowland[1] suggests the use of a cylindric lens, which from the context I take to be the same as IV, *a*, but does not say that he actually so used it. Besides, I know that I, *d*, has independently occurred to Wadsworth, but I have seen no account of its use in this way.

The material of the grating, various photographic processes, and manipulations, and even means of intensifying the brilliancy of the arc, flame, or other source, might all be discussed under the general title of this paper, but at present I have nothing to contribute on any of these important points.

University of Virginia,
April 1903.

[1] *Physical Papers*, p. 489.

THE SPECTRUM OF *o CETI*.[1]

By Joel Stebbins.

On account of the great instrumental power required for the observation of the spectra of faint objects, changes in the spectra of long-period variable stars have not been well studied. In fact, there is no star which undergoes a large variation in brightness whose spectrum has been systematically followed from maximum to minimum, or *vice versa*. It is proposed to give here the results of a study of the spectrum of *o Ceti*, or *Mira*, made, at the suggestion of Director Campbell, with the thirty-six-inch refractor of the Lick Observatory, from June 1902 to January 1903. During this period the star faded in brightness from 3.8 to 9.0 magnitude. The first photograph of the spectrum was obtained about three weeks after the predicted time of maximum, and a series of plates was secured covering the interval to minimum. No negatives were obtained after the star had again begun to increase in brightness.

The most important articles concerning the spectrum of *Mira* are those of Vogel,[2] Sidgreaves,[3] and Campbell.[4] Neither Vogel nor Sidgreaves followed the star long enough to find much change in its spectrum, and Campbell's work was mainly in connection with observations of the star for radial velocity, with the Mills spectrograph.

INSTRUMENTS AND METHODS.

The spectrograph used in my observations was the one employed by Messrs. Campbell and Wright in their work on

[1] "Dissertation in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in the University of California," *Lick Observatory Bulletin* No. 41.

[2] H. C. Vogel, "Ueber das Spectrum von *Mira Ceti*," *Sitzungsberichte der Berliner Acad.*, p. 143, 1896.

[3] Walter Sidgreaves, "The Spectrum of *o Ceti* as Photographed at Stonyhurst College Observatory," *Monthly Notices*, 58, 344, April 1898.

[4] W. W. Campbell, "Note on the Spectrum of *o Ceti*," Astrophysical Journal, 9. 31, January 1899.

Nova Persei.[1] designated as "Spectrograph I." It is the regular Mills spectrograph converted into a one-prism instrument. It gives good definition, on the same photograph, of the region from $\lambda 3700$ to $\lambda 5600$. The length of this range of spectrum is 28 mm. Although this dispersion is only about one-fifth of that of the three-prism instrument, it is enough to yield a very fair determination of velocity.

When the seeing is good, an exposure of ten minutes will give a satisfactory negative of a star to which the *Draper Catalogue* assigns the photographic magnitude 5.0. If *o Ceti* were of about the tenth photographic magnitude at minimum, it would require, roughly, an exposure of one hundred times as long as a fifth-magnitude star, or about sixteen hours. Some of the light being in bright lines, we should expect a still longer exposure to be necessary. This estimate agrees with my experience. About July 1, 1902, an hour's exposure, just before daylight, could be made on the star, and it was bright enough to photograph in that time. An exposure of six hours at the time of minimum, using the fastest photographic plate obtainable, was not sufficient to produce a measurable image, although much could be seen in a qualitative way. On January 5, 1903, the date of my last negative, an exposure of five hours, beginning soon after sunset, was possible. Early in March 1903, the spectrum was again bright enough to record itself with a fairly short exposure; and in the absence of the writer, Messrs. Reese and Curtis were ready to make an attempt, but bad weather prevailed.

A Huggins reflecting slit is used with this instrument. A ninth-magnitude star can be followed accurately.

No flexure of the instrument was noticed on any of the earlier plates. The comparison spectrum was inserted at least four times during each exposure, and the definition was satisfactory. Several long exposures in September and October were imperfect, but the trouble was finally removed by taking greater precautions in tightening the various clamps and screws.

As there were no means of controlling electrically the

[1] *L. O. Bulletin* No. 8.

temperature of the instrument, changes of temperature must have effected the definition. The spectrograph when in use was covered with two thicknesses of woolen blanket.

The same emulsion of Cramer's "Crown" plates was used for most of the work. A few exposures in July and August were made on Cramer's "Isochromatic Instantaneous," but the "Crown" plates were more sensitive.

TABLE I.

o Ceti. 13E, 1902, July 16. $\lambda = 2123.10 + \frac{332873}{220.653 - R}$.

Description	Micrometer R.	$R_0 - R$	$\frac{1}{R_0 - R}$ 0.00	λ From Formula	Correction to Formula	Reduction to Sun	λ
	20.706	199.947	500138	3787.91	*+0.14*		*3788.05*
	21.560	199.093	502278	3795.05	*+0.10*		*3795.15*
Bright, faint	21.950	198.703	503264	3798.33	+0.10	+0.36	3798.79
	26.124	194.529	514062	3834.27	*+0.09*		*3834.36*
Bright	26.301	194.352	514530	3835.83	+0.06	+0.36	3836.25
Bright, faint	28.241	192.412	519718	3853.10	+0.05	+0.37	3853.52
	29.012	191.641	521809	3860.06	*0.00*		*3860.06*
	32.111	188.542	530380	3888.61	*+0.06*		*3888.67*
Bright	32.208	188.445	530659	3889.52	+0.03	+0.37	3889.92
	32.870	187.783	532530	3895.75	*+0.05*		*3895.80*
	33.300	187.353	533752	3899.82	*+0.03*		*3899.85*
Bright, faint	33.950	186.703	535610	3906.00	−0.02	+0.37	3906.39
	36.230	184.423	542232	3928.04	*+0.04*		*3928.08*
	40.364	180.289	554665	3969.43	*−0.02*		*3969.41*
H. line of Sun	40.345	180.308	554607	3969.24	0.00	+0.38	3969.62
Bright	40.479	180.174	555019	3970.61	0.00	+0.38	3970.99
5 wide	41.381	179.272	557812	3979.91	−0.01	+0.38	3980.28
5 wide	41.665	178.988	558697	3982.85	−0.01	+0.38	3983.22
3	42.396	178.257	560988	3990.48	−0.01	+0.38	3990.85
Bright place	43.116	177.537	563263	3998.05	−0.01	+0.38	3998.42
4	43.251	177.402	563692	3999.48	−0.01	+0.38	3999.85
Bright place	43.518	177.135	564541	4002.30	−0.01	+0.38	4002.67
	43.810	176.843	565473	4005.41	*0.00*		*4005.41*
Bright place	43.996	176.657	566069	4007.39	−0.02	+0.38	4007.75
4 wide	44.170	176.483	566627	4009.25	−0.02	+0.38	4009.61
4	45.282	175.371	570220	4021.21	−0.02	+0.38	4021.57
5 very wide	45.610	175.043	571288	4024.76	−0.02	+0.38	4025.12
4	45.850	174.803	572073	4027.38	−0.03	+0.38	4027.73
4 fine	46.436	174.217	573997	4033.78	−0.03	+0.38	4034.13
	47.546	173.107	577677	4046.03	*−0.05*		*4045.98*
3	48.394	172.259	580521	4055.50	−0.03	+0.39	4055.86
	49.130	171.523	583012	4063.79	*+0.15*		*4063.94*
	49.849	170.804	585466	4071.96	*−0.05*		*4071.91*
4 wide	50.414	170.239	587409	4078.43	−0.04	+0.39	4078.78
3 poor	50.850	169.803	688918	4083.45	−0.04	+0.39	4083.80
3 poor	51.274	169.379	590392	4088.36	−0.04	+0.39	4088.71

All of the plates taken in this work were measured and reduced with the iron spark-spectrum as comparison. From the ultra-violet to $\lambda 4415$ there are many strong and sharp iron lines. From $\lambda 4415$ to $\lambda 5600$ the iron lines are fainter, and many of them have companions. The region from $\lambda 4800$ to $\lambda 5600$ of the iron spectrum was photographed with three prisms. It was found that there were several lines in this region which had no companions, and which could be used without hesitation. Close double lines were assumed to have positions depending upon the relative intensities of their components.

The plates were measured with one of the measuring microscopes in use for the regular line of sight work. Each plate was measured with both violet left and violet right in the eyepiece.

A plate of the sky was taken with a very long slit and the amount of the curvature of the comparison lines was determined in the same manner as by Campbell.[1] The curvature corrections were found to be insignificant.

The reductions from micrometer readings to wave-lengths were based on the Cornu-Hartmann formula, much of the computation being done with a Brunsviga calculating machine. The wave-lengths of the comparison lines were taken from Rowland's table. The example on page 343 shows the complete reduction of a plate after the constants of the formula had been derived. Italicized figures are used for the comparison lines.

DATA OF THE OBSERVATIONS.

In Table II is given a list of the plates secured with Spectrograph I. Mr. Wright obtained three plates in 1901, and he kindly turned them over to me for measurement and discussion. The width of the slit is expressed in terms of the divisions on the head of the screw, one division corresponding to 0.025 mm. One plate, taken on September 16, was discarded on account of great flexure during the exposure. Something of interest was found on all other plates, except 57 A.

The brightness of the star at the time when each plate was

[1] ASTROPHYSICAL JOURNAL, 8, 144, 1898.

taken is shown in Fig. 1. The observations of visual magnitude are given at the end of this paper. As the plates are well distributed along the light curve until about the time of the minimum, no great change in the spectrum could have escaped observation.

ABSORPTION SPECTRUM.

As is well known, *o Ceti* and the other long-period variables have absorption spectra of Secchi's third type. Some observers have found the region from $H\gamma$ towards the red crossed by a series of dark bands, with edges sharp towards the violet, and they report that from $H\gamma$ to the ultra-violet the dark-line spectrum is very similar to that of the Sun. At first glance this seems to be verified by my plates, but a closer study shows the details to be very different. Fig. 2 shows an eightfold enlargement of the solar spectrum, photographed with Spectrograph I on a lantern-slide plate. The absorption spectrum of the star, Figs. 3–6, is seen to resemble little that of the Sun. The date of Fig. 4 should be August 11, instead of August 4, as is printed.

In comparing the star spectrum with the solar spectrum, it was not found best simply to measure the plates and then look for coincidences in Rowland's table. It is easy to find a line in the table which agrees in position with the one on the plate, but the intensities may be very different. The method adopted was to compare a plate of *o Ceti* with one of the sky. The two negatives, film sides together, were examined under the microscope with a low power.

The strong calcium lines *g*, H, and K, are present in the spectrum of the star, the *g* line being much more intense than in the solar spectrum. The strong iron lines of the Sun are not so prominent in *o Ceti*; in fact, they do not show with low dispersion. Of the large number of lines in the star spectrum, there are very few which coincide with lines of like intensity in the solar spectrum.

The absorption spectrum of *Mira* was measured accurately on seven plates. Each plate was measured and reduced independently, so that a faint line might be measured on one plate without

being noticed on any of the others. This does not mean that it had developed, or that its intensity had changed. Such lines, of which many were measured, could be easily obscured by an irregular arrangement of the silver grains. The best method of verifying changes is to examine different plates simultaneously, in pairs, under the microscope. This has been done, and all changes in intensity or character have been noted.

TABLE II.

Plates Secured With Spectrograph I.

Plate No.	Astronomical Date	Mt. Hamilton Sidereal Time, Middle of Exposure	Length of Exposure	Brightness of *o Ceti*, Mag.	Photo-Plate	Slit Width	Seeing	Remarks
	1902							
2B	June 27	22^h 09^m	0^h 45^m	3.8	Crown	1.4	Poor	
3C	27	22 39	0 10	3.8	Crown	1 4	Poor	
4E	27	22 46	0 03	3.8	Crown	1 4	Poor	
7C	July 6	22 47	1 15	4.1	Crown	1 4	Good	
8D	6	23 28	0 05	4 1	Crown	1.4	Good	
9E	6	23 35	0 02	4.1	Crown	1 0	Good	
13E	16	23 07	2 00	4.6	Iso.	1.4	Fair	
14F	16	0 12	0 05	4.6	Iso.	1 0	Fair	Shows bright lines only
18F	22	23 22	2 30	5.1	Iso.	1.4	Fair	
21F	29	23 30	2 40	5 3	Iso.	1.4	Fair	
25F	Aug. 4	0 07	3 00	5 4	Iso.	1 4	Good	
26D	4	1 44	0 07	5.4	Crown	1.0	Good	Shows bright lines only
27D	11	0 58	2 35	5.6	Crown	1.5	Fair	
28D	25	0 53	2 35	6 4	Crown	1 5	Poor	
33D	Sept. 6	0 45	3 00	7.0	Crown	1.5	Poor	
39D	22	2 20	4 40	7 1	Crown	1.6	Poor	Flexure during the exposure
42A	Oct. 4	1 47	5 53	7 8	Crown	1 8	Fair	Slight flexure
48A	26	4 10	5 40	8.5	Crown	1.8	Fair	Flexure
54A	Nov. 25	3 10	5 00	9.2	Crown	2.5	Poor	
57A	Dec. 21	3 18	5 25	9.0	Crown	3.0	Poor	Nothing found on this plate
	1903							
58A	Jan. 2	3 12	5 17	9.0	Crowo	3.0	Poor	
59A	5	3 36	4 51	9.0	Crown	2.5	Fair	
	1901							
2211C	Aug. 3	1 15	0 20	. . .	Iso.	1.2	Good	
2212D	3	1 32	0 03	.	Iso.	1.2	Good	Bright lines only
2235D	17	1 26	1 20	. .	Iso.	1.3	. . . .	

In Table III are given the wave-lengths of the absorption spectrum as derived from all the measures. The numbers expressing the intensity indicate, in a general way, the relative strength of the lines. It was intended, on each plate, to assign intensity 1 to the faintest lines distinguishable, and intensity 10 to the strong line at λ 4255. On this scale the *g*, H, and K lines might be 100 or 500, it matters little. The intensities assigned are merely relative, and no doubt the scale varies much in different parts of the spectrum. The lines on each plate were

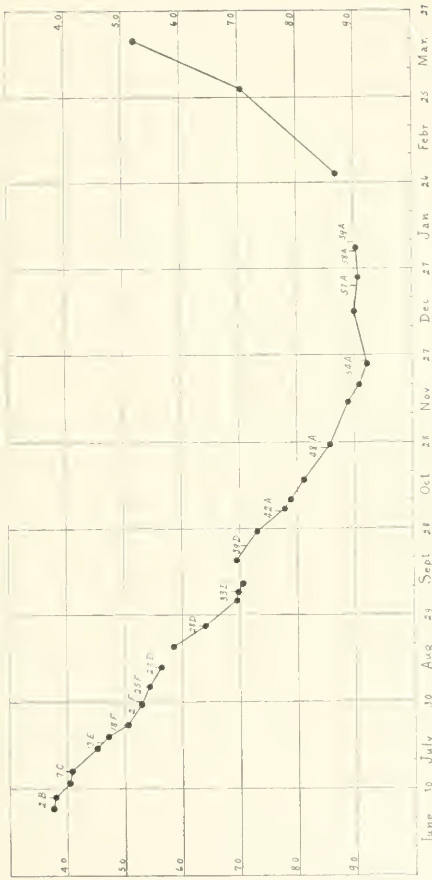


FIG. 1.—*o Ceti.* Light-Curve, 1902–3.

TABLE III.

Absorption Spectrum of *o Ceti.*

Intensity	Observed λ	Residuals in 0.01 t.m.							Remarks
		7C	13E	18F	25F	27D	28D	2235D	
	3936.—	..	..	..	..	—	—	..	K line of Sun, very strong. Adjacent bright lines interfere with measurement of λ
5 wide	3945.09	04	.	..	..	*01*	*02*	..	
5 wide	3949.28	14	..	..	..	—	*14*	..	
2 fine	3951.65	..	..	..	..	..	—	..	
2	3957.27	..	..	..	..	..	—	..	
4	3958.98	—	..	..	..	*03*	03	..	
5 wide	3962.55	—	..	..	..	00	00	..	H line of Sun, very strong. Adjacent bright lines interfere with measurement of λ
	3969.6-	—	—	..	..	..	—	..	
4 wide	3980.20	..	08	..	..	..	*09*	..	
5 wide	3983.18	10	04	..	*19*	*01*	05	..	
5	3990.64	..	21	..	*24*	*04*	07	..	
2 fine	3997.61	—	..	..	..	..	..	..	
4	3999.72	09	13	..	*02*	*07*	*22*	11	
4 wide	4005.6-	..	..	..	—	..	..	..	Perhaps several
5 wide	4009.63	..	*02*	..	05	17	..	*20*	Perhaps several
4	4010.04	..	..	..	..	..	—	..	
3	4012.18	..	..	..	*15*	..	23	*09*	
1	4018.91	..	..	..	..	..	—	..	
4 wide	4021.41	*14*	16	..	00	*08*	10	*04*	
5 wide	4025.08	17	04	..	*17*	08	03	*14*	
3 wide	4027.68	20	05	..	10	..	*09*	*26*	
2 wide	4030.16	..	..	..	03	..	*04*	..	
6	4032.22	*19*	..	..	15	*05*	38	*27*	
7	4034.04	05	09	..	*01*	*02*	03	*11*	
1 fine	4035.71	..	..	..	—	..	..	..	
2	4036.22	..	..	..	..	..	—	..	
3	4045.16	..	..	..	*03*	02	..	..	
2 fine	4047.02	—	..	..	..	..	..	..	
	4043.—	..	..	..	—	—	..	—	Several lines run together
	4050.—	..	..	..	—	—	..	—	
3	4053.25	..	..	..	..	..	..	—	
6	4055.71	..	15	..	*01*	*09*	*19*	12	
2	4058.14	..	.	..	12	..	..	*11*	
2 wide	4060.43	..	..	..	03	..	..	*03*	Too wide and faint to measure. Probably several
	4064.	..	..	..	..	—	—	..	
4 wide	4068.24	..	..	..	..	..	..	—	Too wide and faint to measure. Probably several
	4072.—	..	..	..	..	—	—	..	
5 wide	4072.28	.	..	..	09	*01*	..	*09*	
2 fine	4077.24		—	..	08	*11*	02	—	On plates 7C, 13E and 2235D these two lines run together
7 wide	4079.51	—	—	..	*02*	04	*03*	—	
4 wide	4083.85	*15*	*05*	..	12	*36*	06	39	
3	4088.88	20	*17*	..	*07*	11	*04*	*05*	
1 fine	4091.33	..	—	..	..	..	..	..	
5	4093.55	..	18	..	00	*12*	*21*	13	
3	4097.08	..	..	..	01	13	*08*	*04*	
3 fine	4100.56	..	..	..	..	..	—	..	
4	4105.88	..	..	..	*07*	..	..	08	
5	4110.34	..	*07*	..	*09*	..	08	09	
5	4112.97	..	*07*	..	07	..	..	..	
4 fine	4116.14	..	..	..	..	..	..	—	
4 fine	4117.71	..	00	..	*15*	..	18	*01*	
	4117.9-	.	..	..	..	..	–	..	Several
	4119.1-	..	..	..	..	..	—	..	

TABLE III.—*Continued.*

Absorption of Spectrum of *o Ceti.*

Intensity	Observed λ	Residuals in 0.01 t.m.							Remarks
		7C	13E	18F	25F	27D	28D	2235D	
4 fine	4121.44	..	..	..	..	..	..	—	
4 fine	4122.62	..	..	..	..	..	..	—	
6	4124.66	01	21	..	*15*	..	*02*	*06*	
2 fine	4126.99	..	..	..	..	..	—	..	
3	4129.04	..	..	..	—	..	..	..	
3	4130.60	..	..	..	*04*	05	..	..	
4 fine	4133.00	..	..	..	..	.	—	..	
5	4135.32	*17*	05	..	*12*	*10*	17	15	
3	4137.77	11	..	..	..	..	..	*11*	
2	4140.96	..	..	..	00	..	..	*01*	
4 wide	4144.—	—	..	..	—	—	..	—	
3	4150.73	..	..	..	*12*	..	..	12	
3	4153.40	..	*04*	..	*01*	..	..	04	
3	4156.78	..	12	..	..	..	..	*11*	
4	4160.89	..	..	..	..	..	..	—	
3	4165.82	..	..	..	..	..	..	—	Poor
3	4169.27	..	..	..	17	..	00	*18*	
3	4173.56	..	—	..	..	..	..	..	
7	4175.41	11	28	..	*01*	03	*23*	*05*	
4	4178.32	*10*	13	..	07	*16*	23	*04*	
2	4180.96	..	..	..	..	*08*	07	..	
3	4183.73	..	..	..	—	..	..	..	
4 wide	4188.17	..	..	..	*16*	14	*23*	26	
5 wide	4191.79	*01*	08	..	07	*08*	*36*	32	
3	4199.79	..	..	..	..	..	..	—	
2 wide	4211.19	..	..	..	*27*	27	..	..	Poor
4 wide	4214.86	..	..	..	12	..	*11*	..	
	4227.84	00	28	..	—	*10*	*18*	—	*g* line of Sun, very strong
5	4235.37	..	*10*	..	04	..	07	..	
3	4242.82	..	..	..	*03*	..	..	04	
3	4251.60	..	02	..	*11*	..	..	10	
10	4255.46	06	33	..	*08*	*26*	*06*	01	
1	4259.64	..	..	..	—	..	..	..	
1 wide	4261.87	..	..	..	—	..	..	..	Poor
4	4273.15	..	*02*	..	00	..	..	02	
8	4275.84	00	17	..	*06*	*03*	*04*	*06*	
5	4285.28	..	—	..	..	..	..	.	
6	4290.72	*22*	..	..	13	03	*11*	17	
2 fine	4292.45	—	..	..	..	..	..	..	
1	4295.07	..	..	..	..	..	..	—	
2 fine	4297.56	..	..	..	..	..	..	—	
3	4300.16	06	..	..	..	..	..	*06*	
2	4304.06	..	—	..	..	..	..	..	
3	4307.23	*02*	*12*	..	..	..	..	15	
3	4310.57	..	07	..	..	..	..	*06*	
	4314.29	..	..	00	00	..	01	00	Head
2 fine	4320.03	—	..	..	..	..	..	..	
3	4326.97	06	..	..	*10*	..	..	05	
2	4331.13	..	..	..	—	..	..	..	
2	4348.12	.	..	..	—	..	..	..	
	4353.60	..	18	08	*06*	—		*18*	Head
4	4380.31	*03*	..	..	*06*	..	*25*	35	
5	4385.49	*12*	*09*	..	*04*	14	09	02	
4	4390.71	22	02	..	*14*	*11*	*25*	24	
	4395.89	24	..	24	*13*	—	—	*36*	Head

TABLE III.—*Continued.*

Absorption of Spectrum of *o Ceti.*

Intensity	Observed λ	Residuals in 0.1 t.m.							Remarks
		7C	13E	18F	25F	27D	28D	2235D	
	4422.4	..	*2*	1	*1*	1	0	1	Head
4	4461.1	..	..	..	—	..	..	..	
	4463.0	..	*4*	*1*	*2*	3	4	0	Head
	4505.7	..	0	*1*	2	..	..	*1*	Head
4 wide	4514.7	..	..	..	..	—	..	..	
3 wide	4519.8	..	..	..	..	—	..	..	
2	4530.7	.	..	..	—	..	..	..	
4	4536.8	..	*2*	*3*	*1*	..	5	..	
	4548.8	..	*4*	2	*4*	..	..	6	Head
	4585.0	..	*2*	1	*3*	4	3	*3*	Head
	4626.8	..	*1*	*4*	*2*	6	2	0	Head
	4669.6	..	*1*	*3*	*5*	3	[21]	6	Head. Measure of Plate 28D rejected for discordance
5	4709.2	..	2	0	..	1	..	*3*	
	4714.2	..	*4*	3	1	..	..	..	Head
10	4739.5	..	1	2	*3*	1	3	*4*	
	4760.0	..	*2*	4	3	*4*	6	*5*	Head
	4804.2	..	0	*4*	*3*	0	5	1	Head
	4842.8	..	..	..	..	2	*2*	..	Head
	4954.1	..	2	*3*	*3*	4	0	0	Head
	5167.0	..	2	*8*	2	..	..	5	Head
	5308.3	..	—	..	..	..	..	.	Head
	5358.8	..	—	..	..	..	..	..	Head
	5439.4	..	—	..	..	..	..	..	Head
	5447.9	..	2	0	7	..	..	5	Head
	5498.0	..	—	..	..	..	..	..	Head
	5568.7	..	..	..	..	...	..	—	Head

assigned intensities when the plate was measured; and when the different results were brought together, a sort of mean of the estimates was taken for each line. Where the estimates did not agree well, the line was compared directly on two or more plates to see if any change had taken place.

The residuals in the table were formed by subtracting the mean wave-lengths from the values given by each plate. All plates were assigned equal weight in forming the means. Periods (. . . .) signify that when the plate was measured nothing was noticed in the spectrum at that place. A dash (—) denotes that the line was observed, but could not be measured accurately. When a line was measured on only one plate there is no residual, and the dash indicates the plate referred to.

Plate 18 F was measured only from λ 4300 towards the red.

The comparison on 7 C being underexposed to the red of λ 4415, the plate was not measured in that region.

From the residuals in Table III it is found that the probable error of the wave-length of a dark line derived from one plate is ±0.10 tenth-meter. When a line was measured on six plates, the probable error of its mean position is therefore ±0.04 t.m. In computing the probable error, the residuals from all lines to the violet of λ4400 were used.

Many of the lines being broad or unsymmetrical and difficult to measure, the residuals from different plates are not larger than would be expected. While the agreement of the results with each other is not a test as to systematic errors, yet I feel sure that the wave-lengths of but few of the dark lines measured on several plates can be in error by as much as 0.2 t.m.

A glance at the residuals shows that there is no evidence of variable velocity in the line of sight. Those from Mr. Wright's plate No. 2235 D, which was taken in 1891, indicate that the velocity was about the same during the corresponding phase of the star's light curve, a year previous. The residuals for each plate have been averaged; and if there were any change in the velocity it would appear in the mean residuals. The arithmetical means of the residuals to the violet of λ4400, and the corresponding velocities in kilometers per second, are as follows:

TABLE IV.

Date	Plate No.	t.m.	km
1902, July 6	7 C	+0.02	+1
16	13 E	+0.06	+4
Aug. 4	25 F	−0.03	−2
11	27 D	−0.02	−1
25	28 D	−0.03	−2
1901, Aug. 17	2235 D	+0.02	+1

Since many of the lines were not measured on all of these plates, the average residual of one plate is not strictly the average deviation of the measures on that plate from the mean of all the plates, but no sensible error is introduced by this assumption. When it is remembered that all lines, good and poor, were

included in the means, and that these plates were taken with a one-prism instrument giving a dispersion of but one-fifth of that of the regular Mills spectrograph, the range of only 6 km on six plates is very satisfactory. None of the plates taken later in 1902 are good enough to afford comparable conclusions as to the velocity; and it was thought best to measure only first-class plates in this connection.

The result that the star's velocity was constant over a certain period was derived without assuming the coincidence of any of the star lines with lines of the solar spectrum, or with other known lines. A determination of the actual radial velocity requires an assumption as to what the wave-lengths of the star lines would be if they were not affected by velocity. Agreements of star and solar lines were looked for, by direct comparison of the star plates with a solar plate, and the only coincidences which seem certain are given in Table V. The wave-lengths and other data of the solar lines are from Rowland's table.

The H and K lines could not be measured accurately; the double manganese line at λ4030.92 gives a discordant result, which was rejected, and it looks as though both a strontium and a titanium line make up a stellar line at λ4079.51. The velocity of +66 km was therefore derived from measures of only six lines, and none of these are fine and sharp enough to give the best determination.

TABLE V.

Coincidences of Dark Lines in the Spectra of o *Ceti* and the Sun.

o *Ceti*		Sun λ	Displacement t.m.	Velocity, km	Sun Substance and Intensity
Intensity and Character	λ				
Strong	3936.—	33.82	———	———	*Ca* 1000 (K)
5 wide	3945.09	44.16	+0.93	+71	*Al* 15
5 wide	3962.55	61.67	+0.88	+67	*Al* 20
Strong	3969.6–	68.62	———	———	*Ca* 700 (H)
6	4032.22	30.92	[+1.30]	[+96]	*Mn* 4 and 5
7	4034.04	33.22	+0.82	+61	*Fe-Mn* 7
7 wide	4079.51	77.88	———	———	*Sr* 8
		78.63	———	———	*Ti* 3
Strong	4227.84	26.90	+0.94	+67	*Ca* 20 (*g*)
10	4255.46	54.50	+0.96	+68	*Cr* 8
8	4275.84	74.96	+0.88	+62	*Cr* 7
				Mean +66 km	

Campbell in 1898 found +62 km with the three-prism instrument. The interval between August 29, 1898, the date of his first plate that year, and August 25, 1902, is equal to 130 days more than four periods of 331 days. The radial velocity has therefore been observed to be constant over about two-fifths of the period of light change.

To determine more coincidences, I have measured one of the high-dispersion plates of *o Ceti*, taken by Dr. Campbell in 1897. Although the plate is underexposed, about seventy lines were measured between λ4300 and λ4420, and more than twenty coincidences were found by direct comparison with a solar plate. The results are given in Table VI. As was done for the one-prism plates, the intensities are expressed on an arbitrary scale of 10.

TABLE VI.

Absorption Spectrum of *o Ceti*. High Dispersion. Plate 576 B. December 15, 1897.

o Ceti		Sun λ	Displacement t.m.	Velocity km	Remarks
Intensity and Character	λ				
Several	4299.7	.....	......	......	Narrower in ⊙
	4300.8	.....	......	......	
3	4301.58	.....	......	......	Closer double in ⊙
3	4302.17	.....	.....	......	
2	4303.56	02.69	+0.87	+60.6	*Ca* 4
5, double?	4307.10	.....	......	......	Not ⊙
3	4308.16	.....	......	......	Not ⊙
4	4308.94	——	——	——	07.91 *Ca* 3, 08.08 *Fe* 6
4	4310.83	.....	......	......	Not ⊙
1	4312.90	.....	.....	.....	Not ⊙
Several	4315.2–	.....	......	......	Not ⊙
	4316.1–	.....	......	......	
2	4319.77	18.82	+0.95	+65.9	*Ca*, *Mn?* 4
Bright place	4325.93	.....	......	......	Same in ⊙
6	4326.85	25.94	+0.91	+63.0	*Fe* 8
1	4327.96	.....	......	......	Not ⊙
1	4328.69	.....	......	......	Not ⊙
1	4329.53	.....	......	......	Not ⊙
1	4330.30	.....	......	......	Not ⊙
5	4331.13	30.19	+0.94	+65.0	*V* 0 Stronger than in ⊙
5	4333.92	32.99	+0.93	+64.4	*V* 0
Several	4334.68	.....	......	......	Not ⊙
	4335.29	.....	......	.....	
Poor 2	4335.91	.....	......	......	Possibly weak lines in ⊙
Poor 1	4336.38	.....	......	......	
1	4337.07	.....	......	......	Not ⊙
2	4338.16	37.22	+0.94	+65.0	*Fe* 5

TABLE VI.—*Continued.*

o Ceti Intensity and Character	*o Ceti* λ	Sun λ	Displacement t.m.	Velocity km	Remarks
2	4338.61	37.72	+0.89	+61.5	*Cr* 3
1	4339.11	.....	......	.....	Not ⊙
3	4345.56	44.67	+0.89	+61.4	*Cr* 4
4	4348.27	.....	......	......	Not ⊙
1	4352.16	51.22	+0.94	+64.8	*Cr* 3
Poor 2	4352.90	51.93	+0.97	+66.8	*Cr* 3
Head of band	4353.7–	.....	......	......	Not ⊙. Mean of 4 plates with one prism 4353.6
5	4353.91	52.91	[+1.00]	[+68.9]	*Fe* 4. This line always gives + residual in other stars
3	4356.97	.....	......	......	Perhaps ⊙, not certain
1	4349.70	.....	.....	.....	All weak lines in star. No coincidences certain
1	4360.12	.....	......	......	
1	4360.05	.....	......	......	
1	4361.73	.....	......	......	
Poor 1 wide	4363.05	.....	......	......	
2	4365.56	.....	......	......	
2	4367.10	.....	......	.....	
1	4367.87	.....	......	......	
4	4369.07	.....	......	......	Not ⊙
1	4370.48	.....	......	......	Not ⊙
1	4371.23	.....	... .	......	Not ⊙
4	4372.22	——	——	——	(71.14 *Zr* 1, 71.22 –1, 71.44 *Cr* 4) same appearance
1	4373.17	.....	......	......	Not ⊙
Several to violet of	4376.4–	.... .	......	......	Different in ⊙
Bright place	4376.68	.....	......	......	Not ⊙. This develops into a bright line
5	4377.06	76.11	+0.95	+65.1	*Fe* 6
1 wide	4378.06	.....	.....	......	Different in ⊙
1 wide	4379.17	.....	......	......	
10	4380.27	79.40	+0.87	+59.5	*V* 4
1	4383.67	.....	......	......	Not ⊙
6	4384.62	83.72	+0.90	+61.6	*Fe* 15
7 double?	4385.90	——	——	——	84.87 *V* 3, 85.14 *Cr* 2
5	4390.26	89.41	+0.85	+58.1	*Fe* 2 } Stronger in star than in ⊙
4	4391.06	90.15	+0.91	+62.2	*V* 2 } Stronger in star than in ⊙
Poor 2 wide	4392.76	91.92	+0.84	+57.4	*Cr* 1 } Stronger in star than in ⊙
7	4396.20	——	——	——	95.20 *Ti* 3, 95.41 *V* 2
2	4401.61	00.74	+0.87	+59.2	*V* 1
10 wide	4405.89	04.93	+0.96	+65.3	*Fe* 10
2	4407.02	.....	......	......	Not ⊙
4	4407.72	06.81	+0.91	+61.9	*V* 2
4	4408.70	——	——	——	07.81 V 2, 07.87 *Fe* 4
10	4409.41	——	——	——	08.36 *V* 2, 08.58 *Fe* 3, 08.68 *V* 2
Several to violet of	4416.7–	.....	......	......	Different in ⊙
4	4417.59	16.64	+0.95	+64.4	*V* 0. Stronger in star than in ⊙
Head of band	4422.5–	.....	......	......	Not ⊙. Mean of 6 plates with 1 prism, 4422.4
	Mean of 20 lines +62.7 km				

A comparison of Table VI with the corresponding part of Table III is a test of the reliability of the results obtained with one prism.

TABLE VII.

One Prism		Three Prisms	
Description	λ	λ	Description
3	4300.16	99.7– 00.8–	Several
2	4304.06	.....	Not measured
3	4307.23	07.10	5
3	4310.57	10.83	4
Head	4314.29	.....	Not measured
2	4320.03	19.77	2
3	4326.97	26.85	6
2	4331.13	31.13	5
2	4348.12	48.27	4
Head	4353.6–	53.7–	Head
4	4380.31	80.27	10
5	4385.49	84.62 85.90	6 7
4	4390.71	90.26 91.06	5 4
Head	4395.89	.....	Not measured
Head	4422.4–	22.5–	Head

As might have been expected, many lines observed as single with one prism are really made up of two or more components. This comparison also shows that the wave-lengths of Table III are probably not quite so accurate as the residuals indicate.

A brief summary of the number of dark lines of different elements observed in the spectrum may be of interest.

TABLE VIII.

Element	No. of Lines
Ca	6
Fe	11
Cr	9
V	11
Al	2
Sr	1
Mn	3 } ?
Ti	2 } ?

There can be no doubt of the presence of the first four elements in the list, and the aluminum and strontium lines are

prominent; but manganese and titanium must be considered as doubtful.

On account of the varying instrumental conditions it is easy to fall into error in judging as to changes in the intensity and character of dark lines. If all the negatives were of the same density and of uniform excellence, it would be easy to note such changes. There is one dark line which showed changes of which the reality is certain. This is the *g* calcium line at λ4227.84. Figs. 3–5 show how it broadened as the star grew faint. Measures of its width are necessarily rough, and must depend much upon the judgment of the observer. The measures of two plates are as follows:

TABLE IX.

Date	Plate	Width
1902, June 27	2 B	2 t.m.
September 6	33 D	9 t.m.

Other plates taken between these dates gave intermediate values. The intensity of continuous spectrum, in the neighborhood of λ4227, is almost the same on Plates 2 B and 33 D.

The general impression formed from examining the series of plates is that many other lines also grew broader as the brightness declined, but this is not certain. The effects of greater width of the slit, reduced intensity of the resulting negative, and flexure and temperature changes resulting from longer exposure, would all tend to make the lines wider and less sharply defined. The H and K lines are not shown on most of the plates, and nothing can be said as to changes in their character.

A few lines not visible on the early plates became prominent later. Four such lines have the following positions:

λ3990.64	λ4093.55
4045.16	4097.08

Lines of the solar spectrum which certainly coincide with these have not been found. The residuals in Table III indicate when these lines first appeared.

BANDS.

The prominent bands in the spectrum of *Mira* have been considered by some observers as a series of dark bands, with sharp edges towards the violet, and shading off toward the red. Others think them to be bright flutings like those of the arc spectrum of carbon. For convenience, they will be considered, in this paper, as dark absorption bands. On the plates of *o Ceti* and other third-type stars taken with Spectrograph I the bright portions of the banded spectrum are certainly brighter relative to the region above $H\gamma$, where there are no bands, than are the corresponding portions of the spectrum of a solar-type star. However, the dark portions are fainter than the same places in the solar type of spectrum.

In measuring the plates the micrometer wire was set on the division between a dark and a bright portion of the spectrum, so that the measurements remain the same whether the bands be considered as bright or dark. While the line of separation is probably sharp in most cases, its exact position is not easy to determine. No doubt the effect of irradiation plays an important part in direct visual observations. Since the bands considered as bright have their sharp edges towards the red, the effect of irradiation would be to make the observed wave-lengths of the heads too large. Corresponding to this visual error is the effect of spreading of the image on a photographic plate, causing an error in the same direction, and a slight allowance was made for it in executing the measures. At my request, Dr. Reese made several settings on the head of a band. The difference between his measures and mine was 0.3 t.m., my estimate being that much farther to the red. Although this was the only comparison made with another observer, it seems highly improbable that two persons should disagree by as much as one tenth-meter in the measurement of a sharply defined head.

No attempt was made to determine the position of the more diffuse ends of the bands. In many cases the intensity changes gradually from head to head, there being heavy absorption at the sharp edge, the spectrum growing uniformly brighter to the next line of demarcation.

Sidgreaves, on comparing the spectrum of *o Ceti* with that of *a Herculis* and other third-type stars, found a difference in the wave-length of the same band in different stars. For the head at λ 5447 in *o Ceti*, he found the position of λ 5458 in *a Herculis*, and intermediate values for *a Orionis* and *β Pegasi*. He suspected these discrepancies to be due to instrumental causes, but of this he was not certain. His work being done with an objective prism, he had no comparison spectrum with which to test the reality of the observed differences.

Along with the work on *o Ceti* in 1902, plates were taken of *a Herculis*, *β Pegasi*, *ρ Persei*, *a Ceti*, and *a Orionis*. As these stars are all bright, the exposures on them were comparatively short, from five minutes to one hour. Isochromatic plates were used, so that a range of spectrum as far to the red as λ 5600 was covered. They were measured only for the positions of the heads of the bands. In the spectrum of *Mira* there are few dark lines shown in the region of the bands, but in the others many lines were recorded. Where there are more lines the bands are less prominent, and in the case of *a Ceti* and *a Orionis*, only three bands could be measured. All of these stars are on the regular Mills spectrograph program, and their radial velocities have been determined from their dark line spectra in the region of $H\gamma$. As the wave-lengths of the bands must be affected by the radial velocities, it is necessary to apply to each observed wave-length a corresponding correction. In Table X is given, first the observed wave-lengths of the head of each band, already corrected for the orbital motion of the Earth. The accompanying correction is that which must be applied to each wave-length in order to reduce it to what it would be if the star's radial velocity referred to the Sun were zero. The mean result from the five stars has been taken after applying these corrections.

All bands visible on the plates were measured; and if the position of a band is not given for a star, it was not apparent. The results for any star are from all the plates of that star, except in the case of *a Herculis*, where one of the plates was a "Crown" plate and showed nothing to the red of λ 4954.

The results obtained by Sidgreaves, Vogel, and others are

inserted for comparison. The corrections given by Sidgreaves in *Monthly Notices*, **59**, 509, have been applied to his first published wave-lengths. Vogel's photographs did not extend to the red of λ4800. The results of visual observations of the bands in third-type stars are taken from Frost's Scheiner.

TABLE X.

Measures of Bands.

o Ceti Several Plates +62 km	*α Herculis* 2 and 1 Plate 34 km	*ρ Persei* 2 Plates +27 km	*β Pegasi* 2 Plates +8 km	*α Orionis* 4 Plates +18 km	*α Ceti* 2 Plates 25 km	Corrected Mean, Excluding *o Ceti*	*o Ceti* Corrected for +62 km	Sidgreaves, *o Ceti*	Vogel, *o Ceti*	Vogel and Others, 3d-Type Stars, Visual
4314.3 −0.9	......	......	......	......	......	......	4313.4	..	....	....
4353.6 −0.9	......	......	......	......	......	.....	4352.7	4352	.. ..	....
4395.9 −0.9	......	......	......	......	......	.....	4395.0	4395	....	....
4422.4 −0.9	4421.0 −0.5	......	......	......	......	4421 5	4421.5	4421	4422	....
4463.0 −0.9	......	......	......	......	......	......	4462.1	4460	4462	...
4505.7 −0.9	......	......	......	......	......	......	4504.8	4504	4506	....
4548 8 −0.9	......	......	......	......	......	......	4547.9	4546	4545	....
4585.0 −0.9	4583.9 +0.5	4584.8 −0.4	......	.....	.....	4584.4	4584.1	4583	4581	4608
4626.8 −1.0	4625.7 +0.5	4626.8 −0.4	......	......	.....	4626.3	4625.8	4625	4622	....
4669.6 −1.0	4667.1 +0.5	4667.9 −0.4	......	......	......	4667.6	4668.6	4669	4666	....
4714.2 −1.0	4713.6 +0.5	......	......	......	......	4714.1	4713.2	4714	4710	....
......	4736.6 +0.5	.....	......	......	.....	4737 1	......	4736	....	....
4760.0 −1.0	4760.0 +0.5	4761.3 −0.4	4761.8 −0.1	. ...	.. ...	4761 0	4759 0	4758	4755	4767
4804 2 −1.0	4803.5 +0.5	4804.9 −0.4	4805.4 −0.1	......	.....	4804.6	4803.2	4803	....	....
4842.8 −1.0	......	......	.. .	... ..	......	.... .	4841.8	4842	....	....
......	......	4848.8 −0.4	......	.....	......	4848.4	......	.	....	....
......	......	......	......	......	......	......	......	4892	....	....
4954.1 −1.0	4953.0 +0.6	4954.7 −0.4	4954 6 −0.1	4955.4 −0.3	4954.7 +0.4	4954.5	4953.1	4951	. .	4963
......	......	......	......	......	......	......	......	4998	.	....
......	......	......	......	......	......	......	......	5046	. .	....
......	......	......	......	......	......	......	......	5074	.	....
......	......	......	......	......	......	......	......	5098	..	....
......	......	......	......	......	......	......	......	5135	.. .	...
5167.0 −1.1	5165.0 −0.6	5166.6 −0.5	5166.3 −0.1	5165.8 −0.3	5165.4 +0.4	5165.8	5165.9	5162	. .	5169
......	......	......	......	......	......	......	......	5237	....	5244
5308.3 −1.1	......	......	......	......	......	......	5307.2	5306	...	....
5358.8 −1.1	......	......	......	......	......	......	5357.7	5356	. ..	....
......	......	......	......	......	......	......	......	5406	....	....
5439.4 −1.1	......	......	......	......	......	......	5438.3	. ..	. .	....
5447.9 −1.1	5446.9 +0.6	5447.9 −0.5	5446.7 −0.2	5446.3 −0.3	5445.9 +0.5	5446.8	5446 8	5447	...	5453
5498.0 −1.1	5496.5 +0.6	5497.1 −0.5	......	......	......	5496.8	5496.9	5498	.. .	....

Several objects measured as bands by the writer were not recorded by Sidgreaves; but they are not prominent, and we should not expect different observers with different instruments to agree as to details. Sidgreaves recorded more bands than I did to the red of λ 4954. The plates taken with Spectrograph I are underexposed in this region, and only the more prominent bands show.

A comparison of the corrected positions for *o Ceti* with the

means for the other stars shows that there is little difference in the wave-lengths, except that due to different radial velocities.

It is interesting to note what effect the application of the corrections for radial velocities has upon the agreement of the results. Residuals have been formed by subtracting the mean positions from those of each of the five stars. The sum of the squares of the residuals is reduced from 12.34 to 5.46 by the application of the corrections. This diminution of the residuals shows the validity of the assumption that the positions of the bands are the same in the five stars observed, and that the bands in each star have the same displacement, due to velocity, as have the fine, dark lines in the $H\gamma$ region.

However, the case is not clear with *o Ceti*. The correction for + 62 km changes the sum of the squares of the residuals, obtained by subtracting the mean wave-lengths of the other stars from those of the *o Ceti*, from 10.84 to 10.09. The corrections change the algebraic sum of the residuals from + 5.8 t.m. to − 5.3 t.m. This result is not surprising when it is remembered that all poorly defined bands were included in forming the residuals.

Judging by the accordance of the individual results, and allowing for any chance of personal error in the measures, it seems unlikely that the position of a sharply defined head of band should be in error by as much as one tenth-meter.

As far as I know, the identification of the bands of the third-type stars has not been accomplished. If the division at λ 5165.9 be considered as the head of a bright band, it is in close agreement with the head of the third carbon band at λ 5165.3, as measured by Kayser and Runge. The head at λ 4737.1, measured in *a Herculis*, may also correspond to their fourth carbon band, λ 4737.2. These two coincidences are the only ones I have found and we certainly need more evidence before drawing any conclusions from them.

VARIATIONS OF INTENSITY IN THE CONTINUOUS SPECTRUM.

Sidgreaves found that the continuous part of some regions of the star's spectrum changed in relative intensity as the star grew fainter. These changes have been verified in the present work.

In order to estimate the relative brightness of different parts, it was necessary to have a standard scale which should look like the spectrum of the star. An exposure on the sky of about one minute gives a strong solar spectrum not unlike that of a solar-type star. The telescope with Spectrograph I attached was pointed to the north pole at about noon on a clear day, and a series of twenty-five plates with carefully timed exposures varying from 1^s to 80^s was taken. These plates, all of the same emulsion, were developed simultaneously in the same tray, so that all would receive the same photographic treatment. The 1^s exposure gave a faint image and the 80^s an overexposed one. As on any one plate the amount of the silver deposit varies in different parts of the spectrum, the region between the iron lines at λ 4046 and λ 4072 was adopted as standard.

These plates were used for the purpose of comparing the intensities on different plates of *o Ceti* and also on different parts of the same plate. The method of using the scale plates was as follows: The plate of the star was placed film up on the table of the measuring microscope. The different scale plates were then successively placed film down on the star plate and viewed with a low power. The scale plate whose spectrum equaled the star spectrum in density was selected, and the number of seconds which it had taken to produce the image on the scale plate was called the intensity of the star spectrum. Of course, the lines in the star spectrum interfered with the estimates, but it was usually possible to find a space where there were apparently no absorption lines. In all cases the darkest patch of spectrum in the immediate vicinity on the negative was used.

Since we do not know the exact relation between time of exposure and density of the photographic image, the estimates made with the scale plates do not give us absolute intensities. There are also many errors introduced by instrumental causes. Among other sources of error is that due to the non-achromatism of the large objective. The proportions of light of different wave-lengths which enter the slit must vary with the quality of the seeing, and the care used in guiding. The star image which falls on the slit is not homogeneous, but is composed of a central

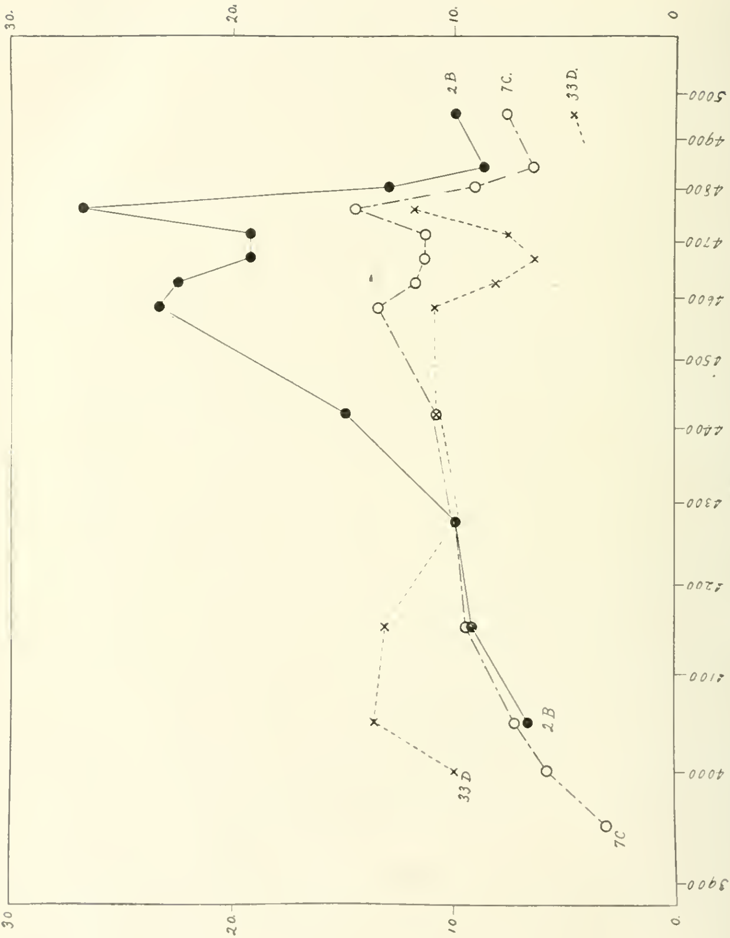

30.
20.
10.
0
5000
4900
4800
4700
4600
4500
4400
4300
4200
4100
4000
3900
2 B
7 C.
33 D.
2 B
33 D
7 C

FIG. 7.

PLATE XV.

FIG. 2.—Solar Spectrum (sky).

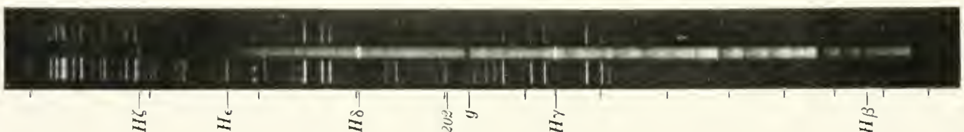


FIG. 3.—o *Ceti*, 1902 July 6. 7 C.

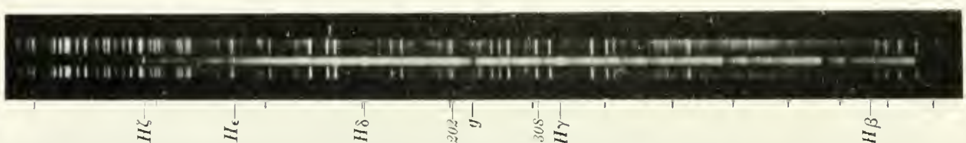


FIG. 4.—o *Ceti*, 1902 August 4. 27 D.

disk of $H\gamma$ light surrounded by concentric circles of light of longer and shorter wave-lengths.

The effects of different conditions of seeing and guiding were determined from plates taken of other stars, and while some differences were noticed, they were small compared with the changes observed in the case of *o Ceti*.

Fig. 7 gives the "intensity curves" of *o Ceti* from three plates, which were all of the same emulsion. The intensities of the same portions of the star spectrum were estimated on each plate by means of the scale. These estimates were plotted, and, in the figure, the lines join the estimates without reference to the intensities of the other parts of the spectrum. The images on the different plates being as a whole unequal in density or blackness, the portion near λ 4275 was made equal to 10, and the estimated intensities of other portions were changed to correspond. All estimates to the red of λ 4400 refer to the brighter portions, which may be heads of bright bands.

Sidgreaves's plates showed that as the star declined in brightness, the intensity of the bright portions of the spectrum between λ 4300 and λ 5000 grew less relatively to that of the region near λ 5500. My plates show a decrease in intensity of the region from λ 4300 to λ 5000 relative to the continuous spectrum from λ 4000 to λ 4300.

Figs. 3 and 5 (Plates XV and XVI) show the changes well. Plate No. 2B is not in excellent focus and is therefore not reproduced.

It should be remembered that the solar spectrum, Fig. 2 (Plate XV) is from a negative of the sky, on a lantern-slide plate whose sensibility curve is very different from that of the Crown plates. The maximum intensity of the continuous spectrum on a Crown plate is about λ 4600 for a solar-type star.

These changes in intensity have been described as changes in the continuous spectrum, but they may be simply the fading out of some bright bands.

BRIGHT LINES.

The most noticeable and interesting features in the spectrum of *o Ceti* are the bright lines. The great brilliancy of some of

the hydrogen lines, when the star was near its maximum, has been recorded by several observers.

The peculiar fact was noticed that $H\alpha$, $H\beta$, and $H\epsilon$ were apparently missing, while others of the hydrogen series were very bright. Photographs, which showed $H\gamma$ and $H\delta$ as intense, gave no trace of $H\beta$ and $H\epsilon$. Sidgreaves, in 1898 and 1899, found something which from its position might be the bright $H\beta$, but he did not consider it as certain. It has been seen bright on some plates at Harvard.[1] Mr. Wright found $H\epsilon$ distinctly bright on a plate which he had taken in August 1901. Figs. 3 to 5 show additional evidence on this point. $H\beta$ and $H\epsilon$ were recorded as bright lines on all the dense negatives taken with Spectrograph I. They seem to have grown stronger relatively to the other hydrogen lines and also to the continuous spectrum as the star grew faint.

An attempt was made to observe $H\alpha$ visually, but without

TABLE XI.

Measures of Bright Lines.

λ	7C	9E	13E	14F	18F	25F	27D	28D	2235D
3751.2	..	..	..	..	..	.		..	..
3771.52	..	..	..	..	..	..	—	..	..
3798.76	*04*	..	03	..	..	..	02	..	..
3836.20	*04*	..	05	..	..	*07*	08	00	*03*
3853.51	..	..	01	..	..	.	00	00	..
3889.91	03	..	01	11	..	*08*	03	02	*12*
3906.36	00	..	03	..	..	*04*	03	09	*11*
3908.18	..	..	..	..	..	..	02	*03*	..
3933.45	..	..	..	..	..	..	*09*	09	..
3939.10	..	..	..	..	..	.	*07*	07	..
3968.49	..	..	..	..	..	.	*10*	10	..
3970.87	11	..	12	..	..	*04*	*01*	03	*20*
4007.74	*01*	..	01	..	..	..	..	..	..
4102.66	*03*	00	08	00	..	*03*	03	*01*	*03*
4202.91	*04*	..	06	..	..	00	*08*	*06*	14
4216.71	..	..	..	..	..	11	*20*	09	..
4234.12	..	..	*03*	..	..	02	*13*	03	10
4308.70	..	..	..	..	..	*02*	*03*	06	..
4341.33	09	*04*	10	13	..	*15*	*05*	00	*04*
4373.61	..	..	..	..	..	..	..		..
4376.78	..	..	..	..	..	*05*	07	*01*	..
4571.82	..	..	..	..	..	*20*	*07*	28	..
4862.34	..	..	23	..	*09*	*14*	02	*03*	..

[1] *Harvard Annals*, **28**, Part I.

PLATE XVI.

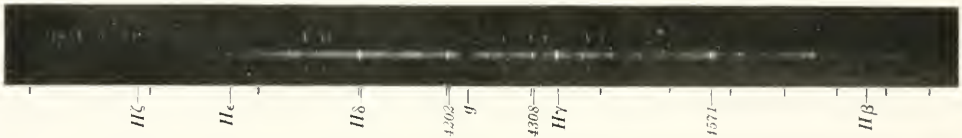


FIG. 5.—*o Ceti*, 1902 September 6. 33 D.

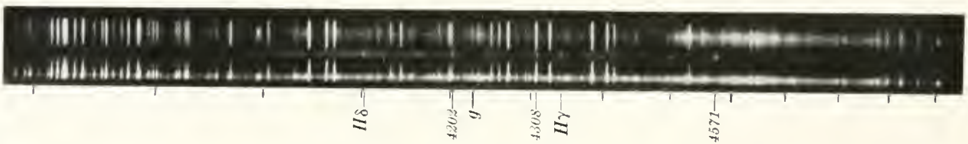


FIG. 6.—*o Ceti*, 1902 October 4. 42 A.

success, on the night of September 6, 1902, when the instrument was changed to adapt it for visual work. Corresponding to the bright well-defined *Ha* of the comparison spectrum there was continuous spectrum, but no bright line in the star. Mr. Wright was present and verified the observation. *Ha* should be looked for when *Mira* is again at a maximum.

Table XI gives the measures of all the bright lines which appeared, sooner or later, during the course of the work. The same scheme of residuals is used as in Table III.

Below is given a list of places which have the appearance of bright lines, but which, in the judgment of the observer, are bright spaces between absorption lines. Some of the places were measured on only one plate, but the wave-lengths are probably as accurate as those of the absorption lines.

λ 3978.7	λ 4213.0
3998.5	4427.9
4002.7	4434.5
4031.5	4512.6
4179.6	4777.6

From the residuals of Table XI the probable error of the wave-length of a bright line, determined from one plate, is found to be ± 0.06 t.m. The wave-lengths of the bright lines show no more change than do those of the dark lines. The average residual for each plate has been computed and there is no sign of variable position. A comparison of these average residuals with those of the dark lines is as follows:

TABLE XII.

Date	Plate No.	Dark Lines	Bright Lines
1902, July 6	7 C	+0.02	+0.01
16	13 F	+0.06	+0.04
August 4	25 F	−0.03	−0.03
11	27 D	0.02	−0.03
25	28 D	−0.03	+0.03
1901, August 17	2235 D	+0.02	−0.04

These average residuals are all so small that the agreement in sign for some cases is of little significance.

Table XIII gives the identification of some of the bright lines. There can be no doubt that the hydrogen series is present. Metallic lines which seem to coincide in five cases are given, but, in spite of the accuracy of the measures, we must consider the question of their identity to be still open.

TABLE XIII.

Identification of Bright Lines.

o Ceti-λ	Tabular λ	Displacement t. m.	Substance	Authority for λ
3751.2-	3750.15	+1.0-	$H\kappa$	Ames
3771.52	3770.7-	+0.82	$H\iota$	Ames
3798.76	3798.0-	+0.76	$H\theta$	Ames
3836.20	3835.6-	+0.60	$H\eta$	Ames
3853.51				
3889.91	3889.15	+0.76	$H\zeta$	Ames
3906.36	3905.66	+0.70	*Si* 12	Rowland
3908.18				
3933.45				
3939.10				
3968.49				
3970.87	3970.18	+0.69	$H\epsilon$	Rowland
4007.74				
4102.66	4101.89	+0.77	$H\delta$	Wright
4202.91	4202.20	+0.71	*Fe* 8	Rowland
4216.71				
4234.12				
4308.70	4308.08	+0.62	*Fe* 6	Rowland
4341.33	4340.63	+0.70	$H\gamma$	Rowland
4373.61				
4376.78	4376.11	+0.67	*Fe* 6	Rowland
4571.82	4571.26	+0.56	*Mg* 5	Rowland
4862.34	4861.53	+0.81	$H\beta$	Rowland

It should be noticed that a bright line developed on each side of each of the strong dark calcium lines, *g*, H and K. Without looking at the plates, this might seem to be due to a double reversal of the calcium lines. Their appearance is not such, however. Mr. Wright also examined the plates with this in mind, and in his judgment the lines are separate bright lines, and the phenomenon is not one of double reversal.

Several attempts were made, in June and July 1902, to secure photographs of the spectrum of *Mira*, with the regular three-prism instrument. It was impossible, however, to make an exposure long enough to record the continuous spectrum. The only features recorded were $H\gamma$ and two other bright lines. $H\gamma$

was single on all the plates. It seemed nearly monochromatic, but was a little sharper on the violet than on the red side. It had the same appearance as that found by Campbell in 1898 at about the same interval after the star's maximum. In 1898 he found that $H\gamma$ was triple from five to two weeks before maximum. Before observations of *o Ceti* were begun in 1902, it had been intended to make polariscopic tests for Zeeman effects in the bright lines; but they were found single on the first photographs and no observations for polarization were attempted. These should certainly be made when it is again possible to observe the star at maximum. Measures of the three-prism plates give the following displacements of $H\gamma$ in tenth-meters.

TABLE XIV.

Date	Plate No.	t.m.
1902, July 2	2446 F	+0.65
2	2447 A	+0.68
21	2470 D	+0.65
August 18	2505 E	+0.61
		Mean +0.65 t.m.

It is interesting to compare the measures of plates taken in 1902 with those of 1898. Campbell also observed the bright lines near λ 4308 and λ 4376. They were also measured on the high-dispersion plate of August 18, 1902. In the following scheme, under the heading "Bright Lines" is given the actual observed displacement of each bright line, corrected for the

TABLE XV.

Line	Campbell, 1898		Stebbins, 1902		
	Three Prisms		Three Prisms.	Bright Lines	One Prism Dark Lines, +66km
	Bright Lines t.m.	Dark Lines, +66km t.m.	t.m.	t.m.	t.m.
$H\delta$ 4101.89	+0.64	+0.85	......	+0.77	+0.90
Fe? 4308.08	+0.60	+0.89	+0.61	+0.62	+0.95
$H\gamma$ 4340.63	+0.64	+0.90	+0.65	+0.70	+0.96
Fe? 4376.11	+0.61	+0.91	+0.66	+0.67	+0.96

earth's motion. Under "Dark Lines" is given an assumed displacement which corresponds to that of the absorption lines in the spectrum.

The dark lines in this region of the spectrum are apparently displaced about 0.25 tenth-meter farther to the red than are the bright lines. The results obtained with one prism are systematically larger than those obtained with three prisms. This may be partly due, in the case of $H\gamma$ and $H\delta$, to overexposure on many of the plates. Since the lines seem to shade off towards the red, greater exposure probably slightly increases the apparent wave-lengths. However, since the displacement of the other hydrogen lines, which where not overexposed, is about the same as that of $H\gamma$ and $H\delta$, this effect is probably not large. The difference between the results of 1898 and 1902 is due, no doubt, to personal errrors.

The bright lines at $\lambda\lambda$ 4308 and 4376, marked as possibly due to iron, are of peculiar interest. They were recorded in Table VI as dark lines. The appearance of each, on some of the plates, is of a bright line with an adjacent dark one on the red side. If the bright lines be due to iron, they are displaced by the same amount as the bright hydrogen lines; and if iron produces the absorption components, the displacements are equal to those of other dark lines.

A glance at the series of plates showed that there were many changes of intensity among the bright lines, both relative to each other and to the continuous spectrum. Lines not visible on the earlier plates became more intense than the hydrogen lines which were so bright near maximum.

In order to estimate the amounts of the changes, a scale was made by the following method: An occulting strip was arranged in front of the photographic plate in the three-prism Mills spectrograph in such a manner that only the line λ 4308.081 of the iron spectrum reached the plate. A two-second exposure gave just a trace of an image of this line, whereas ten minutes produced an overexposed image of greater breadth and blackness than was ever obtained of the bright $H\delta$ star line. The plateholder was moved by small successive steps along in its cell,

and a series of carefully timed exposures, varying from one second to ten minutes, was made. At intervals the current was switched to the other side of the comparison apparatus and an image of the same line due to an exposure of five seconds was recorded on the plate. These extra images served as a rough test of the constancy of the light, and as reference points, for the scale. The exposures were all made on the same plate, and the different images received the same photographic treatment.

The bright lines of *o Ceti* were compared with this scale, and to each line was assigned as intensity, the number of seconds required to produce the equal line in the scale. Independent estimates of the same line made in this manner at different times are accordant with each other.

As the density of the star spectrum, as a whole, varied greatly on the different plates, the intensities of the same line on different plates are not comparable, unless they are referred to a common standard. If the intensity of a bright line, as estimated with the scale, be divided by the intensity of the continuous spectrum on the same plate, we get what we might call the intensity of the bright line referred to the continuous spectrum. The quotients formed in this manner will be referred to as the intensities of the bright lines. It is evident that long and short exposures of the star taken on the same night should give approximately the same intensities of the bright lines. As has already been shown, there is evidence that the intensity of the continuous spectrum varied in different parts as the star faded. The intensities of bright lines were, therefore, all referred to the portion of the continuous spectrum between λ 4102 and λ 4227.

The development of the bright lines is shown in Table XVI. It happens that the faintest bright line to which an intensity was assigned is called 1. On plate 27D the line at λ 3908.18 was estimated with the bright line scale to be of intensity 3.5. The intensity of the continuous spectrum between $H\delta$ and g was called 24. Dividing 3.5 by 24 and multiplying by 10, an arbitrary factor, the resulting intensity of the bright line is 1. On

the plate 2B the scale value of the $H\delta$ line was 250, and the continuous spectrum was of intensity 6. The resulting intensity of $H\delta$ is therefore 420. These numerical operations are crude, but they are as accurate as necessary.

In Table XVI a period (.) indicates that the image on the plate is dense enough, and that the definition is good enough to show the line as bright, had it existed when the plate was taken, but that it was not visible. A dash (—) means that the line did not appear on the plate, but that, judging from the evidence furnished by other plates, it would have been recorded with a longer exposure. Where the observer is unable to state whether the line existed or not, the space has been left blank. Plates numbered 13E, 18F, 21F, and 25F, and those of Mr. Wright, are Isochromatic, and are therefore not strictly comparable with the Crown plates.

The short exposures 3C and 4E give results fairly consistent with the long exposure 2B, taken on the same night. The plates 8D and 9E do not agree at all with the denser negative of 7C. This discordance shows that the estimates are very rough.

Another source of error is the effect of widening the slit for the long exposures. This might easily produce a progressive change in the apparent intensity of the lines relative to the continuous spectrum, but it certainly could not produce such great variation among the lines themselves.

The changes of the lines are shown graphically in Figs. 8–13. The intensities are represented by ordinates. It will be seen how the hydrogen lines, $H\gamma$ and $H\delta$, decreased in intensity as the star grew faint. In Fig. 8, Plate 2B, the intensity of $H\gamma$ has been arbitrarily reduced to accord with the evidence furnished by the other plates taken on the same night. Figs. 8–12 truthfully indicate the development of the bright lines from $H\delta$ to the red. In Fig. 13 and to the violet of $H\delta$ on the other plates the data are insufficient on account of underexposure. The line at λ 4007 was the only one which certainly disappeared during the interval covered by these six plates. The development of the line at λ 4571 was remarkable. This line, which is perhaps due to magnesium, did not appear until the star was of about

TABLE XVI.

Intensities of Bright Lines.

	Crown Plates						Isochromatic Plates				Crown Plates						Isochromatic	
Date	1902 June 27			July 6			July 16	July 22	July 29	Aug. 4	Aug. 11	Au. 25	Sept. 6	Sep 22	Oct. 4	Oct. 26	1901 Aug. 3	Aug. 17
Plate Number..	2B	3C	4E	7C	8D	9E	13E	18F	21F	25F	27D	28D	33D	39D	42A	48A	2211C	2235D
Mag. of Star ...		3.8			4.1		4.6	5.1	5.3	5.4	5.6	6.4	7.0	7.0	7.8	8.5	-	
Intensity of Continuous Spectrum	6	2.0	1.5	9.5	1.4	2.0	7.5	7	7	11	24	17	6.5	7	5	2.0	2.9	9
*H*κ 3751.2–	—	—	—	—	—	—	—	—	—	—	faint	—	—	—				—
*H*ι 3771.52	—	—	—	—	—	—	—	—	—	—	2	—	—	—				—
*H*θ 3798.76	—		—	3	—	—	3	—	—	—	2		—	—				—
*H*η 3836.20	9	10	—	12	-	—	13	8	4	2	6	4	4	—				6
3853.51	.	.	.	.	.	.	3	—	—	—	4	4	—	3				.
*H*ζ 3889.91	33	40	20	34	27	—	41	26	21	14	17	11	13	7	5		14	16
3906.36	4	—	—	7	—	—	6	4	—	2	5	5	6	5				4
3908.18											3	3	—					
3933.45											1	2	—	3				
3939.10											4	5	7	8				
3968.49											4	5	8	5				
*H*ε 3970.87	4	—	—	9	—	—	12	13	13	11	17	14	17	10				4
4007.74	6	—	—	9	—	—	—	4	.	.	.	.	.	.				.
*H*δ 4102.66	420	600	410	220	140	50	290	390	390	260	120	100	90	40	30	13	140	190
4202.91	10	—	—	9	—	—	13	14	19	21	12	21	32	31	30	40	21	17
4216.71	.	.	.	.	.	.	.	.	.	6	4	6	14	14	13			.
4234.12	.	.	.	.	.	.	8	—	7	7	4	6	11	6	12			7
4308.70	.	.	.	.	.	.	.	.	12	13	9	13	35	37	48	60		.
*H*γ 4341.33	420	520	280	130	90	34	160	240	190	170	80	70	70	30	24	13	80	60
4373.61	.	.	.	.	.	.	.	.	.	3	2	5	9	8	6			.
4376.78	.	.	.	.	.	.	.	11	8	8	6	8	15	15	12			.
4571.82	.	.	.	.	.	.	.	.	.	12	7	15	28	60	90	75		.
*H*β 4862.34	17	—	—	10	—	—	8	5	5	6	8	9	11	12	11			3

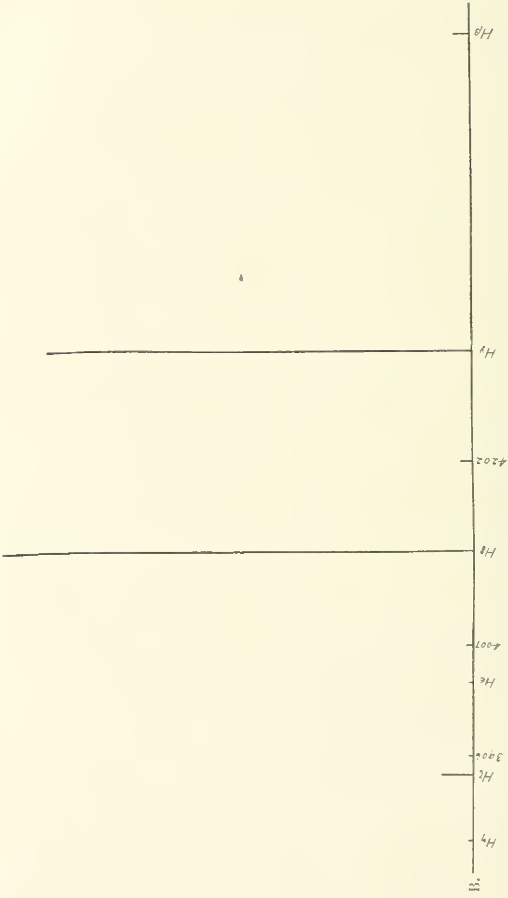


FIG. 8.

magnitude 5.4, and as the star grew fainter it became the most prominent object in the spectrum.

The images on the plates following 48A are not strong enough to give results which can be entered in Table XVI. A brief summary of what was found on the plates may be of interest:

54A. Bright lines 4202, 4308, apparently present, 4571 certain. No trace of any continuous spectrum.

58A. Bright lines 4202, 4308, 4571 all certain. Faint continuous spectrum from λ 4000 to λ 4800.

59A. Bright line 4202 not found. 4308, 4571 present. Faint continuous spectrum from λ 4000 to λ 4150, also from λ 4200 to λ 4585.

On 58A and 59A the continuous spectrum was visible in the $H\gamma$ and $H\delta$ regions, but no trace of a bright line was seen in either case. The evidence furnished by these later plates therefore strengthens the conclusion that the bright hydrogen lines disappeared at minimum. $H\gamma$ and $H\delta$ would certainly have been recorded if they had remained as intense as they were a month preceding minimum. Figs. 8 13 show that $H\gamma$ and $H\delta$ were growing fainter as the star declined, and the lines λ 4202, λ 4308, and λ 4571 were increasing in intensity as far as Plate 48A. I am not prepared to say whether the intensities of these last three lines increased or decreased after that time, but they did not change much. It is certain that no new bright lines appeared which were as prominent as those already under observation, for the plates were carefully examined. In fact the lines on Plate 54A were entirely overlooked on the first examination.

While the method of estimating intensities here presented is subject to many errors, it affords a better idea of the changes than be could given by a mere description. All the estimates with the bright-line and continuous-spectrum scales were made in such a way as to avoid personal bias. The observer did not know, when using the scales, how the final intensities would come out, and the relative changes observed in the continuous spectrum and in the bright lines are real.

Until this star has been successfully followed through all its phases, it is obviously too soon to advance any theory to account for the observed changes in its spectrum or in its brightness.

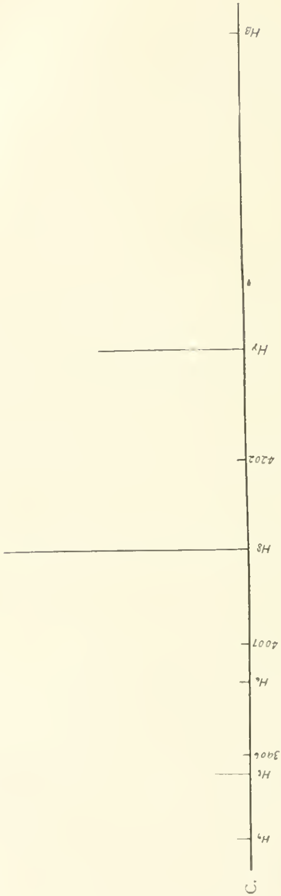

Hθ
Hγ
4202
Hδ
4007
Hε
3906
Hζ
Hη
7C.

Fig. 9.

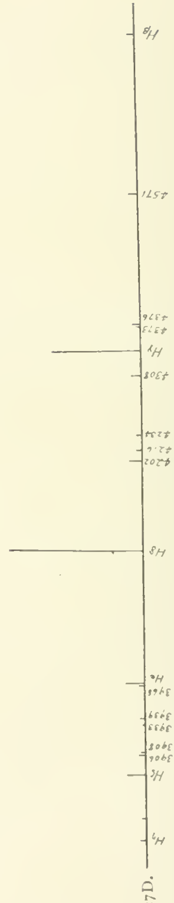

Hβ
4571
4376
4373
Hγ
4308
4234
4216
4202
Hδ
Hε
3968
3939
3933
3908
3906
Hζ
Hη
27D.

Fig. 10.

The light gathering power of the great refractor is not sufficient to follow the spectrum satisfactorily, at least not on the dispersive scale used by me. An ideal equipment for work on variables would consist of a large reflector which could be used for this purpose alone, with spectrographs of various dispersive powers arranged for use in all parts of the spectrum.

The apparent constancy of radial velocity is strong evidence that the variations in brightness are not due to the influence of a companion star, unless, indeed, the companion were of very small relative mass, were moving in a very eccentric orbit and approached very close to the primary. The displaced system of bright lines could not belong to such a companion, as their wave-lengths are apparently constant. The large irregularities in the period of the light curve practically preclude the question of a binary system, though perhaps not absolutely so.

The remarkable distribution of light in the hydrogen series seems as yet impossible to explain. Miss Clerke has explained the apparent absence of $H\epsilon$ by assuming the hydrogen to be at a lower level in the star's envelope than the calcium layer, the H calcium absorption destroying the $H\epsilon$ radiations. This theory may still be true, though the existence of a faint bright $H\epsilon$ has been proven. However, $H\beta$ and $H\alpha$ are much more reduced in intensity than is $H\epsilon$; and with them there is no evidence of over lying absorption strata. It is usually expected that the bright hydrogen series will diminish in intensity from red towards the violet. This appears to hold even for new stars, in which the disturbance has been very sudden, though exception should perhaps be made for $H\alpha$ when new star spectra approach the nebular form. The supposition that in *Mira* the hydrogen series may regularly decrease in brightness from violet to red is not tenable, as the diminution from $H\gamma$ to $H\beta$ is entirely too great. Inasmuch as the bright lines are strongly shifted (to the violet with reference to the dark-line system) from their normal positions apparently by causes other than radial velocity and pressure, it seems probable that the peculiarities of the hydrogen-line intensities are due also to causes now unknown to us. The presence of the bright iron lines λ 4308 and λ 4376, the absence of others

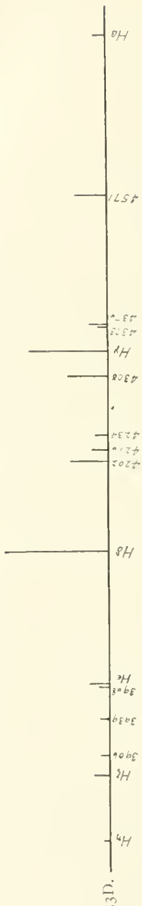


FIG. 11.

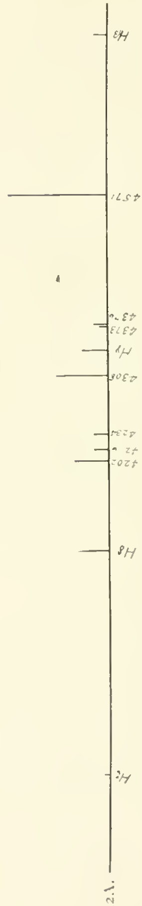


FIG. 12.

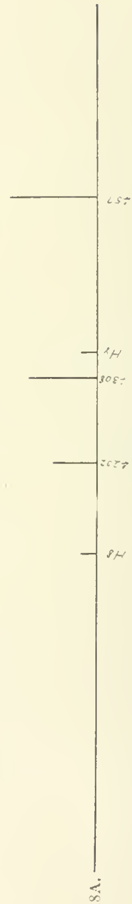


FIG. 13.

of the same element, and the diversity of structure observed by Campbell in the triple $H\gamma$ and $H\delta$ bands are perhaps due to similar unknown causes.

The great variations of relative intensity observed in the hydrogen and other bright lines, and in the continuous spectrum, show that the star's decrease in light is produced by other causes than general absorption.

Considering all the evidence, it seems reasonably certain that the star's variation in brightness is due to the action of internal forces.

I beg to acknowledge my indebtedness to Director Campbell, who provided the necessary apparatus and made valuable suggestions during the course of the work; to Messrs. Wright and Reese, for continual advice and assistance, and to Dr. H. D. Curtis, for enlarging the negatives for reproduction.

OBSERVATIONS OF THE BRIGHTNESS OF *o CETI* IN 1902–1903.

The brightness of *o Ceti* was observed by Argelander's method about once a week, beginning with June 23, 1902. At first the star's magnitude was determined with the naked eye or opera glass. After it had become too faint to be seen with an opera glass, the three-inch finder of the twelve-inch telescope was used.

In order to avoid prejudice, the observations in "grades" were made, recorded, and then laid aside, to be reduced a month or two later. When observing, I did not know the magnitude of any of the comparison stars, and in but few cases did I remember what the estimate on any previous night had been.

The magnitudes of all stars brighter than 7.00 were taken from the *Harvard Photometry*, *Harvard Annals*, **45**. For the fainter stars the adopted magnitudes are those of H. M. Parkhurst, given in **29**, 144 of the same publications. No stars that are not in one of these two sources were used for the comparisons.

The value of one "grade" has been determined from the observations to be 0.11 magnitude. However, most of the comparisons were made and reduced in such a manner that the resulting magnitude is independent of the value of a grade. For the sake of uniformity, all estimates were made in grades, and it

was tried to keep the value of one grade as nearly constant as possible. In reducing observations where *o Ceti* was compared with two stars, one brighter and one fainter, the magnitude of the variable has been taken proportionally between them. In taking the mean of several estimates, an estimate derived from two stars has been given twice the weight of an estimate based

TABLE XVII.

Reference Letters and Magnitudes Adopted for the Comparison Stars.

Letter	Mag.	*B. D.*	Name	R. A. 1900	Decl. 1900
a	3.64	−10° 240	η *Ceti*	$1^h 03^m.6$	−10 43′
b	3.64	+ 2° 422	γ *Ceti*	2 38.1	+ 2 49
c	3.71	−16 295	τ *Ceti*	1 39.4	−16 28
d	3.86	− 8° 244	θ *Ceti*	1 19.0	− 8 42
e	3.90	− 0 406	δ *Ceti*	2 34.4	− 0 06
f	3.91	+ 2° 317	α *Piscium*	1 56.9	+ 2 17
g	3.96	−11 359	ζ *Ceti*	1 46.5	−10 50
h	4.30	+ 7 388	ξ^2 *Ceti*	2 22.8	+ 8 01
i	4.33	+ 9 359	μ *Ceti*	2 39.5	+ 9 42
j	4.46	+ 8° 273	*o Piscium*	1 40.1	+ 8 39
k	4.62	+ 4 293	ν *Piscium*	1 36.2	+ 4 59
l	4.73	+ 8° 455	λ *Ceti*	2 54.4	+ 8 31
m	4.88	+ 2° 290	ξ *Piscium*	1 48.4	+ 2 42
n	5.05	+ 4 418	ν *Ceti*	2 30.6	+ 5 09
p	5.22	− 4 260		1 37.7	− 4 11
q	5.54	− 1° 353	75 *Ceti*	2 27.1	− 1 28
r	5.63	− 3° 336	66 *Ceti*	2 07.7	− 2 52
s	5.64	− 1° 322	70 *Ceti*	2 17.1	− 1 20
t	5.66	− 7° 393	67 *Ceti*	2 12.0	− 6 53
u	5.73	− 1° 377	84 *Ceti*	2 36.1	− 1 08
w	5.76	+ 1 410		2 12.8	+ 1 17
x	6.00	− 2 375	63 *Ceti*	2 06.5	− 2 18
y	6.30	− 3 374	71 *Ceti*	2 20.0	− 3 14
z	6.47	− 5° 438		2 14.6	− 4 48
A	6.71	+ 0° 370		2 10.1	+ 0 15
B	6.8−[1]	− 4 394		2 20.0	− 4 20
C	7.27	− 3 372			
D	7.33	− 3° 340			
E	7.86	− 2 389			
F	8.12	− 4° 379			
G	8.44	− 3° 375			
H	8.65	− 4 390			
I	8.80	− 3 371			
J	8.95	− 3° 363			
K	9.35	− 3 347			
L	9.44	− 3 355			
M	9.48	− 3 373			
N	9.99	− 3 362			

[1] Harvard magnitude 7.06, has 8.5 magnitude companion. Combination of the two assumed 6.8 magnitude.

upon a single comparison star. As a hundredth of a magnitude is of little significance in observations of this kind, the final means have been rounded off to the nearest tenth of a magnitude, but the light-curve, shown in Fig. 1, was drawn with the means taken to hundredths. Evidence of the Purkinje phenomenon shows in the change from opera glass to the three-inch finder.

TABLE XVIII.

Observations of the Brightness of *o Ceti.*

Astronomical Date	G. M. T.	Comparisons	Mag.	Mean	Astronomical Date	G. M. T.	Comparisons	Mag.	Mean
1902					1902				
June 23	23.5	v — b	3.64		Aug. 25	$22^h.2$	x 2 v 2 B	6.42	6.4
		v 2 g	3.74		Sept. 3	20.8	B 2 v 2 D	7.06	
		a 2 v	3.86				v A	6.71	6.9
		v 2 f	3.69		6	21.6	B 2 v 2 D	7.06	
		v = d	3.86	3.8			y 6 v 4 C	6.88	7.0
27	22.8	b 2 v	3.86		9	22.5	v 2 C	7.05	7.0
		f 1 v	4.02		17	19.5	v = B	6.80	
		v 2 g	3.74				y 8 v 3 C	6.98	
		v 2 c	3.49				v 3 D	7.00	6.9
		v 4 e	3.46		27	19.8	v = C	7.27	
		d 2 v	4.08	3.8			v = D	7.33	7.3
July 2	22.9	b 2 v 2 e	3.77		Oct. 5	18.5	v = C	7.27	
		v = f	3.91				E 4 v 2 F	8.04	7.8
		g 2 v 2 i	4.14		8	19.0	E 5 v 2 F	8.05	
		d 2 v 3 m	4.27	4.0			C 2 v 3 G	7.74	7.9
6	22.8	v = e	3.90		15	19.3	v 2 G	8.22	
		f 1 v 3 m	4.15				E 5 v 2 F	8.05	8.1
		d 2 v 2 h	4.08		27	19.0	F 3 v 2 J	8.01	
		g 2 v 3 i	4.11	4.1			v 1 H	8.54	
14	22.2	e 3 v 2 m	4.49				v 2 I	8.58	8.6
		i 1 v 1 l	4.53		Nov. 11	19.8	v = J	8.95	
		v = k	4.62				v = I	8.80	
		v = j	4.46	4.5			H 2 v 4 M	8.93	8.9
18	23.1	v = k	4.62		17	17.8	I 2 v 3 M	9.07	
		v = m	4.88				J 3 v 6 L	9.11	9.1
		j 2 v 3 n	4.70	4.7	24	19.8	J 5 v 5 N	9.47	
22	23.1	v = n	5.05	5.0			v 4 L	9.00	
29	22.6	v = p	5.22				v 2 K	9.13	
		n 2 v 2 t	5.36				I 2 v 2 M	9.14	9.2
29	23.2	m 5 v 5 s	5. 6	5.3	Dec 12	19.5	J 2 v 5 L	9.09	
Aug. 4	23.1	v = t	5.66				v = I	8.80	9.0
		n 2 v 2 s	5.34		24	17.8	J 2 v 2 K	9.15	
		p 3 v 2 q	5.41	5.4			v 5 L	8.89	9.1
11	22.7	v = s	5.64		1903				
		v = n	5.73		Jan. 3	16.8	J 3 v 3 K	9.15	
		q 2 v 2 r	5.58	5.6			v = I	8.80	9.0
18	23.8	r 2 v 2 x	5.81		29	16.2	v = J	8.95	
		v = w	5.76				G 2 v 2 H	8.54	8.7
		s 3 v 3 y	5.97	5.9	Feb. 28	16.0	V 2 C	7.05	7.0
25	22.2	v = y	6.30		March 17	15.5	m 5 v 5 q	5.21	5.2
		v = z	6.47						

After January 10, 1903, the observer was in residence at Berkeley, and a few observations of brightness were made there with various instruments.

REMARKS.

June 27: Fifth comparison assigned ½ weight on account of interval of 4 grades.

August 18: First of opera glass series.

September 3: Could see v with naked eye.

September 17: First with three-inch finder.

October 5: Identifications certain.

October 8 and 15: Remembered comparison with E and F on October 5.

January 29: Berkeley. One-inch telescope.

February 28: Berkeley. Two-inch telescope.

March 17: Berkeley. Opera glass.

MAY 1, 1903.

ON THE SPECTRUM OF THE AURORA.

By C. Runge.

In his report on the aurora Paulsen[1] compares its spectrum with the spectrum of the bluish light near the cathode of a vacuum tube filled with oxygen and a little nitrogen and monoxide of carbon. Paulsen comes to the conclusion that there is a close connection between the two spectra:

Les tableaux ci-dessus semblent donc révéler un accord intime entre le

Krypton	Intensity	Aurora	Intensity	Oxygen Tube near Cathode	Intensity
				603.5 589.0	1
				598.0	2
587.1	8			589.0–575.0	3
				575.0–553.7	1
				569.3	1
557.0	8				
556.2	4	558.0–554.4	10	561.8–556.8	12
				543.3	1
				534.3	5
				528.7–513.5	1
				526.5	3
				523.0	5
				520.0–518.3	8
				508.0 506.5	1
				500.0	3
				496.0	2
				492.0	1
				485.5 480.0	5
				476.2	1
467.1	2	470	10	470.2	10
462.4	1	463	10	464.8	10
				458.8	1
		455	10	456.8	1
				450.5	2
450.2	4				
446.4	5	449	10	448.8	2
445.4	4				
440.0	1	441.5–439.0	1	441.6	10
				437.5	1
437.6	3			436.5	2
436.2	2			435.2 433.6	10
432.0	4	436.0–430.5	1		
431.9	2			431.7	2
427.4	4	428.5 425.0	10	428.5 426.0	10

[1] *Rapports présentés au congrès international de physique*, **3**, 438. Paris, 1900.

spectre de l'aurore boréale et celui de la lumière qui entoure la cathode d'un tube contenant de l'oxygene et de l'azote.

The conclusion seems to me misleading. For if we compare the spectrum of the aurora with the spectrum of krypton, the coincidences are at least as striking as in the case considered by Paulsen. In the preceding table the three spectra are written side by side as far as krypton lines have been observed. The first column gives the spectrum of krypton in a vacuum tube without Leyden jar and spark-gap, according to my observations. The second column contains the spectrum of the aurora, and the third the spectrum near the cathode of an oxygen tube, both as given by Paulsen. The wave-lengths are given in $\mu\mu$.

I do not maintain that these coincidences prove the spectrum of the aurora to be that of krypton. I wish, on the contrary, to draw the inference that these comparisons of spectra have very little value as long as the wave-lengths of the auroral lines are not measured more accurately. The only auroral line which has been measured with a considerable amount of accuracy is the green line, and here the coincidence seems to be in favor of krypton, as I pointed out some years ago.[1]

In Scheiner's *Astronomical Spectroscopy*, translated, revised, and enlarged by E. B. Frost, the following determinations of the wave-length of the green line are considered the most accurate:[2]

	λ		λ
1867, Angström	5568	1874, Huggins	5572
1872, Vogel,	5572	1880, Copeland	5573
1872, Wijkander	5573	1882, Gyllenskiöld	5569
1873, Lemström	5570	1894, Campbell	5571.6

My determination of the green krypton line is:

$$\lambda = 5570.417, \text{ mean error } 0.015.$$

The broad band at λ 561.8–556.8 that Paulsen has measured in the spectrum of the oxygen tube is probably the green band of carbon monoxide.

KIRCHRODE BEI HANNOVER,
October 1903.

[1] *Nature*, 59, 29, 1898, p. 326.

[2] I have reduced the wave lengths to Rowland's scale.

TEN STARS WHOSE RADIAL VELOCITIES VARY.

By Edwin B. Frost and Walter S. Adams.

The systematic observations of stars having spectra of the *Orion* type, which have been a part of our program during the past two years, continue to yield, as an interesting by-product, a large proportion of spectroscopic binaries. The present list brings the number so far found with the Bruce spectrograph up to twenty-three (aside from four having spectra of other types). We have at present obtained the minimum number of three good plates for only sixty-three of these *Orion* type stars, so that the ratio of those whose radial velocities are variable is at the least greater than 1 : 3 for those so far observed by us. The fact must be considered that the lines in the spectra of many of these stars are so broad and ill-defined that only rough determinations of radial velocity are possible, whence variations of small amplitude must escape detection with the present appliances. Further, the interval of time covered by our observations, particularly of the fainter stars included in our present list, is too short to permit the recognition of variations having periods longer than a few days or weeks. Finally, three observations are by no means sufficient to establish the constancy of the radial velocity of any star, even during a short interval of time. Accordingly the striking inference must be drawn that one out of every two or three stars with spectra of the *Orion* type constitutes a dual (or perhaps multiple) system.

Most of the spectrograms referred to below were obtained with the dispersion of one prism and with the triple camera lens of 607 mm focus. They are designated as series IB, followed by the current number. Since our last communication, in the June number of this Journal, the outer temperature case has been altered by the insertion of two doors, so that it can be used with one prism or two or three prisms, as may be desired, and the temperature can be maintained practically as steadily as when

all doors are closed and the three prisms are used according to the original design of the spectrograph.

Reference has already been made[1] to the advantages derived from the use of low dispersion for stars with this type of spectrum. It would probably not be too much to say that fully ha' of the stars in the list below can be studied to better advantage with one prism than with three. A notable illustration of this is *ι Orionis*, in the spectrum of which, upon negatives taken with high dispersion, the lines are barely recognizable as very faint brightenings in the continuous spectrum, and are practically immeasurable. Accordingly, while it may appear that in the list of measures given below rather large differences are to be found between the values obtained by the different observers from the same plate, these differences are to be considered as due rather to the inherent character of the spectra than to the fact that insufficient scale has been employed to enable accurate measurement.

π Andromedae ($\alpha = 0^h\ 32^m$; $\delta = +33°\ 10'$; Mag. = 4.4).

Plate	Date	G. M. T.	Taken by	Velocity F	Velocity A.	No. of Lines F.	No. of Lines A.	Velocity Mean
				km	km			km
IB 92	1903, Sept. 25	$17^h\ 31^m$	F.	− 6	+ 3	2	5	− 2
106	Oct. 10	14 7	A.	+31	+33	4	3	+32
129	Oct. 17	19 30	F.	+58	+62	5	5	+60

The lines in the spectrum of this star are rather sharper than in the case of most stars of this class. The first plate is poor, but the other two are rated in our notes as good.

ξ Cassiopeiae ($\alpha = 0^h\ 37^m$; $\delta = +49°\ 58'$; Mag. = 4.8).

Plate	Date	G. M. T.	Taken by	Velocity F.	Velocity A.	No. of Lines F.	No. of Lines A.	Velocity Mean
				km	km			km
IB 107	1903, Oct. 10	$14^h\ 50^m$	A.	−20	− 8	3	3	−14
153	Oct. 24	17 34	A.	−33	−37	5	3	−35
175	Nov. 7	14 35	F.	0	− 9	3	5	− 5

[1] "Some Miscellaneous Radial Velocity Determinations with the Bruce Spectrograph," Astrophysical Journal, 18, 67, 1903.

The lines in the spectrum of this star, although not excessively broad, are very ill-defined and diffuse, and the measures are probably subject to more uncertainty than those for any star in the list.

o Orionis ($\alpha = 5^h\ 17^m$; $\delta = -0°\ 29'$; Mag. $= 4.6$).

Plate	Date	G. M. T.	Taken by	Velocity		No. of Lines		Velocity Mean
				F.	A.	F.	A.	
				km	km			km
C 18	1903, Feb. 19	$15^h\ 59^m$	A.	....	+31	..	4	+31
C 26	Feb. 25	13 33	F.	+29	....	4	..	+29
A 413	Mar. 6	14 25	A.	....	+31	..	6	+31
IB 75	Sept. 5	21 51	A.	+18	+21	4	5	+19
87	Sept. 18	21 46	A.	+29	+34	6	6	+32
103	Sept. 26	21 55	A.	....	+27	..	6	+27
114	Oct. 10	20 34	A.	....	+32	..	3	+32
133	Oct. 17	22 47	F.	....	+27	..	7	+27
156	Oct. 24	20 05	A.	....	+33	..	5	+33

The values given by C 18 and C 26 of the above list are entitled to low weight in view of the quality of the spectra. The chief evidence of variation in the star's velocity is furnished by IB 75: the excellent character of this plate and the satisfactory agreement of the two sets of measures upon it lead us to the conclusion that the variation is real. The spectrum is very well adapted, for a star of this type, to accurate measurement, most of its lines, in particular those due to helium, being strong, narrow, and well defined.

χ *Aurigae* ($\alpha = 5^h\ 26^m$; $\delta = +32°\ 8'$; Mag. $= 5.0$).

Plate	Date	G. M. T.	Taken by	Velocity		No. of Lines		Velocity Mean
				F.	A.	F.	A.	
				km	km			km
IB 73	1903, Sept. 5	$19^h\ 48^m$	A.	+31	+25	5	5	+28
112	Oct. 10	18 46	A.	+15	+ 9	5	6	+12
120	Oct. 16	20 16	A.	+11	+15	5	5	+13

The spectrum of this star is similar to that of *o Orionis*, although its lines are rather less sharply defined. λ 4267 is exceptionally strong.

ι *Orionis* ($\alpha = 5^h\ 30^m$; $\delta = -5°\ 59'$; Mag. = 3.0).

Plate	Date	G. M. T.	Taken by	Velocity F.	Velocity A.	No. of Lines F.	No. of Lines A.	Velocity Mean
				km	km			km
IB 76	1903, Sept. 5	$22^h 29^m$	A.	+18	+23	2	4	+21
97	Sept. 25	21 58	F.	+44	+35	4	4	+40
104	Sept. 26	22 33	A.	+59	+55	2	3	+57
134	Oct. 17	23 19	F.	....	+35	..	3	+35
147	Oct. 23	23 37	F.	+42	+41	3	3	+42
157	Oct. 24	20 38	A.	....	....	..	..	....
167	Oct. 30	20 00	A.	+89	+91	2	2	+90

The spectrum of this star appears to be decidedly complex. In the case of the majority of the plates the helium lines and $H\gamma$ consist of exceedingly broad and diffuse lines upon which are superposed maxima sometimes to the number of two or three, and usually of considerable intensity. The effect of these is greatly to complicate the determinations of velocity. Sufficient evidence has not yet been obtained to determine whether these maxima are due to the other member or members of the system producing the variation in velocity, or to physical conditions in the star. The determinations of velocity given in the table are, in all cases, derived from the broad diffuse lines, although it has also been our practice to measure the position of the various maxima as well. In the case of IB 157, however, the complication is so great that it has seemed best to us to omit the value obtained from it until further study of the star's spectrum has enabled us to form more definite conclusions as to the relationships involved in the various lines.

ν *Orionis* ($\alpha = 6^h\ 2^m$; $\delta = +14°\ 47'$; Mag. = 4.4).

Plate	Date	G. M. T.	Taken by	Velocity F.	Velocity A.	No. of Lines F.	No. of Lines A.	Velocity Mean
				km	km			
A 393	1903, Jan. 22	$20^h 23^m$	F.	+82	+80	2	4	+81
IB 173	Oct. 31	23 32	F.	+20	+23	6	5	+21
187	Nov. 14	21 31	A.	+11	+13	5	5	+12

The first plate, taken with three prisms and camera A, had such broad and hazy lines that it was laid aside without measure-

ment. The second plate, however, indicated so different a radial velocity that an attempt was made to measure the first, with the above result, which is only very rough. On the second plate measures were made on the carbon line at λ4267 and *Mg* λ4481, in addition to *Hγ* and three helium lines.

18 *Aquilae* ($\alpha = 19^h\ 2^m$; $\delta = +10°\ 55'$; Mag. = 5.1).

Plate	Date	G. M. T.	Taken by	Velocity		No. of Lines		Velocity Mean
				F.	A.	F.	A.	
				km	km			km
IB 46	1903, June 13	$18^h\ 10^m$	A.	+15	+10	3	3	+12
52	July 4	18 37	F.	−17	−25	3	3	−21
68	Sept. 5	14 12	A.	..	−28	..	4	−28
77	Sept. 12	14 28	A.	− 7	− 3	4	6	− 5

We have found the lines in this star difficult to set upon, and the difference in the values obtained by the two observers is unusually large.

2 *Lacertae* ($\alpha = 22^h\ 17^m$; $\delta = +46°\ 2'$; Mag. = 4.8).

Plate	Date	G. M. T.	Taken by	Velocity		No. of Lines		Velocity Mean
				F.	A.	F.	A.	
				km	km			
IB 64	1903, Aug. 8	$17^h\ 50^m$	F.	− 82	− 89	4	4	− 86
71	Sept. 5	17 43	A.	− 19	− 13	4	5	− 16
	Second Component			−199	−171	3	3	−185
88	1903, Sept. 25	13 27	F.	+ 1	+ 1	4	4	+ 1

The spectrum on the first plate is much fainter than on the other two, but on re-examination it gave indications of the presence of lines due to a second luminous component, which were measurable on the second plate. These additional lines cannot be seen on Plate 88.

6 *Lacertae* ($\alpha = 22^h\ 26^m$; $\delta = +42°\ 37'$; Mag. = 4.6).

Plate	Date	G. M. T.	Taken by	Velocity		No. of Lines		Velocity Mean
				F.	A.	F.	A.	
				km	km			km
IB 50	1903, June 26	$19^h\ 45^m$	F.	−23	−24	4	3	−24
65	Aug. 8	18 47	F.	−14	−14	4	4	−14
83	Sept. 18	18 6	A.	− 6	0	3	4	− 3

In this spectrum the silicon lines appear in addition to those of hydrogen and helium, but settings could be made on only one of these. All the plates are rated as good.

1 Hev. *Cassiopeiae* ($\alpha = 23^h\ 25^m$; $\delta = +58°\ 0'$; Mag. = 4.8).

Plate	Date	G. M. T.	Taken by	Velocity		No. of Lines		Velocity Mean
				F.	A.	F.	A.	
				km	km			km
IB 151	1903, Oct. 24	$15^h\ 48^m$	A.	−58	−75	5	5	−66
168	Oct. 30	21 07	A.	..	−60	..	3	−60
169	Oct. 31	15 27	F.	−75	−65	4	4	−70
177	Nov. 7	16 52	F.	+ 3	− 6	3	3	− 2

Plate 168 was obtained through clouds and is underexposed. The lines in the spectrum are very broad and diffuse, and are perhaps complicated by maxima.

AN *ORION* STAR WITH A GREAT RADIAL VELOCITY.

Five spectrograms have yielded the following values for the velocity of the star:

ξ *Persei* ($\alpha = 3^h\ 53^m$; $\delta = +35°\ 30'$; Mag. = 4.1).

Plate	Date	G. M. T.	Taken by	Measured by	No. of Lines	Velocity
						km
IB 93	1903, Sept. 25	$18^h\ 27^m$	F.	A.	2	+88
101	Sept. 26	20 38	A.	A.	3	89
119	Oct. 16	19 20	A.	A.	2	80
141	Oct. 23	18 41	F.	A.	2	80
160	Oct. 24	23 17	A.	A.	2	88
					Mean	+85

The spectrum of this star is excessively difficult of accurate measurement, owing to the breadth, and still more to the extremely ill-defined character of its lines. Consequently we do not regard the above range of 9 km in the measures as a real variation. Observations covering a longer interval may, however, show the star to be a spectroscopic binary, and, in view of the very low radial velocities which seem in general to be characteristic of stars having the *Orion* type of spectrum, a result of this nature is, in fact, rather to be expected.

BRIGHT-LINE SPECTRA.

Recent plates which we have obtained of the following stars of the *Orion* class show them to contain bright lines:

c Persei,	$\alpha = 4^h\ 1^m$;	$\delta = +47°\ 27'$;	Mag. = 4.3.	3 plates by F.
25 *Orionis,*	5 20	+1 45	4.6.	2 plates by F. and A.
β Piscium,	22 59	+3 17	4.6.	4 plates by F.

As is characteristic of the stars with this peculiar variety of the *Orion* type spectrum, the hydrogen lines have a double bright component, which is superposed about centrally on the broader dark line or band. Bright $H\beta$ is especially conspicuous on the plates, partly from its intrinsic intensity, and partly because of the weakness of the continuous spectrum at that region on our plates. Bright $H\delta$ is only faintly visible on the broad absorption line, following the usual decline in intensity of such bright lines as the violet is approached. The helium lines at λ4388 and 4472 and *Mg* 4481 are dark, but are dim and diffuse in each star. The separation of the components of the bright hydrogen lines is least for *β Piscium*, and is very wide for 25 *Orionis*.

Eight plates of *c Persei* were used in the *Draper Catalogue*, and the spectrum was classed as A. Five each were included for 25 *Orionis* and *β Piscium*, which were respectively assigned to classes B and A. The exposure times were probably not suited to bring out the peculiarities of the spectra. These stars were not included among those studied by Miss Maury and by Miss Cannon in Vol. 28 of the *Harvard Annals*.

The determination of the true radial velocity is very difficult for such spectra, but, as far as we have yet measured the plates, we have not obtained certain evidence of a variation of the radial velocities of these three stars.

YERKES OBSERVATORY,
November 15, 1903.

FURTHER OBSERVATIONS ON THE SPECTRUM OF THE SPONTANEOUS LUMINOUS RADIATION OF RADIUM AT ORDINARY TEMPERATURES.[1]

By Sir William Huggins and Lady Huggins.

In the plate accompanying our paper on the spectrum of the glow of radium bromide,[2] at least seven lines are seen to agree, both in position and in intensity, with corresponding lines in the band spectrum of nitrogen. We called attention to other lines, of which some traces may be detected on the plate, and we suggested that with a longer exposure a more complete spectrum would be obtained. One strong line in the radium bromide glow spectrum, about λ 3914, has no similar line corresponding to it in the band spectrum of nitrogen as given on the plate.

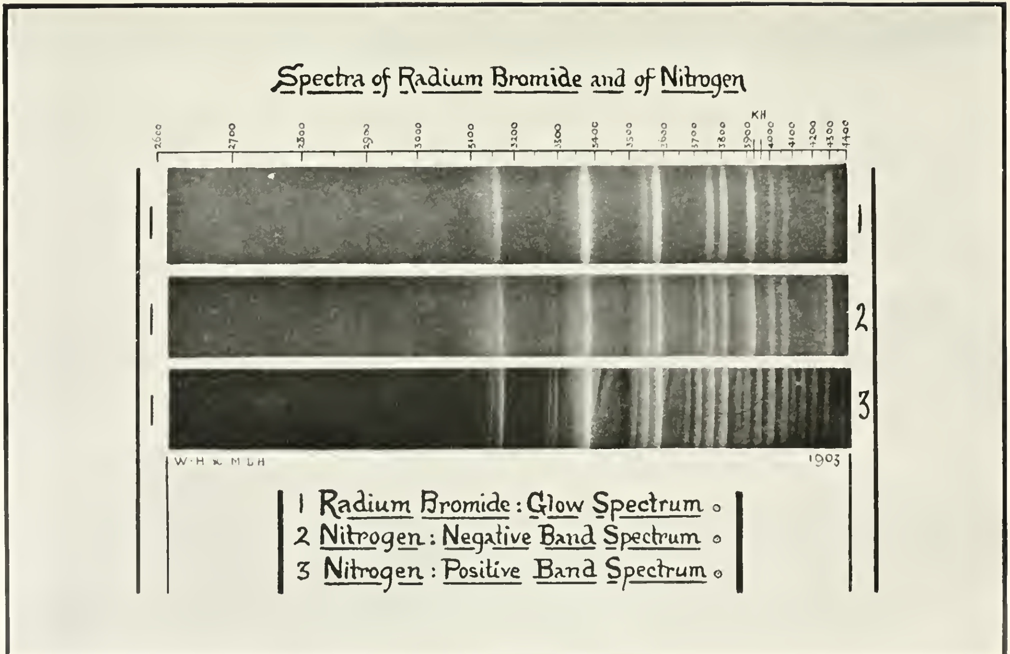
We have since taken photographs, with longer exposures, of two specimens of radium bromide, one prepared by Buchler & Co., and the other received from the Société Centrale de Produits Chimiques. In these photographs lines only faintly glimpsed in our earlier photographs can be seen distinctly. A photograph taken of the French radium bromide with an exposure of 216 hours is reproduced on the accompanying plate.

The coincidence of the spectrum with the band spectrum of nitrogen is shown to be even more complete by the presence of a faint trace of the next more refrangible band, beginning at λ 2976.7. In addition, some of the fainter single lines of the nitrogen spectrum now come out in the radium bromide spectrum.

At the same time that the coincidence down to minuter details with the nitrogen band spectrum is brought out, the strong outstanding line, about λ 3914, is now seen to be accompanied by a second, but less intense, outstanding line at about λ 4280; neither of which is present in the ordinary band spectrum of nitrogen, which was the one reproduced on the plate of our first paper.

[1] From advance proofs of a paper communicated to the Royal Society, October 29, 1903.

[2] Astrophysical Journal, 18, 151, 1903; *Proc. R. S.*, 72, 196, 1903.


Spectra of Radium Bromide and of Nitrogen
2600
2700
2800
2900
3000
3100
3200
3300
3400
3500
3600
3700
3800
3900
KH
4000
4100
4200
4300
4400
1
2
3
W·H & M·L·H
1903
1 Radium Bromide : Glow Spectrum
2 Nitrogen : Negative Band Spectrum
3 Nitrogen : Positive Band Spectrum

This nitrogen band spectrum is the one distinguished by Deslandres as that of the positive pole, but it appears at all parts of a vacuum tube, and is also produced when a suitable induction coil discharge, without capacity, is taken across air at the ordinary density. The nitrogen spectrum that was measured by Ames was taken by using an end-on vacuum tube closed with a quartz plate; in his list no lines are given at the places of the two outstanding lines in the glow spectrum.

When, however, the spectrum is taken of the aureole about the negative pole of a vacuum tube containing a residuum of atmospheric air, the ordinary, or positive-pole spectrum becomes enriched by a new spectrum of bands; and in this additional spectrum the heads of the two strongest bands in the photographic region, occur at the positions of the two outstanding lines of the radium glow spectrum.[1] On the plate are given, below the more complete radium spectrum now obtained, the ordinary band spectrum of nitrogen, and also the same spectrum enriched with the bands peculiar to the aureole of the negative pole. This latter spectrum corresponds to that of the radium glow. The peculiar conditions, whatever they may be, which determine the presence of these additional negative-pole bands must find their counterpart in the nitrogen molecules when under stimulation by the radium bromide. The additional bands which show themselves in the spectrum of nitrogen when taken from the glow at the negative pole of a vacuum tube are usually believed to be associated with the stimulation of the very rapidly moving corpuscles of the cathode stream. Accordingly the presence of these negative-pole bands in the spectrum of nitrogen when excited by radium naturally suggests whether the β rays, which are analogous to the cathode corpuscles, may not

[1] DESLANDRES'S measures, reduced to Rowland's scale, of the heads of these two bands are λ 3914.4 and 4279.6 (*Thèses*, 1888, GAUTHIER-VILLARS, and *Comptes Rendus*, **101**, 1256). ÅNGSTROM and THALÉN give 4281.6 for the less refrangible band (*Nova Acta Upsal.* (3), **9**, 1875). HASSELBERG'S measure for the head of the less refrangible band is 4378.6 (*Mem. de l'Acad. St. Petersb.*, **32**, No. 15). PERCIVAL LEWIS on "Some New Fluorescence and Afterglow Phenomena in Vacuum Tubes Containing Nitrogen" (ASTROPHYSICAL JOURNAL, **12**, 8) found fluorescent nitrogen to give a band spectrum; and, in some conditions of the fluorescence, the most intense bands were those of wave-lengths 3576.9 and 3371.2.

be mainly operative in exciting the radium glow. On this surmise it would be reasonable to expect some little extension of the glow outside the radium itself. We are unable to detect any halo of luminosity outside the limit of the solid radium bromide; the glow appears to end with sudden abruptness at the boundary surface of the radium. It may be that it is only at molecular distances, and at the moment of their formation, that the rays can excite the nitrogen molecules.

As the glow spectrum is produced by the influence of the radium on nitrogen at the atmospheric pressure, it seemed to be of interest to find out whether the negative-pole spectrum could be obtained in air at the ordinary pressure. It has already been stated that when a suitable discharge of an induction coil, without capacity in the circuit, is taken between electrodes in air, the ordinary band spectrum of nitrogen appears. Separate photographs, therefore, were taken of the parts of the discharge in the close neighborhood of the two electrodes, which were about three-eighths of an inch apart. The bands peculiar to the negative-pole of a vacuum tube were found upon the plate taken of the negative electrode.

As the radium glow consists of light from nitrogen molecules stimulated into luminosity by the presence of the more active radium molecules, it was reasonable to suppose that the bromine molecules, chemically associated with the latter, might also be sufficiently stimulated to reveal their presence by the lines in the spectrum, peculiar to them. Photographs were accordingly taken of the poles of a vacuum tube containing traces of atmospheric air together with bromine vapor. The band spectrum of nitrogen appeared alone upon the plates when no capacity was introduced; but with the intercalation of a jar, the lines of bromine came out in the photopraphs, in addition to the lines of air. The experiment was then repeated at atmospheric pressure by enclosing platinum electrodes in a glass bulb communicating with the atmosphere by a narrow tube. Photographs of the coil discharge taken between them revealed the ordinary band spectrum of nitrogen. A few drops of bromine were then introduced into the bulb, filling it with bromine vapor. Photographs were again

taken of the discharge in the air now heavily laden with bromine, but the spectrum remained precisely the same as before the bromine was introduced, namely, that of nitrogen only.

We find in this experiment possibly a sufficient reason for the absence of any of the lines of bromine in the glow spectrum: it may be that stimulation from the active radium molecules affects preferentially the nitrogen molecule, so that this molecule can be shaken into luminosity by a stimulation which is insufficient to excite the bromine molecule to a comparable extent.

The experiment then suggested itself whether, under similar conditions of discharge, radium itself, when placed upon the electrodes, would be able to show its presence by its characteristic lines in the spectrum of the discharge taken between them. The result was negative; as in the case of bromine, no lines other than those of nitrogen appearing upon the plate. A small jar was then put into the circuit and another photograph taken, when the complete spectrum of radium came out strongly, but without the band spectrum of nitrogen.

If, as suggested by Rutherford, the a rays are connected with helium, the experiment seemed worth making of taking a photograph of the spectrum arising from their bombardment upon a zinc sulphide screen. It seemed possible, though not very probable, that the encounters of these bodies, at the enormous speed at which they travel, with the molecules of air, and their final collision with the screen, might on that hypothesis give rise to some of the radiations peculiar to helium and so produce its spectrum on the plate. Fortunately the strong continuous spectrum due to the fluorescence of the screen ends abruptly in the violet a little before the place, at λ 3889, of the strongest line of helium in the photographic region, and so leaves the spectrum quite free for the detection of this line, even if it were only faintly present. The result of the experiment, so far as concerns helium, was negative; which must not of course be interpreted as excluding the presence of helium, but only as showing that, if present, the conditions are not favorable to the appearance of its spectrum.[1]

[1] M. Henri Becquerel has quite recently investigated the scintillation observed on a phosphorescent screen when excited by radium. He comes to the conclusion:

On the first photograph that was taken, the two strongest lines of the nitrogen band spectrum were faintly seen, but a photograph with a new screen and a longer exposure showed no trace of the nitrogen bands. In the first case it might be that some very minute particles of radium bromide had attached themselves to the screen, and by their independent glow had given rise to the lines of nitrogen which were on the photographic plate.

About one centigram of French radium bromide, which was in the form of small particles, was put into a very small glass tube scarcely larger than was necessary to contain it. The tube was securely closed and left for two months. As the α rays being unable to escape, would probably occupy the interstices between the radium bromide particles, it seemed desirable to examine whether as helium, or still in some precedent condition, they would show their presence in the glow spectrum. The tube was exposed, immediately in front of the slit, for 168 hours. The spectrum shows a strong continuous spectrum from the fluorescence of the glass, and faintly the bands of nitrogen, but no other lines with certainty. We intend to photograph again the spectrum of the glow from this tube, after a longer time has passed for an accumulation of the α rays, and of the gas-like emanation.

When the radium bromide is covered with a plate of quartz, the continuous spectrum, due to the fluorescence of the quartz, is not only strong, but extends a long way into the ultra-violet. It can be traced on the photograph as far as $\lambda 2500$.

After a few hours the quartz darkens under the action of the radium bromide, the brown stain extending through the complete substance of a plate one-tenth of an inch in thickness. The stain is due probably to the reduction of silicon.

Experiments were made in the hope of throwing light upon the shift found in the photograph of the radium glow spectrum,

(1) "Ce sont les rayons α qui provoquent la phosphorescence scintillante;" (2) "Ces faits établissent sinon une démonstration, du moins une grande présomption en faveur de l'hypothèse qui attribuerait la scintillation à des clivages provoqués irrégulièrement sur l'écran cristallin par l'action continue plus ou moins prolongée des rayons α." *Comptes Rendus*, **137**, 633, 634; October 27, 1903.

reproduced on the plate of our first paper. As subsequent photographs of this spectrum were entirely free from any trace of shift, the shift found on the first plate must have been accidental. Repeated photographs, taken with the spectroscope in different positions, failed to show the smallest trace of shift from flexure. The only suggestion we can make in explanation is that the piece of solid radium bromide accidentally shifted in its cell, so as no longer to be directly under the slit, and in consequence the collimator lens was not wholly filled with light.

The results of the experiments described in this paper would appear to show generally, if analogy with electric stimulation may be assumed, that the radium stimulation, whether we take the operative cause to lie in the β rays, or in the encounters of nitrogen molecules with the active molecules of radium — by which, for the first time, a spectrum of bright bands in the ultra-violet region has been obtained at ordinary temperatures, and without the intervention of an electric discharge — from the very circumstance of its being of such a nature as to give rise to the band spectrum of nitrogen, is not of a kind which can elicit from either the molecules of bromine, or of radium their characteristic line spectra.

The question suggests itself whether or not the same inability may hold in respect of the helium molecule, which is easily stimulated by an electric discharge; we have not as yet made experiments on this point.

Reviews

Lehrbuch der Physik. Von O. D. Chwolson. Bd. I, pp. 791. Translated from the Russian into the German by H. Pflaum. Braunschweig: Vieweg, 1902.

The appearance of a new compendium of physics, filling no fewer than four octavo volumes, is certain to make one curious as to the features which distinguish it from other treatments of a similar nature. And since in this particular case, alas! no similar treatment exists in the English language, we are driven to the larger German treatises, such as Wüllner and Müller-Pouillet, for a basis of comparison. For, while the *Text-book of Physics* by Poynting and Thomson may well be reckoned in this class, and while, in a very true sense, it will cover the entire field of physics, it is nevertheless largely devoted to the discussion of discrete problems, and begins by omitting the entire subject of dynamics.

As compared with Wüllner's *Lehrbuch*, which may perhaps be taken as a typical, connected discussion of the whole subject, the most striking contrasts are perhaps the following:

1. Chwolson gives a much more elaborate philosophical introduction, devoting more than fifty pages to questions such as the classification of the sciences, the distinction between theoretical, mathematical, and experimental physics, the characteristics of a good hypothesis, etc. The author has evidently a keen appreciation of the unity of his science, for this same philosophical vein runs through the entire volume. To illustrate, the spectrometer, essentially an instrument for measuring angles, and usually handled under the head of optics, here finds discussion in a chapter on "Measuring Instruments," where it logically belongs.

2. As compared with Wüllner or with Violle's *Cours de Physique*, the volume under review contains much more elaborate references to the literature of each topic. These valuable bibliographies are placed at the end of each chapter, and are there grouped under the headings of the sections, so that really each section is provided with its own bibliography. The amount of Russian work here cited will be surprisingly large to those who still imagine that the energies of this

great nation are exclusively devoted to the arts of war and to the care of Siberian exiles.

3. The mathematical discussion is the very simplest — too simple, we venture to think. Differential equations, even the most elementary, are studiously avoided. In America, certainly no student would be reading so advanced a treatment of general physics without the preparation to enjoy more elegant mathematical methods. As illustration may be cited the derivation — or rather the lack of derivation — of the equations for damped vibrations, p. 156.

4. Most American readers will probably agree that some of the chapters are too full. For example, the ninth chapter devotes twelve octavo pages to the "dimensions" of physical quantities. Is it not easily possible to put all the essentials of this subject into one quarter of this space?

Your reviewer is not alone in thinking that a great service would be rendered English-speaking students by any competent physicist who will give them a treatise on general physics, along the lines of Chwolson — a treatise which will fill the great gap lying between the college and university text-books, on one hand, and the great compendium of Winkelmann, on the other hand.

H. C.

A Popular History of Astronomy During the Nineteenth Century. Fourth edition. By AGNES M. CLERKE. London: A. & C. Black, 1902. Pp. 489.

WITH the fourth edition Miss Clerke brings her well-known history down to the summer of 1901. As with previous new editions, fresh material has been wisely chosen and skilfully woven into the text, so that neither its value as a book of reference nor its literary charm has been impaired. With great wealth of material to be sifted, it is really remarkable that so little that is questionable has been incorporated and that so few researches of significance have been overlooked. Perhaps the most important omission is that of Nichols's investigations on the heat-radiation of the stars, which were made during the summers of 1898 and 1900. One has the impression also that German astronomers do not receive their full share of attention. At any rate, the great preponderance of English over German references seems disproportionate to the relative merits of German and English investigators. Of a short list of errors, typographical and otherwise, in the third edition, called

to the reviewer's attention by Dr. Schlesinger, the greater part have been corrected in the fourth edition. Among those that remain might be mentioned that of the parallax of 61 *Cygni*, which is still given as between $0\rlap{.}''43$ and $0\rlap{.}''47$, although more recent determinations by Davis, Kapteyn, Wilsing, and others make it practically certain that the average parallax of the two components is less than $0\rlap{.}''40$. There is still an error in correcting the series contained in footnote 5 on page 71 — the first member being $1\frac{1}{2}$, instead of $\frac{1}{2}$ as given.

The illustrations are much improved in quality and are supplemented by a half-tone of the comet of May 1901, reproduced from a photograph taken at the Cape of Good Hope. To Table IV has been added a list of radial and tangential velocities of selected stars.

S. B. BARRETT.

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